Math 551 **Introduction to Scientific Computing** **Summer I 2019**

Homework Set 2 Due Thursday, 30 May 2019

1. Find $P_2(x)$ for $f(x) = e^x \cos x$ expanded about $x_0 = 0$. Then find a bound on the error $|f(x) - P_2(x)|$ in using P_2 to approximate f on $[0, 1]$.

2. The floating point representation of a number is $x = \pm(0.a_1a_2 \ldots a_n)_\beta \times \beta^e$, where $a_1 \neq 0$, $-M \leq e \leq M$. Suppose $\beta = 2$, $n = 8$, and $M = 4$.

   (a) Find the smallest positive (x_{min}) and largest positive (x_{max}) floating point numbers that can be represented. Give the answers in decimal form (base 10).

   (b) Find the floating point number in this system that is closest to π.

3. Find the two roots of $x^2 - 50x + 1 = 0$, performing all calculations in 5 decimal digit arithmetic (i.e. $\beta = 10$, $t = 5$). Thus, round the answer of each arithmetic operation to 5 significant digits.

4. Recall that the *machine epsilon* of a computer is the smallest positive floating point number *eps* such that $fl(1+eps) > 1$. We can determine *eps* on a given machine, for a given floating point precision, by evaluating the expression

$$(1 + x) - 1 \qquad (*)$$

   for decreasing values of x. The smallest representable positive x for which $(*)$ is nonzero is *eps*. On a binary machine it is enough to consider the sequence $x_n = 2^{-n}$ for $n = 1, 2, \ldots$ (Why?).

   Write a MATLAB code to determine *eps* on the machine you are using, and compare it with the value of *eps* in MATLAB (type 'eps' in MATLAB to see this value). What is the relationship between the two. (Note: you may find it useful to first issue the MATLAB command '*format long e*' so that you are sure of when an expression computes identically to 0). Include a copy of your code.

5. Consider evaluating the integrals

$$y_n = \int_0^1 \frac{x^n}{x + 10}\, dx$$

   for $n = 1, 2, \ldots, 30$.

   (a) Show analytically that $y_n + 10y_{n-1} = 1/n$.

   (b) Show that $y_0 = \log 11 - \log 10$ and then use it with the recursion

$$y_n = \frac{1}{n} - 10y_{n-1}$$

   to numerically generate y_1 through y_{30}

   (c) Show for $n \geq 0$ that $0 \leq y_n \leq 1$, and discuss the results in (b) in light of this.