Math 128 **Exam #1** **Fall 2017** **SPECIAL CODE: 101701**

Name Key with work

IBP
Integration by Parts

Signature: ________________________________

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Academic Honesty Statement: By signing my name above, I acknowledge that I understand each of the following behaviors:

> *using a calculator or cell phone (or any other communication technology); referring to a piece of paper or object with helpful information on it (cheat sheet, notes, etc...); looking at a test or answer sheet that is not my own; allowing another student to look at my test or answer sheet; communicating with other students (verbally or nonverbally); taking the test for another student; taking my bubble sheet of answers with me when I've finished; talking while waiting to hand in my test materials to the proctors; sharing a calculator*

to be a form of academic dishonesty (cheating). I am also pledging not to engage in any of these behaviors. I understand that if I do engage in these behaviors, the consequences will be failure of the exam and a formal charge of academic dishonesty to the Ombuds Office.

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Please shut off all cell phones, ear phones, computers, beepers, smart watches, etc…

Please put everything away except a #2 pencil and a calculator that is NOT your cell phone. You may write on the test. There are twenty five multiple choice questions and each question is worth four points.

1. On the bubble sheet, where it says "Name," please print your last name, leave a space, and then print your first name in the rectangles. Then fill in the bubbles underneath.

2. On the bubble sheet, where it says "Identification Number," please write your entire Student ID number in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in your ID # correctly.

4. On the bubble sheet, where it says "Special Codes," please write the numbers: 101701 in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in the special code correctly.

Please make sure you bubble in your answers carefully on the bubble sheet and ***circle your answers on your test booklet****.*

Formulas:

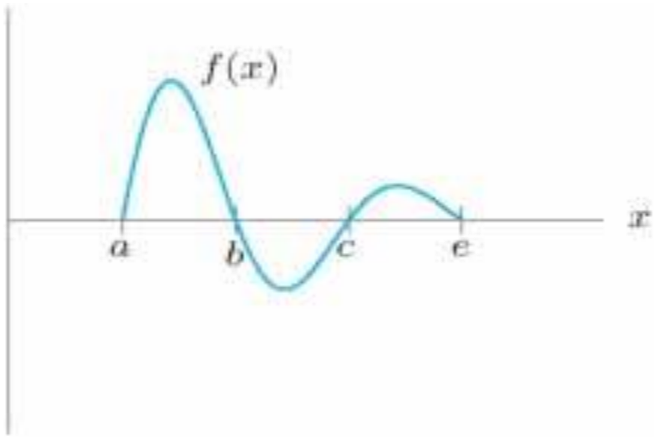
$PV = \int_0^M S(t)e^{-rt}dt$

$FV = PVe^{rM}$

$$\int_a^b \frac{P'(t)}{P(t)}dt = \ln\left(\frac{P(b)}{P(a)}\right)$$

$$\underbrace{\int udv = uv - \int vdu}_{\text{IBP}}$$

1.) Consider the graph of $f(x)$ below:



Which of the following is the largest? + / −

(A) $\int_a^e f(x)dx$ + but smaller than (B)

(B) $\int_a^b f(x)dx$ + and clearly larger than (D)

(C) $\int_b^c f(x)dx$ −

(D) $\int_c^e f(x)dx$ +

2.) A bacteria colony initially has a population of 8 million. Suppose that the population is growing at rate of $f(t) = 2^t$ million bacteria per hour. What is the best approximation of the population, in millions, after 3 hours?

(A) 17.34

(B) 19.23

(C) 18.10

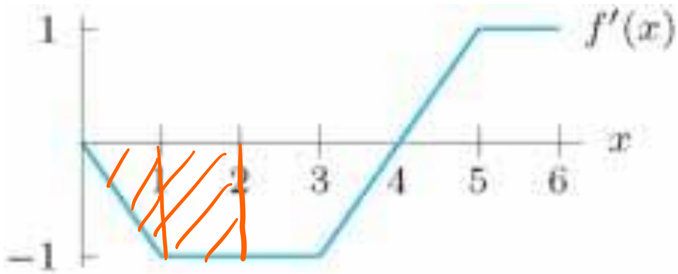
(D) 21.78

calculator

$$\underbrace{8}_{t=0} + \underbrace{\int_0^3 2^t\,dt}_{\text{change over } [0,3]} \approx 18.0989$$

Integral of the Rate of Change

3.) Consider the graph of $f'(x)$ below:



Suppose $f(0) = 2$. What is the value of $f(2)$?

(A) -1

(B) 0.5

(C) 4

(D) 1

$$F(2) = F(0) + \boxed{\int_0^2 f'(x)\,dx}$$
$$= 2 + \left(-\frac{3}{2}\right)$$
$$= \boxed{\frac{1}{2}}$$

4.) A company determines that its marginal revenue (in dollars per day) is given by $R'(t) = 150e^t$. The company's marginal cost is given by $C'(t) = 120 - 0.3t$ (in dollars per day). Find the total profit over the first five days (from $t = 0$ to $t = 5$).

(A) \$21,516

(B) \$21,667

(C) \$21,508

(D) \$21,519

Profit = Revenue − Cost

$$P(t) = R(t) - C(t)$$

$$\Rightarrow \underbrace{P'(t)}_{\text{marginal profit}} = \underbrace{R'(t)}_{\text{marginal revenue}} - \underbrace{C'(t)}_{\text{marginal cost}}$$

$$P'(t) = R'(t) - C'(t)$$
$$= 150e^t - (120 - 0.3t)$$
$$= 150e^t - 120 + 0.3t$$

profit over first 5 days $= \int_0^5 \underbrace{P'(t)}_{\text{rate of change of profit}}\,dt$

$$= \int_0^5 150e^t - 120 + 0.3t\,dt \underset{\text{calculator}}{\approx} \boxed{21{,}515.7}$$

5.) The rate that ice is forming on a pond is modeled by $F'(t)$, measured in inches per hour and t is measured in hours since 9AM. What does the following integral represent?

$$\int_0^2 F'(t)dt = \boxed{3.4}$$

(A) There is not enough information.

(B) Between 9AM and 11AM, 3.4 inches of ice formed.

(C) At 11AM, the pond has 3.4 inches of ice.

(D) Between 9AM and 11AM, ice formed at a rate of 3.4 inches per hour.

$t = 2$ hrs 11AM

$t = 0$ hrs 9AM

$$\int_0^2 \underbrace{F'(t)}_{\text{rate of change of ice formation}} dt = \text{AMOUNT OF ICE FORMED between 9Am } (t=0) \text{ and 11 Am } (t=2)$$

$= 3.4$

6.) What is the area enclosed by the curves $y = x^2 - 2x$ and $y = x$?

(A) 9

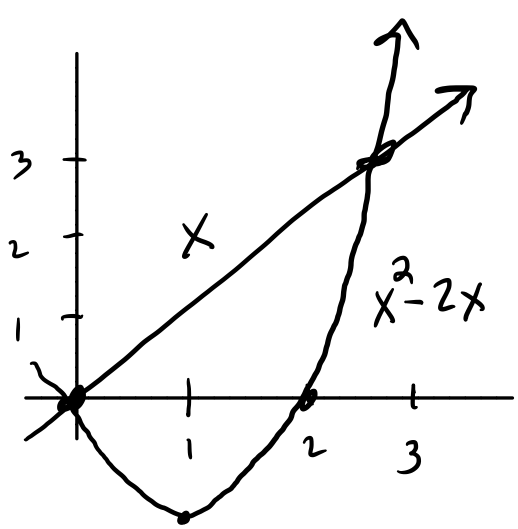
(B) $\frac{4}{3}$

(C) $\frac{5}{3}$

(D) $\frac{9}{2}$

intersection: $x^2 - 2x = x \Rightarrow x^2 - 3x = 0$

$\Rightarrow x(x-3) = 0 \quad x = 0, 3$



$$\text{area} = \int_0^3 x - (x^2 - 2x)\,dx$$

$$= \int_0^3 3x - x^2\,dx$$

$$= \left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^3$$

$$= \left(\frac{27}{2} - 9\right) - (0 - 0)$$

$$= \frac{27 - 18}{2} = \boxed{9/2}$$

7.) Find the average value of the function $f(x) = \frac{3}{\sqrt{x}}$ on [4,9].

(A) 6

(B) 7

(C) 7.6

(D) 1.2

$$\text{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{9-4}\int_4^9 3x^{-1/2}\,dx = \frac{3}{5}\int_4^9 x^{-1/2}\,dx$$

$$= \frac{3}{5}\left.\frac{x^{1/2}}{1/2}\right|_4^9 = \frac{6}{5}\left(9^{1/2} - 4^{1/2}\right)$$

$$= \frac{6}{5}(3-2) = \frac{6}{5} = \boxed{1.2}$$

8.) A firm's profit function is modeled by $P(x) = \frac{1}{1+x}$ dollars, where x is the number of units sold. What is the average profit for the first 50 units sold?

(A) $\frac{\ln(51)}{50}$ dollars

(B) $\frac{\ln(26)}{50}$ dollars

(C) 26 dollars

(D) 51 dollars

over 0-50

$$\text{avg} = \frac{1}{50-0}\int_0^{50} \frac{1}{1+x}\,dx$$

$$= \frac{1}{50} \ln|1+x| \Big|_0^{50}$$

$$= \frac{1}{50}[\ln(51) - \ln(1)] = \frac{1}{50}[\ln(51) - 0]$$

$$= \boxed{\frac{\ln(51)}{50}}$$

9.) Suppose the demand function is $D(q) = -q^2 + q + 5$ and the supply function is $S(q) = q^2 - q + 1$, at what price and quantity is the market in equilibrium?

(A) $p = 8,\ q = 3$

(B) $p = 4.4,\ q = \sqrt{2}$

(C) $p = 1.6,\ q = \sqrt{2}$

(D) $p = 3,\ q = 2$

$D(q)$ intersects $S(q)$ at (q^*, p^*)

$$-q^2 + q + 5 = q^2 - q + 1$$

$$2q^2 - 2q - 4 = 2(q^2 - q - 2)$$

$$= 2(q-2)(q+1) = 0$$

$$\Rightarrow q = -1 \text{ or } \boxed{q^* = 2}$$

must be positive since $q^* > 0$

p^*? $p^* = D(q^*)$

$= -4 + 2 + 5$

$= 3$

same!

or

$p^* = S(q^*) = 4 - 2 + 1$

$= 3$

$$\boxed{(q^*, p^*) = (2, 3)}$$

10.) Suppose the demand function is given by $D(q) = 100 - 4q$. What is the consumer surplus when the equilibrium quantity is q = 10?

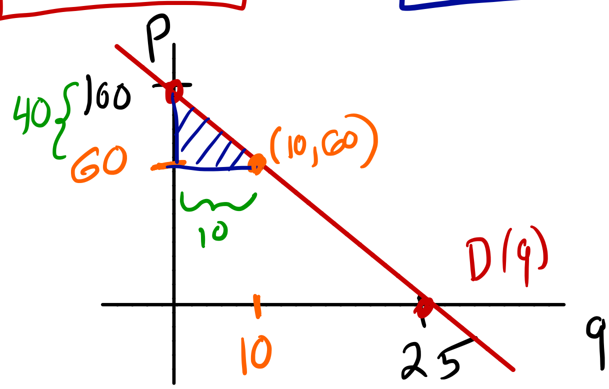
(A) 175

(B) 200

(C) 210

(D) 225

$$\underbrace{(q^*, p^*) = (10, 60)}_{\text{equilibrium pt}}$$



Consumer surplus $= \text{area}(\ \ ) = \frac{1}{2} \cdot 10 \cdot 40$

$$= \frac{400}{2} = \boxed{200}$$

11.) Find the producer surplus when the market is in equilibrium if the supply function is given by $S(q) = q^2 + 4q + 7$ and the demand function is given by $D(q) = 82 - 6q$.

(A) 127

(B) 75

(C) 335

(D) 133

equilibrium: $S(q) = D(q)$

$q^2 + 4q + 7 = 82 - 6q \Rightarrow q^2 + 10q - 75 = 0$

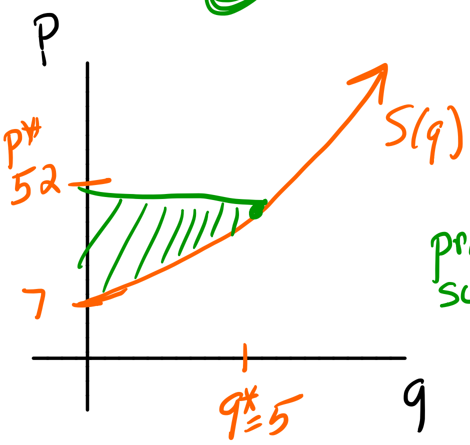
$(q+15)(q-5) = 0$

$q = -15$ NO! $q > 0$ since it is a quantity!

$\boxed{q^* = 5}$

$q^* = 5$: $S(5) = 25 + 20 + 7 = \boxed{52}$

$D(q) = 82 - 30 = \boxed{52} = p^*$



producer surplus $= \int_0^5 52 - S(q)\,dq$

$= \int_0^5 52 - (q^2 + 4q + 7)\,dq \underset{\text{calculator}}{\approx} 133.3 \quad \left(\frac{400}{3}\right)$

12.) Suppose a company is expected to earn \$500 per year, at a continuous rate, over the next ten years with a continuous interest rate of 4%. The rights to the earnings are the company are for sale now for \$5,000. Is it advised to buy the earning rights to this company?

(A) Yes, purchase the company for \$5,000 because it will earn \$6148 over ten years.

(B) No, do not purchase the company for \$5,000 because it will earn less than \$5,000 over ten years.

(C) Yes, purchase the company for \$5,000 because it will earn \$7459 over ten years.

(D) No, do not purchase the company for \$5,000 because it is worth \$4121 today.

ans: Compute the PV of the \$500/yr continuous payments:

$PV = \int_0^{10} 500 e^{-0.04t}\,dt \underset{\text{calculator}}{\approx} \$4121$

So the company is only worth \$4,121 TODAY, so you should not purchase it for \$5K.

13.) Consider the table below:

Population of California (in millions)

Year	2003	2004	2005	2006	2007
Population	29.2	30.3	35.8	37.6	39.6

What is the relative increase between 2006 and 2007?

(A) 5%

(B) 5.3%

(C) 6.1%

(D) 5.9%

$$\text{relative increase} = \frac{P(2007) - P(2006)}{P(2006)}$$

$$= \frac{39.6 - 37.6}{37.6} = \frac{2}{37.6}$$

$$\approx 0.05319$$

14.) If the monthly mortgage payment for a home that cost \$116,500 is \$833.33 with an interest rate of 6% compounded continuously, what is the length of the loan?

(A) Between 12 and 18 years

(B) Between 19 and 24 years

(C) Between 25 and 30 years

(D) Between 31 and 36 years

Find m = length of loan in YEARS

$$PV = \int_0^m \boxed{S} e^{-rt}\, dt$$

$$\boxed{S} = 12 * 833.33/\text{month} = \frac{12 \cdot 2500}{3} = \boxed{10000}$$

$$116500 = \int_0^m 10000\, e^{-0.06t}\, dt$$

or

$$11.65 = \int_0^m e^{-0.06t}\, dt \Rightarrow \frac{1}{-0.06}\left(e^{-0.06}\Big|_0^m\right) = 11.65$$

$$\Rightarrow e^{-0.06m} = 11.65(-0.06) + 1$$

$$\Rightarrow m = \frac{1}{-0.06}\ln(11.65\cdot(-0.06) + 1)$$

$$\approx 20.01 \text{ years}$$

15.) The relative rate of change of a population of fish is modeled by $\frac{P'(t)}{P(t)} = \frac{8t-25}{9t+7}$
By what percent did the population change over from $t = 0$ to $t = 10$ years?

(A) Increases by 79%
(B) Decreases by 23%
(C) Increases by 21%
(D) Decreases by21%

$$\ln\left(\frac{P(10)}{P(0)}\right) = \int_0^{10} \frac{P'(t)}{P(t)}\,dt = \int_0^{10} \frac{8t-25}{9t+7}\,dt \approx -0.231$$

calculator

$$\ln\left(\frac{P(10)}{P(0)}\right) \approx -0.231$$

$e \qquad e$

$$\frac{P(10)}{P(0)} \approx e^{-0.231} \Rightarrow P(10) \approx e^{-0.231} P(0)$$

$$\approx \underbrace{0.7937}_{\text{decrease by} \approx 0.21} P(0)$$

or $\approx -21\%$

16.) Evaluate the integral below:

$$\int \left(\sqrt{x} + \frac{1}{x^4}\right) dx$$

(A) $2x^{3/2} - \frac{1}{5x^5} + C$

(B) $\frac{2}{3}x^{3/2} - \frac{1}{3x^3} + C$

(C) $\frac{2}{3}x^{3/2} - \frac{1}{5x^5} + C$

(D) $2x^{3/2} - \frac{1}{3x^3} + C$

$$= \int x^{1/2} + x^{-4}\,dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^{-3}}{-3} + C$$

$$= \boxed{\frac{2}{3}x^{3/2} - \frac{1}{3}x^{-3} + C}$$

17.) What is the anti-derivative of $f(t) = \frac{1}{t} - 2e^t + sin\left(\frac{t}{4}\right)$?

(A) $\ln|t| - 2e^t - \frac{1}{4}cos\left(\frac{t}{4}\right) + C$

(B) $\ln(t) - 2e^{t-1} - 4\cos\left(\frac{t}{4}\right) + C$

(C) $\frac{1}{t^2} - 2e^t - 4cos\left(\frac{t}{4}\right) + C$

(D) $\ln|t| - 2e^t - 4cos\left(\frac{t}{4}\right) + C$

$$F(t) = \int f(t)\,dt$$
$$= \int \frac{1}{t} - 2e^t + \sin\left(\frac{1}{4}t\right)dt$$
$$= \ln|t| - 2e^t - \frac{1}{1/4}\cos\left(\frac{1}{4}t\right) + C$$
$$= \boxed{\ln|t| - 2e^t - 4\cos\left(\frac{1}{4}t\right) + C}$$

18.) Evaluate the integral below:

$$\int x^3\sqrt{x^4+7}\,dx = \int (x^4+7)^{1/2}x^3\,dx$$

(A) $\frac{1}{6}(x^4+7)^{3/2} + C$

(B) $\frac{2}{3}(x^4+7)^{3/2} + C$

(C) $-\frac{1}{2}(x^4+7)^{-1/2} + C$

(D) $\frac{1}{4}x^4 \cdot \frac{2}{3}(x^4+7)^{3/2} + C$

$w = x^4+7$

$dw = 4x^3dx$

$\frac{1}{4}dw = x^3dx$

$$= \int w^{1/2}\frac{1}{4}dw$$
$$= \frac{1}{4}\frac{w^{3/2}}{3/2} + C$$
$$= \frac{1}{6}w^{3/2} + C$$
$$= \boxed{\frac{1}{6}(x^4+7)^{3/2} + C}$$

19.) Evaluate the integral below.

$$\int (5x^2 + x^{-3})dx = 5\frac{x^3}{3} + \frac{x^{-2}}{-2} + C$$

$$= \boxed{\frac{5}{3}x^3 - \frac{1}{2}x^{-2} + C}$$

(A) $\frac{5}{3}x^3 + \frac{1}{2}x^{-2} + C$

(B) $10x - 3x^{-4} + C$

(C) $\frac{5}{3}x^3 - \frac{1}{2}x^{-2} + C$

(D) $-\frac{5}{3}x^3 + \frac{1}{2}x^{-2} + C$

20.) Find $f(x)$ with initial conditions given below:

$$f'(x) = (14x + 3)\sin(7x^2 + 3x) \text{ with } f(0) = -5$$

$F(x) = \int F'(x)dx$ upto a constant

$= \int \sin(\underbrace{7x^2+3x}_{w}) \underbrace{(14x+3)dx}_{dw}$

$w = 7x^2+3x$

$dw = (14x+3)dx$

$= \int \sin w \, dw$

$= -\cos w + C$

$\boxed{F(x) = -\cos(7x^2+3x) + C}$

(A) $f(x) = \cos(7x^2 + 3x) - 6$

(B) $f(x) = -\cos(7x^2 + 3x) - 4$

(C) $f(x) = -\cos(7x^2 + 3x) - 5$

(D) $f(x) = \cos(7x^2 + 3x) - 5$

C? $-5 = F(0) = -\cos(0) + C$

$= -1 + C$

$\Rightarrow \boxed{C = -4}$

$$\int u\,dv = uv - \int v\,du$$

IBP

21.) Evaluate the integral below:

$$\int x^3 \ln(4x)dx = \frac{1}{4}x^4 \ln(4x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$$

$u = \ln(4x)$ $\quad v = \frac{1}{4}x^4$

$du = \frac{1}{4x} \cdot 4\,dx$ $\quad dv = x^3\,dx$

$= \frac{1}{x}\,dx$

$$= \frac{1}{4}x^4 \ln(4x) - \frac{1}{4}\int x^3\,dx$$

$$= \boxed{\frac{1}{4}x^4 \ln(4x) - \frac{1}{16}x^4 + C}$$

(A) $\frac{1}{4}x^4 \cdot \frac{1}{2}(\ln(4x))^2 + C$

(B) $\frac{1}{4}x^4 \ln(4x) - \frac{1}{16}x^4 + C$

(C) $\frac{1}{4}x^4 \ln(4x) - \frac{1}{4}x^4 + C$

(D) $\frac{1}{4}x^4 \ln(4x) + \frac{1}{16}x^5 + C$

22.) When trying to evaluate the integral below using u-substitution, which of the following would be the resulting integral?

$$\int x^2(x^3 - 3)^{16}dx$$

(A) You cannot use the method of u substitution to evaluate this integral.

(B) $\frac{1}{3}\int u^2(u-3)^{16}\,du$

(C) $\frac{1}{3}\int u^{16}\,du$

(D) $3\int (u^3 - 3)^{16}\,du$

$$\int \underbrace{(x^3-3)}_{u}{}^{16}\ \underbrace{x^2\,dx}_{\frac{1}{3}du}$$

$u = x^3 - 3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$$= \int u^{16}\frac{1}{3}\,du$$

$$= \boxed{\frac{1}{3}\int u^{16}\,du}$$

IBP TWICE!

23.) Evaluate the integral below:

1st IBP:

$$\int \underbrace{(x^2+2x)}_{u}\underbrace{e^{3x}dx}_{dv}$$

$u = x^2+2x$

$dv = e^{3x}dx$

(A) $\frac{1}{3}(x^2+2x)e^{3x} - \frac{1}{9}(2x+2)e^{3x} + \frac{2}{27}e^{3x} + C$

(B) $\frac{1}{3}(x^2+2x)e^{3x} - \frac{1}{9}(2x+2)e^{3x} - \frac{2}{27}e^{3x} + C$

(C) $\frac{1}{3}(x^2+2x)e^{3x} - \frac{1}{9}(2x+2)e^{3x} + \frac{2}{9}e^{3x} + C$

(D) $\frac{1}{3}(x^2+2x)e^{3x} - \frac{2}{3}(2x+2)e^{3x} - \frac{2}{3}e^{3x} + C$

practice!

Improper

24.) Evaluate the integral below:

$$\int_1^{\infty} -14e^{-14x}\ dx$$

$= -14\int_1^{\infty} e^{-14x}dx = -14\boxed{\lim_{b\to\infty}\int_1^b e^{-14x}dx}$

$= -14\cdot\frac{1}{14e^{14}} = \boxed{-\frac{1}{e^{14}}}$

$\lim_{b\to\infty}\int_1^b e^{-14x}dx = \lim_{b\to\infty}\frac{1}{-14}e^{-14x}\Big|_1^b = -\frac{1}{14}\lim_{b\to\infty}\left[\frac{1}{e^{14b}} - \frac{1}{e^{14}}\right]$

goes to 0

$= -\frac{1}{14}\left[0 - \frac{1}{e^{14}}\right] = \frac{1}{14e^{14}}$

(A) $\frac{1}{e^{14}}$

(B) $-\frac{1}{e^{14}}$

(C) 0

(D) 1

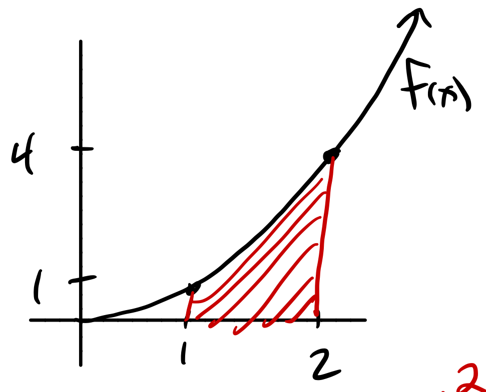
25.) Use the Fundamental Theorem of Calculus to find the area under $f(x) = x^2$ from $x = 1$ to $x = 2$.

(A) 9/7

(B) 7/3

(C) 3

(D) 4



$$\text{area} = \int_1^2 x^2\,dx = \frac{1}{3}x^3 \Big|_1^2$$

$$= \frac{1}{3}(2^3 - 1^3) = \boxed{\frac{7}{3}}$$