Math 551 **Introduction to Scientific Computing** **Summer I 2019**

Homework Set 1 Due Friday, 24 May 2019

1. Consider the polynomial $f(x) = x^2 - x - 2$.

   (a) Find $P_1(x)$, $P_2(x)$ and $P_3(x)$ for $f(x)$ about $x_0 = 0$. What is the relation between $P_3(x)$ and $f(x)$? Why?

   (b) Find $P_1(x)$, $P_2(x)$ and $P_3(x)$ for $f(x)$ about $x_0 = 2$. What is the relation between $P_3(x)$ and $f(x)$? Why?

   (c) In general, given a polynomial $f(x)$ with degree $\leq m$, what can you say about $f(x) - P_n(x)$ for $n \geq m$?

2. Find both $P_2(x)$ and $P_3(x)$ for $f(x) = \cos x$ about $x_0 = 0$, and use them to approximate $\cos(0.1)$. Show that in each case the remainder term provides an upper bound for the true error.

3. Consider $f(x) = e^x$, and find a general formula for the Taylor polynomial $P_n(x)$ for f about $x_0 = 0$.

   (a) Using the remainder term, find a minimum value of n necessary for $P_n(x)$ to approximate $f(x)$ to within 10^{-6} on $[0, 0.5]$.

   (b) Prove that $f(x)$ analytic on $(-\infty, \infty) = \mathbb{R}$.

4. Given a function $f(x)$, use Taylor approximations to derive a second order *one-sided* approximation to $f'(x_0)$ is given by

$$f'(x_0) = af(x_0) + bf(x_0 + h) + cf(x_0 + 2h) + O(h^2)\,.$$

   What is the precise form of the error term? Using the formula approximate $f'(1)$ where $f(x) = e^x$ for $h = 1/(2^p)$ for $p = 1 : 15$. Form a table with columns giving h, the approximation, absolute error and absolute error divided by h^2. For each indicate to which values they are converging. Finally, verify that the last column appears to be converging to a value derived using the error term.

5. MATLAB: Download and modify the m-file *fp_example.m* with

```
N= (1:20)';   h=2.^(-N);
```

   Also, add a *title* to the graph containing **your** full name. Run the script, printout a hardcopy of the graph and hand it in.