**Math 128** **Exam #1** **Fall 2015** **SPECIAL CODE: 101501**

Name key with work

IBP - Integration by Parts

Signature: ________________________________

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Academic Honesty Statement: By signing my name above, I acknowledge that I understand each of the following behaviors:

> *using a calculator or cell phone (or any other communication technology); referring to a piece of paper or object with helpful information on it (cheat sheet, crib sheet, bill of a baseball cap, etc...); looking at a test or answer sheet that is not my own; allowing another student to look at my test or answer sheet; communicating with other students (verbally or nonverbally); taking the test for another student; taking my bubble sheet of answers with me when I've finished; talking while waiting to hand in my test materials to the proctors*

to be a form of academic dishonesty (cheating). I am also pledging not to engage in any of these behaviors. I understand that if I do engage in these behaviors, the consequences will be failure of the exam and a formal charge of academic dishonesty to the Ombuds Office.

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Please shut off all cell phones, ear phones, computers, beepers, etc…

***Please put everything away except a #2 pencil and a calculator that is NOT your cell phone. You may write on the test. There are twenty five multiple choice questions and each question is worth four points.***

1. On the bubble sheet, where it says "Name," please print your last name, leave a space, and then print your first name in the rectangles. Then fill in the bubbles underneath.

2. On the bubble sheet, where it says "Identification Number," please write your entire Student ID number in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in your ID # correctly.

4. On the bubble sheet, where it says "Special Codes," please write the numbers: 101501 in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in the special code correctly.

5. Lastly, on the bubble sheet, in the margin above your name, please neatly print "Exam #1 Fall 2015", your section number (01, 02 or 03), and sign your name.

*Please make sure you bubble in your answers carefully on the bubble sheet and* ***circle your answers on your test booklet****.*

1.) Evaluate the integral: $\int(3x^8 - 7x^3 + 6)dx$ $= \frac{3x^9}{9} - 7\frac{x^4}{4} + 6x + C$

$= \boxed{\frac{1}{3}x^9 - \frac{7}{4}x^4 + 6x + C}$

(A) $\frac{1}{3}x^9 - \frac{7}{3}x^4 + 6x + C$

(B) $9x^9 - \frac{7}{4}x^4 + 6x + C$

(C) $9x^9 - \frac{7}{3}x^4 + 6x + C$

(D) $\frac{1}{3}x^9 - \frac{7}{4}x^4 + 6x + C$

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2.) Evaluate the integral: $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx$ with $f(9) = 29$.

(A) $\frac{2}{3}x^{3/2} - \sqrt{x} + 14$

(B) $\frac{2}{3}x^{3/2} - 2\sqrt{x} + 17$

(C) $\frac{2}{3}x^{3/2} - 2\sqrt{x} + 29$

(D) $\frac{2}{3}x^{3/2} - \sqrt{x} + 29$

$f(x) = \int \sqrt{x} - \frac{1}{\sqrt{x}}\,dx = \int x^{1/2} - x^{-1/2}\,dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} + C = \boxed{\frac{2}{3}x^{3/2} - 2x^{1/2} + C}$

C? $29 = f(9) = \frac{2}{3}(9)^{3/2} - 2(9)^{1/2} + C$

$= 18 - 6 + C$

$C = 29 - 18 + 6 = \boxed{17}$

---

3.) Evaluate the following integral. $\int_0^a (8x - x^2)dx$ $= \left(8\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^a$

(A) $8 - 2a$

(B) $4a^2 - \frac{1}{3}a^3$

(C) $8a^2 - \frac{1}{3}a^3$

(D) $4a^2 - a^3$

$= \left(4a^2 - \frac{1}{3}a^3\right) - (0 - 0)$

$= \boxed{4a^2 - \frac{1}{3}a^3}$

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4.) Evaluate the integral: $\int \frac{x}{\sqrt{25-x^2}}dx$ $= \int (\underbrace{25-x^2}_{w})^{-1/2} x\,dx$

(A) $-\sqrt{25-x^2} + C$

(B) $\sqrt{25-x^2} + C$

(C) $-\frac{1}{5}\sqrt{25-x^2} + C$

(D) $\frac{1}{25}\sqrt{25-x^2} + C$

$w = 25 - x^2$

$dw = -2x\,dx$

$-\frac{1}{2}dw = x\,dx$

$= \int w^{-1/2}\left(-\frac{1}{2}\right)dw = -\frac{1}{2}\int w^{-1/2}\,dw$

$= -w^{1/2} + C = \boxed{-(25-x^2)^{1/2} + C}$

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5.) Find the anti-derivative of $f'(x) = e^{6x}$ with $f(0) = 0$.

(A) $\frac{1}{6}e^{6x}$

(B) $e^{6x} - 6$

(C) $\frac{1}{6}e^{6x} - \frac{1}{6}$

(D) $6e^{6x} - 6$

$F(x) = \int f'(x)\,dx = \int e^{6x}dx = \frac{1}{6}e^{6x} + C$

$C$? $0 = F(0) = \frac{1}{6}e^{0} + C = \frac{1}{6} + C \Rightarrow C = -1/6$

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6.) Evaluate the integral: $\int \frac{5e^{1/x}}{3x^2}dx$ $= \frac{5}{3}\int \frac{e^{1/x}}{x^2}dx = \frac{5}{3}\int e^{w}(-dw)$

(A) $\frac{5e^{1/x}}{x^3} + C$

(B) $10xe^{1/x} + C$

(C) $-\frac{5e^{1/x}}{3} + C$

(D) $\frac{5e^{1/x}}{3} + C$

$w = \frac{1}{x}$

$dw = \frac{-1}{x^2}dx$

$-dw = \frac{1}{x^2}dx$

$= -\frac{5}{3}e^{w} + C$

$= \boxed{-\frac{5}{3}e^{1/x} + C}$

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IBP

$x \to 1$

7.) Evaluate the integral: $\int -7x\sin(4x)\,dx$ $= -7\int \underbrace{x}_{u}\underbrace{\sin(4x)dx}_{dv}$

(A) $-\frac{7}{4}x\cos(4x) - \frac{7}{16}\sin(4x) + C$

(B) $\frac{7}{4}x\cos(4x) - \frac{7}{16}\sin(4x) + C$

(C) $-\frac{7}{4}x\cos(4x) + \frac{7}{16}\sin(4x) + C$

(D) $\frac{7}{4}x\cos(4x) + \frac{7}{16}\sin(4x) + C$

$= -7\left[-\frac{1}{4}x\cos(4x) - \int -\frac{1}{4}\cos(4x)dx\right]$

$= \boxed{\frac{7}{4}x\cos(4x) - \frac{7}{16}\sin(4x) + C}$

$u = x$ $\quad v = -\frac{1}{4}\cos(4x)$

$du = dx$ $\quad dv = \sin(4x)dx$

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8.) Evaluate the integral: $\int \left(\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}\right)dx$ $= \int x^{-1} + 2x^{-2} + 3x^{-3}\,dx$

$= \ln|x| + 2\frac{x^{-1}}{-1} + 3\frac{x^{-2}}{-2} + C$

$= \boxed{\ln|x| - 2x^{-1} - \frac{3}{2}x^{-2} + C}$

(A) $\frac{2}{x^2}+\frac{6}{x^3}+\frac{12}{x^4}+C$

(B) $2x+2\ln|x^2|+3\ln|x^3|+C$

(C) $\ln|x|-\frac{2}{x}-\frac{3}{2x^2}+C$

(D) $\ln|x|+2\ln(x^2)+3\ln|x^3|+C$

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IBP

2 times

$x^2 \to x \to 1$

9.) Evaluate the integral: $\int 11x^2e^{2x}dx$ $= 11\int \underbrace{x^2}_{u}\,\underbrace{e^{2x}dx}_{dv} = 11\left[\frac{1}{2}x^2e^{2x} - \boxed{\int xe^{2x}dx}\right]$

$u = x^2$ $\quad v = \frac{1}{2}e^{2x}$

$du = 2x\,dx$ $\quad dv = e^{2x}dx$

$\boxed{= 11\left[\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C\right]}$

$\int \underbrace{x}_{u}\,\underbrace{e^{2x}dx}_{dv} = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx = \boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C}$

$u = x$ $\quad v = \frac{1}{2}e^{2x}$

$du = dx$ $\quad dv = e^{2x}dx$

(A) $11x^2e^{2x} - 11xe^{2x} + \frac{11}{2}e^{2x} + C$

(B) $\frac{11}{2}x^2e^{2x} - \frac{11}{2}xe^{2x} - \frac{11}{4}e^{2x} + C$

(C) $\frac{11}{2}x^2e^{2x} - \frac{11}{2}xe^{2x} + \frac{11}{4}e^{2x} + C$

(D) $\frac{11}{4}x^2e^{2x} - \frac{11}{4}xe^{2x} + \frac{11}{4}e^{2x} + C$

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10.) Evaluate the integral: $\int \left(4\sqrt{x} - \frac{1}{2\sqrt{x}}\right)dx$ $= \int 4x^{1/2} - \frac{1}{2}x^{-1/2}dx$

$= 4\frac{x^{3/2}}{3/2} - \frac{1}{2}\frac{x^{1/2}}{1/2} + C$

$= \boxed{\frac{8}{3}x^{3/2} - x^{1/2} + C}$

(A) $\frac{8}{3}x^{3/2} - \sqrt{x} + C$

(B) $\frac{2}{\sqrt{x}} + \frac{1}{4x^{3/2}} + C$

(C) $2x^{3/2} + \frac{1}{4}\sqrt{x} + C$

(D) $6x^{3/2} - \frac{1}{2}\ln|\sqrt{x}| + C$

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IBP

$x \to 1$

11.) Evaluate the integral: $\int_0^6 xe^{-x}dx$ $= \left(-xe^{-x} - e^{-x}\right)\Big|_0^6$

(A) $-7e^{-6}$ $= (-6e^{-6} - e^{-6}) - (0 - 1) = \boxed{-7e^{-6} + 1}$

(B) $-5e^{-6} + 1$

(C) $-7e^{-6} - 1$ $\int \underbrace{x}_{u}\underbrace{e^{-x}dx}_{dv} = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C$

(D) $-7e^{-6} + 1$

$u = x$ $\quad v = -e^{-x}$

$du = dx$ $\quad dv = e^{-x}dx$

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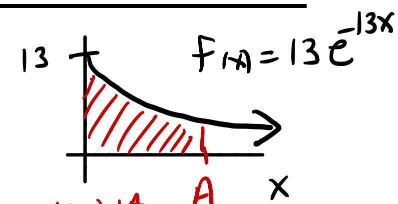


12.) Evaluate the integral: $\int_0^\infty 13e^{-13x}dx$

(A) 1 $= \lim_{A\to\infty} 13 \int_0^A e^{-13x}dx$

(B) $-1$ $= \lim_{A\to\infty} \left(13 \cdot \frac{-1}{13} e^{-13x}\right)\Big|_0^A = \lim_{A\to\infty} \left(-e^{-13x}\right)\Big|_0^A$

(C) 0

(D) Does not exist. $= \lim_{A\to\infty} \left(-e^{-13 \cdot A} - (-e^0)\right) = \lim_{A\to\infty} \left(\frac{-1}{e^{13A}} + 1\right) = 1$

$\to$ goes to 0 as $A \to \infty$

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13.) The population of a town in 2010 was 12,000 people. If the population is changing with a rate of $r(t) = 710e^{0.02t}$, where t is measured in years since 2010, what is the total change in population between 2010 and 2017?

$\underbrace{P(7)}_{2017} = \underbrace{P(0)}_{2010} + \int_0^7 P'(t)dt = 12{,}000 + \underbrace{\int_0^7 710\, e^{0.02t}dt}_{\text{change in population}}$

(A) 5,335 people per year

(B) 17,335 people per year

(C) 17,335 people

(D) 5,335 people

$710 \int_0^7 e^{0.02t}dt = 710 \frac{1}{0.02} e^{0.02t}\Big|_0^7 \approx \boxed{5334.72}$

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14.) Which of the following statements is true about $\int_0^3 (x^3 - 4x)dx$ ?

$f(x) = x^3 - 4x = x(x^2 - 4)$

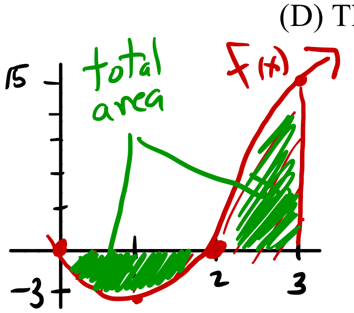
$= 0$ at $0, \pm 2$

(A) The integral value is $\frac{9}{4}$, but the total area is $\frac{41}{4}$.

(B) The integral value is 0 because the area above the x-axis cancels the area below the x-axis.

(C) The area and the evaluated integral represent the same number, which is $\frac{9}{4}$.

(D) The area and the evaluated integral represent the same number, which is $\frac{41}{4}$.

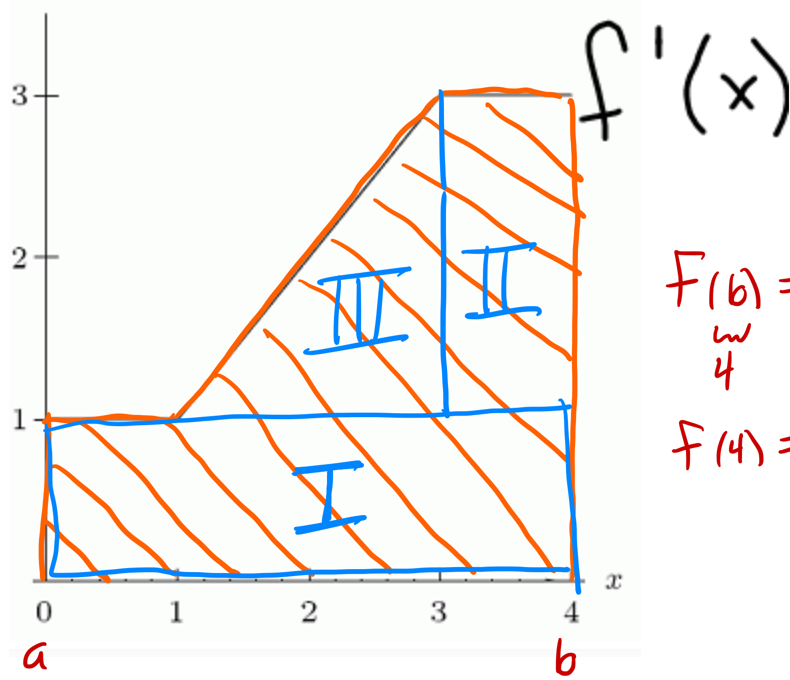



$\int_0^3 x^3 - 4x\, dx = \left(\frac{1}{4}x^4 - 2x^2\right)\Big|_0^3 = \left(\frac{3^4}{4} - 18\right) - (0 - 0) = \frac{81 - 72}{4} = \boxed{9/4}$

total area $= \underbrace{\int_0^3 |f(x)|dx}_{|f(x)| \text{ in case } f(x) < 0} = \int_0^2 \underbrace{-(x^3 - 4x)}_{\geq 0} dx + \int_2^3 x^3 - 4x\, dx$

$\underset{\text{(calculator)}}{=} 4 + 6\frac{1}{4} = \boxed{\frac{41}{4}}$

15.) If the following graph represents $f'(x)$ and $f(0) = 3$. Find $f(4)$.




$F(b) = f(a) + \int_a^b f'(x)dx$

$\underset{4}{\smile}$ $\underset{0}{\smile}$

$f(4) = 3 + \boxed{\int_0^4 f'(x)dx}$

integral equals change in $f(x)$ over $[0,4]$

(A) 8
(B) 3
(C) 11
(D) 5

$F(4) = 3 + \text{I} + \text{II} + \text{III} = 3 + 4 + 2 + \frac{1}{2}\cdot 2\cdot 2 = \boxed{11}$

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16.) Find the average value of the function $f(x) = 3x^2 - 4$ on the interval [0,4].

(A) 12
(B) 13
(C) 4
(D) 16

$avg = \frac{1}{b-a}\int_a^b F(x)\,dx$ $[a,b]$

$= \frac{1}{4-0}\int_0^4 3x^2 - 4dx = \frac{1}{4}(x^3 - 4x)\Big|_0^4 = \frac{1}{4}\left[(64-16)-(0-0)\right] = \frac{48}{4} = \boxed{12}$

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17.) If the supply curve is given by $p = S(q)$ and the demand curve is given by $p = D(q)$ where $(p*, q*)$ represents the equilibrium point. Which of the following represents the consumer surplus?

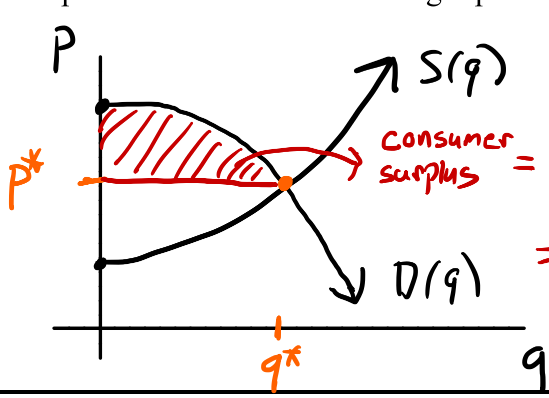
$(q^*, p^*)$

(A) $\left(\int_0^{q*} D(q)dq\right) - p*q*$

(B) $p*q* - \left(\int_0^{q*} S(q)dq\right)$

(C) $\left(\int_0^{q*} S(q)dq\right) - p*q*$

(D) $p*q* - \left(\int_0^{q*} D(q)dq\right)$

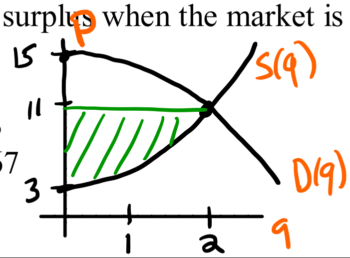



$= \int_0^{q^*} D(q) - p^* dq$

$= \int_0^{q^*} D(q)dq - \int_0^{q^*} p^* dq$

$= \int_0^{q^*} D(q)dq - q^* p^*$

18.) Given the supply curve $\overbrace{p = 3 + 2q^2}^{S(q)}$ and the demand curve $\overbrace{p = 15 - q^2}^{D(q)}$, find the producer surplus when the market is in $\boxed{\text{equilibrium.}}$




(A) \$22
(B) \$5.33
(C) \$10.67
(D) \$16

$(q^*, p^*) = (2, 11)$ $\quad 3 + 2q^2 = 15 - q^2 \Rightarrow q^2 = 4$

$q = \pm 2$ must be +

$$\text{Area} = \int_0^{q^*} p^* - S(q)\,dq = \int_0^2 11 - (3 + 2q^2)\,dq$$

$$= \int_0^2 8 - 2q^2\,dq = \left(8q - \frac{2}{3}q^3\right)\Big|_0^2 = \left(16 - \frac{16}{3}\right) - (0 - 0)$$

$$= \frac{32}{3} = \boxed{10\frac{2}{3}}$$

$P = \begin{cases} 3+8 \\ 15-4 \end{cases} = 11$

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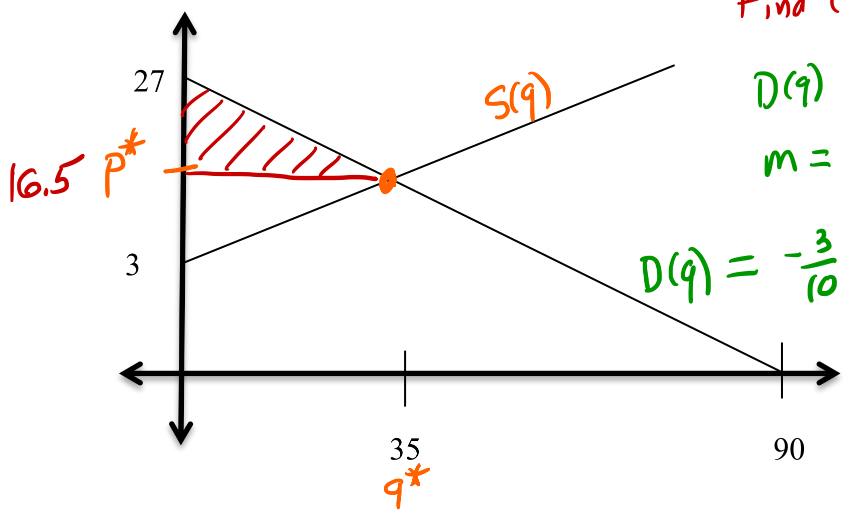
19.) Given the graph of the supply and demand functions below, find the consumer surplus when the market is in equilibrium:




Find $(q^*, p^*)$: $q^* = 35$, $p^*$?

$D(q)$ is a line thru $(0, 27)$ and $(90, 0)$

$$m = \frac{\Delta p}{\Delta q} = \frac{0 - 27}{90 - 0} = \frac{-27}{90} = -\frac{3}{10}$$

$$D(q) = -\frac{3}{10}q + 27 \Rightarrow p^* = D(q^*)$$

$$= -\frac{3}{10} \cdot 35 + 27$$

$$= \frac{-105 + 270}{10} = \frac{165}{10} = 16.5$$

$$\text{consumer surplus} = \text{area triangle} = \frac{1}{2} \cdot 35 \cdot (27 - 16.5) = 17.5 \cdot 10.5$$

$$= \boxed{183.75}$$

(A) \$236.25
(B) \$262.50
(C) \$183.75
(D) \$210

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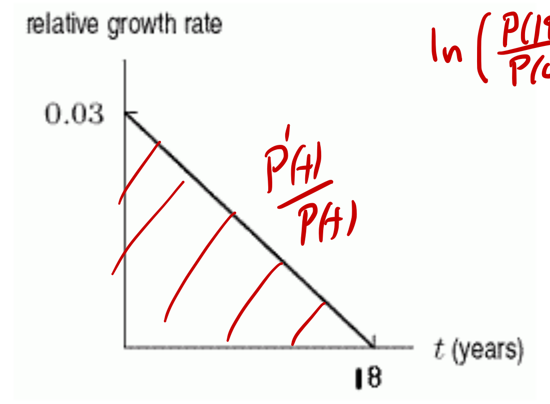
20.) A graph of the relative growth rate of a population is given in the following figure: By what percentage does the population change over the 18 year period?




$$\ln\left(\frac{P(18)}{P(0)}\right) = \int_0^{18} \frac{P'(t)}{P(t)}\,dt$$
$$= \text{area} = \frac{1}{2}\cdot 18 \cdot 0.03$$
$$= 0.27$$
$$\Rightarrow \frac{P(18)}{P(0)} = e^{0.27}$$
or
$$P(18) = e^{0.27}P(0)$$
$$\approx \underbrace{1.31}_{1+\boxed{\underbrace{0.31}_{\text{increase}}}} P(0)$$

(A) Increases by 27%
(B) Decreases by 27%
(C) Increases by 31%
(D) Decreases by 54%

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21.) The population of a small town is 12,000 people in 2010. In 2015, the population of the town is 14,500 people. Assuming that there is a continuous relative rate of change per year, write a formula to model this population, where t is measured in years since 2010.

$$r = \text{relative change over } [2010, 2015] = \frac{14500-12000}{12000} = \frac{2500}{12000} = \frac{5}{24} \approx 0.2083$$

$$P(t) = P(0)e^{rt} = 12000e^{0.2083t}$$

(A) $P(t) = 12{,}000 + 2{,}500t$

(B) $P(t) = 12{,}000e^{0.8275t}$

(C) $P(t) = 12{,}000e^{0.038t}$

(D) $P(t) = 12{,}000e^{0.208t}$

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22.) You want to start a family in five years, and you will need to add an addition to your home in four years to make room. You will need $120,000 in four years. If you add money to an investment earning 5% continuously, at what rate should you deposit money into the account to reach this goal?

$$FV = e^{rM}\,PV$$
$$120000 = e^{0.05\cdot 4}\int_0^4 S e^{-0.05t}\,dt$$
$$\Rightarrow S = \frac{120000}{e^{0.2}\int_0^4 e^{-0.05t}\,dt} \approx \boxed{27{,}099.9}$$

(A) $27,605 per year
(B) $26,188 per year
(C) $27,100 per year
(D) $24,562 per year

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$PV = \int_0^M S e^{-rt} dt$

23.) I have just bought a home with a purchase price, after a down payment, of $300,000. I can afford to pay $2000 per month on the mortgage, which has an annual interest rate of 4.6% compounded continuously. How long will the mortgage run?

$M = \#\text{ years} \quad S = 12 \cdot 2000 = 24000$

(A) 18 years and 8 months
(B) 16 years and 9 months
(C) 24 years and 2 months
(D) 22 years and 7 months

$300000 = \int_0^M 24000\, e^{-0.046t} dt = 24000 \int_0^M e^{-0.046t} dt$

$\frac{300}{24} = \frac{1}{-0.046}\left(e^{-0.046M} - 1\right) \Rightarrow M = \frac{1}{-0.046} \ln\left(\frac{-0.046 \cdot 300}{24} + 1\right) \approx \boxed{18.6\text{ yrs}}$

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24.) You receive an inheritance of $88,000. You decide to deposit the money into an account that accrues 5.7% interest compounded continuously. How much money do you have in this account $\underbrace{\text{in 4 years}}_{FV}$?

(A) $84,560
(B) $98,240
(C) $110,536
(D) $108,064

$FV = PV \cdot e^{rM}$

$= 88{,}000\, e^{(0.057) \cdot 4} \approx \boxed{110{,}535.5086}$

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25.) Suppose I will need $7000 in three years. How much do I need to deposit each week into an account paying 3% annual interest compounded continuously in order to reach my goal?

(A) $43
(B) $47
(C) $38
(D) $41

$S$ - \$/yr

$7000 = e^{(0.03 \cdot 3)} \int_0^3 S e^{-0.03t} dt$

$\text{ans} = \frac{S}{52} \approx 42.88$

$\Rightarrow \boxed{S = \frac{7000}{e^{0.09} \int_0^3 e^{-0.03t} dt}} \approx 2229.91$

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