Math 551 **Introduction to Scientific Computing** **Summer 2019**

Homework Set 4 Due Friday, 14 June 2019

1. Using the m-file *my_newton.m*, estimate $\sqrt{8}$ by finding the positive root of $f(x) = x^2 - 8$. Try three different initial guesses: (i) $x_0 = 2$, (ii) $x_0 = 3$, and (iii) $x_0 = 50$. In all cases take $tol = 1e{-}14$, and include the output from each. Discuss the results in light of Problem #3 from HW #3.

2. Consider the function $f(x) = tan(x) - x$.

   a) Use Newton's method to find the root near $x = 101$. You will observe that this root is difficult to find. Starting with $x_0 = 101$ is not a good initial guess. Use a graphical method to determine roughly where α is, then choose an initial condition x_0 sufficiently close to α in order to achieve convergence. To explain the difficulty compute the quantity $C \approx \frac{1}{2}\frac{|f''(\alpha)|}{|f'(\alpha)|}$. Recall the discussion in class. Discuss your findings.

   b) Reformulate the problem of finding a root of $f(x)$ by finding a function $h(x)$ whose roots are identical to those of $f(x)$ (hint: use the fact that $\tan x = \frac{\sin x}{\cos x}$). Apply Newton's method to the $h(x)$ that you found, with the initial guess $x_0 = 101$. Comment on the convergence in this case as compared to the findings in part a).

3. In HW #1 we found the second order Taylor polynomial $P_2(x) = 1 + x$ for $f(x) = e^x \cos x$ expanded about $x_0 = 0$. An upper bound on the error incurred when using P_2 to approximate f on $[0, 1]$ was given by

$$\max_{x\in[0,1]} \frac{e^x(\sin x + \cos x)}{3} = \frac{e(\sin 1 + \cos 1)}{3} \approx 1.2520,$$

   derived using the remainder term. Find the true maximum error to at least 12 decimal digits of accuracy. How do the bound and this true error compare? Plot the absolute value of the error between P_2 and f over $[0, 1]$. Be sure to label the axes, and put a *title* on the plot. Include a copy of your graph.

4. (Order of Convergence of the Secant method) In class it was stated that the order of convergence for the Secant method is $(1 + \sqrt{5})/2$. Confirm this by applying the Secant method to $f(x) = 10 - e^x$, $x_0 = 1.5$, $x_1 = 1.8$ and tol $= 1e{-}14$. To do so you'll need to write an M-file *my_secant.m*, whose first line should be

```
function [root,vals,iter,ierr] = my_secant(f,x0,x1,tol,itermax)
```

   with the stopping criteria $|f(x_n)| < tol$, the same that we used for Newton's method.

   Use the iterates produced to estimate the order of convergence p. You will need a good approximation to the exact root, for which you should use 2.302585092994045, the most accurate that MATLAB can produce. Include a copy of your m-file.

5. Let A be the 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Use Gaussian elimination to obtain A^{-1} by solving the two systems $Ax_1 = e_1$ and $Ax_2 = e_2$, where e_1 and e_2 are the columns of the 2×2 identity matrix. Note that you can perform both at the same time by considering the augmented system $[A|I]$. Prove that A^{-1} exists if and only if $det(A) \neq 0$.

6. Write a MATLAB m-file to find the LU decomposition of a given $n \times n$ matrix A **with** partial pivoting. The routine should return the updated matrix A and the pivot vector p. Name the m-file function **my_lu.m**, the first few lines of which should be as follows:

```
function [a,p] = my_lu(a)
%
n = size(a,1);
p = (1:n)';
(your code here!)
```

The code above sets n equal to the dimension of the matrix and initializes the pivot vector p. As a test of your code, in MATLAB execute the statements **exactly** as they appear:

```
>>diary my_lu.txt
>>format short e
>>a = [2 2 -3;3 1 -2;6 8 1];
>>[a,p] = my_lu(a)
>>diary off
```

Submit your output and be sure to include a copy of your code.