Math 128 **Exam #2** **Fall 2017** **SPECIAL CODE: 111401**

Name ________________________________

Signature: ________________________________

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Academic Honesty Statement: By signing my name above, I acknowledge that I understand each of the following behaviors:

> *using a calculator or cell phone (or any other communication technology); referring to a piece of paper or object with helpful information on it (cheat sheet, notes, etc...); looking at a test or answer sheet that is not my own; allowing another student to look at my test or answer sheet; communicating with other students (verbally or nonverbally); taking the test for another student; taking my bubble sheet of answers with me when I've finished; talking while waiting to hand in my test materials to the proctors; sharing a calculator*

to be a form of academic dishonesty (cheating). I am also pledging not to engage in any of these behaviors. I understand that if I do engage in these behaviors, the consequences will be failure of the exam and a formal charge of academic dishonesty to the Ombuds Office.

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Please shut off all cell phones, ear phones, computers, beepers, smart watches, etc…

Please put everything away except a #2 pencil and a calculator that is NOT your cell phone. You may write on the test. There are twenty five multiple choice questions and each question is worth four points.

1. On the bubble sheet, where it says "Name," please print your last name, leave a space, and then print your first name in the rectangles. Then fill in the bubbles underneath.

2. On the bubble sheet, where it says "Identification Number," please write your entire Student ID number in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in your ID # correctly.

4. On the bubble sheet, where it says "Special Codes," please write the numbers: 111401 in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in the special code correctly.

Please make sure you bubble in your answers carefully on the bubble sheet and ***circle your answers on your test booklet****.*

Formulas:

Partial Derivatives:

$$f_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

Linear Approximation:

$$f(x,y) \approx f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$$

Normally Distributed Density Function:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Uniformly Distributed Density Function:

$$p(x) = \frac{1}{b-a}$$

Exponentially Distributed Density Function:

$$p(t) = ke^{-kt}$$

Mean:

$$\mu = \int_{-\infty}^{\infty} xp(x)dx$$

Median:

$$\int_{-\infty}^{T} p(x)dx = 0.5$$

1.) Given $p(x) = kx^2$ where $0 \le x \le 2$, find the value of k that will make this function a PDF.

(A) 1/8

(B) 1/2

(C) 3/8

(D) -2

To be a PDF $\int_{-\infty}^{\infty} p(x)\,dx = 1$

Here, $\boxed{\int_0^2 kx^2\,dx} = 1 \rightarrow \frac{8k}{3} = 1$ $\boxed{k = 3/8}$

$$k\int_0^2 x^2 = k\frac{x^3}{3}\Big|_0^2 = \frac{k}{3}\left(2^3 - 0^3\right) = \frac{8k}{3}$$

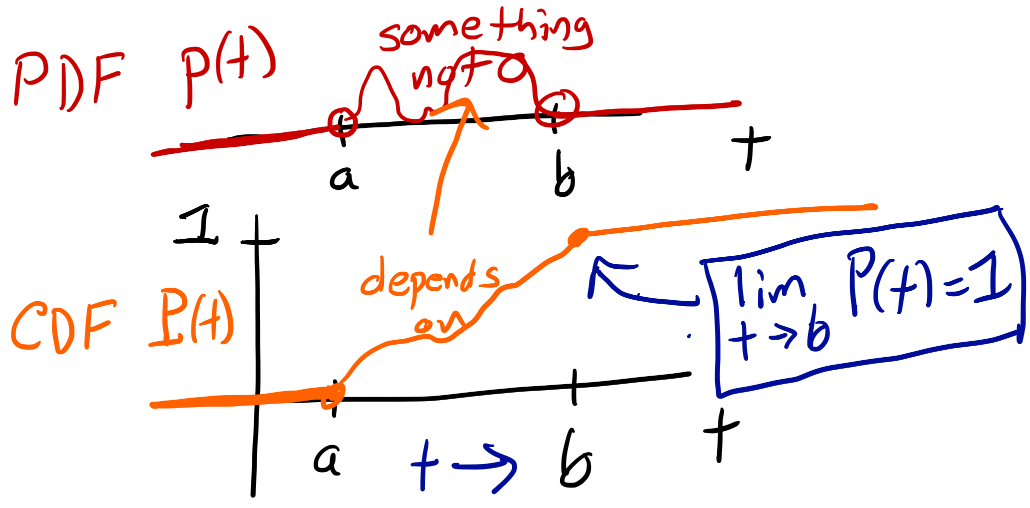
2.) Suppose $P(t)$ is a CDF with PDF $p(t)$ on $a \le t \le b$. Which of the following is always true?

(A) $\cancel{P(a) = -1}$

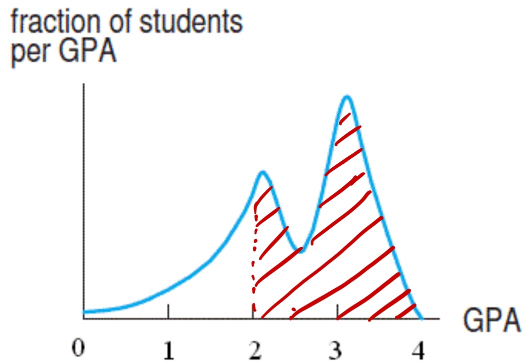
(B) $\cancel{p(t) = 1}$

(C) $\lim_{t \to b} P(t) = 1$

(D) $\cancel{\int_a^b P(t)dt = p(t)}$



3.) The PDF of the GPA of students at a university is shown below.



Approximately what fraction of the students had a GPA of 2 or greater?

(A) ~~1/8~~

(B) 2/3

(C) ~~1~~

(D) ~~1/10~~

$P(X \geq 2) =$ red area

From the picture

$\frac{1}{2} < \text{area} < 1$

4.) The distribution of IQs can be modeled by a Normal Distribution with mean 100 and standard deviation 15. Approximately what fraction of the population has an IQ between 85 and 115?

$\mu = 100$
$\sigma = 15$

(A) 0.6827

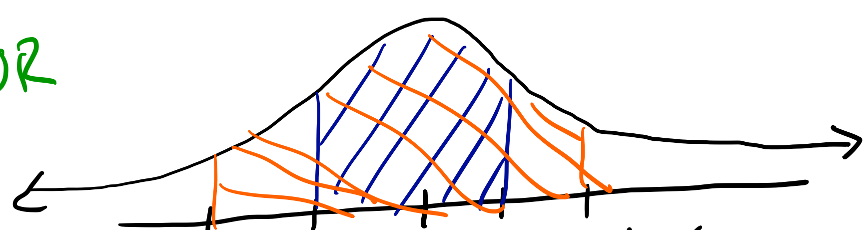
(B) 0.5000

(C) 0.3173

(D) 0.0833

Compute $\displaystyle\int_{85}^{115} \frac{1}{15\sqrt{2\pi}} e^{\frac{-(x-100)^2}{2(15)^2}}\,dx$

OR



$\mu-2\sigma \quad \mu-\sigma \quad \mu \quad \mu+\sigma \quad \mu+2\sigma$

$\boxed{85 \quad 100 \quad 115}$

$\approx 68\%$

$\approx 95\%$

5.) The table below shows the value of a CDF, $P(x)$, of household income distribution in the United States in 2006. What percentage of households made between \$60,000 and \$100,000?

Income x (thousand \$)	20	40	60	80	100	150
$P(x)$ (%)	21.7	45.4	63.0	75.8	84.0	94.0

(A) 84%

(B) 63%

(C) 21%

(D) There is not enough information.

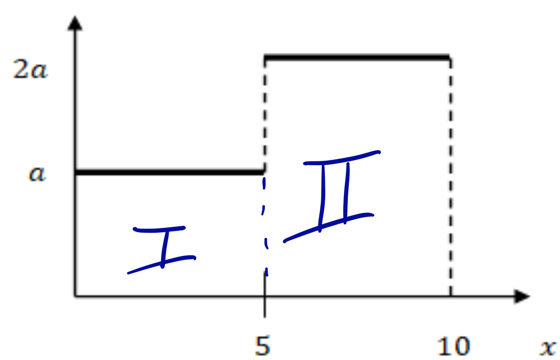
$P(X \le 60K) = 63\%$

$P(X \le 100K) = 84\%$

$\Rightarrow P(60K \le X \le 100K) = (84-63)\%$

$= \boxed{21\%}$

6.) For what value of a does the graph represent a PDF?



(A) 1/15

(B) 1/5

(C) 1/20

(D) 1/10

$1 = \text{area} = \text{I} + \text{II}$

$= 5 \cdot a + 5 \cdot 2a = 15a$

$\Rightarrow a = 1/15$

7.) The annual rainfall in Massachusetts is a random variable with probability density function defined by $\frac{1}{150}(2x+5)$ on $0 \le x \le 10$. Find the mean annual rainfall.

(A) 5.0

(B) 3.194

(C) 1.0

(D) 6.111

$$\mu = \int_{-\infty}^{\infty} x\, p(x)\, dx$$

For us, $\boxed{\mu = \int_0^{10} x \frac{1}{150}(2x+5)\,dx}$ or calculator ≈ 6.111

$$= \frac{1}{150}\int_0^{10} 2x^2+5x\,dx = \frac{1}{150}\left[\frac{2}{3}x^3+\frac{5}{2}x^2\Big|_0^{10}\right]$$

$$= \frac{1}{150}\left[\left(\frac{2}{3}10^3+\frac{5}{2}10^2\right)-(0+0)\right] = \frac{1}{150}\left[\frac{2000}{3}+250\right]$$

$$= \frac{1}{150}\left[\frac{2000+750}{3}\right] = \frac{1}{3}\,\frac{2750}{150} = \frac{1}{3}\,\frac{55}{3} = \frac{55}{9} = 6.1111\ldots$$

8.) Wait times at a traffic light are exponentially distributed with probability density function $p(t) = 0.07e^{-0.07t}$, where t is measured in minutes $0 \le t < \infty$. Find the median wait time at the traffic light.

(A) 9.9 minutes

(B) 7 minutes

(C) 47.9 minutes

(D) 35 minutes

$$p(t) = Ke^{-Kt}$$

(sometimes we use λ instead of K)

$$\Rightarrow K = 0.07$$

Exponential Distribution

mean $\mu = \frac{1}{K}$

median $T = \frac{\ln 2}{K}$

$$\Rightarrow T = \frac{\ln 2}{0.07}$$

$$\boxed{\approx 9.9021}$$

9.) Given that X is a random variable on $12 \leq x \leq 20$ with a CDF of

$$\underline{P}(x) = \frac{1}{8}x - \frac{3}{2}$$

Find the mean.

$$p(x) = \underline{P}'(x) = \frac{1}{8}$$

PDF (CDF)'

(A) 16

(B) 8

(C) 4

(D) 15.5

$$\Rightarrow \mu = \int_{12}^{20} x\, p(x)\, dx = \int_{12}^{20} \frac{1}{8} x\, dx$$

$$= \frac{1}{16} x^2 \Big|_{12}^{20} = \frac{1}{16}\left(20^2 - 12^2\right) = \frac{1}{16}(400 - 144)$$

$$= \frac{256}{16} = \frac{2^8}{2^4} = 2^4 = \boxed{16}$$

10.) The clotting time of blood is normally distributed with mean of 7.35 seconds and standard deviation of 0.35 seconds. What is the probability that the blood clotting time will be less than 7 seconds?

(A) 0.1429

(B) 0.8414

(C) 0.3362

(D) 0.1587

$\mu = 7.35 \quad \sigma = 0.35$

$$P(x < 7) = \int_{-\infty}^{7} \frac{1}{0.35\sqrt{2\pi}} e^{\frac{-(x-7.35)^2}{2(0.35)^2}} dx$$

on a calculator we can use a # that is a number of σ's to the left.

$$\approx \int_{3}^{7} \frac{1}{0.35\sqrt{2\pi}} e^{\frac{-(x-7.35)^2}{2(0.35)^2}} dx$$

$$\approx 0.158655$$

11.) Given the cumulative distribution function below, which of the following would be used to calculate the median?

$$\text{CDF } P(x) = \frac{1}{6}x^2 - \frac{1}{6}x - 1 \quad \text{and} \quad \boxed{\begin{aligned} p(x) &= P'(x) \\ &= 1/3x - 1/6 \end{aligned}}$$

$$3 \le x \le 4$$

(A) $\int_3^4 \left(\frac{1}{3}x - \frac{1}{6}\right) dx = 1$

(B) $\int_3^T \left(\frac{1}{6}x^2 - \frac{1}{6}x - 1\right) dx = 0.5$

(C) $\int_3^4 \left(\frac{1}{6}x^2 - \frac{1}{6}x - 1\right) dx = 1$

(D) $\int_3^T \left(\frac{1}{3}x - \frac{1}{6}\right) dx = 0.5$

median T is defined as

$$\int_{-\infty}^{T} p(x)\, dx = 1/2$$

PDF

or

$$\int_3^T p(x)\, dx = 1/2 \quad \text{or} \quad \boxed{\int_3^T \frac{1}{3}x - \frac{1}{6}\, dx = \frac{1}{2}}$$

12.) Wait times are the bank are exponentially distributed with PDF of $p(t) = 0.04e^{-0.04t}$ on $0 \le t < \infty$. Find the CDF.

(A) $P(x) = -e^{-0.04x}$

(B) $P(x) = -e^{-0.04x} + 1$

(C) $P(x) = -e^{-0.04x} - 1$

(D) $P(x) = 1$

$$P(x) = \int_{-\infty}^{x} p(t)\, dt$$

$$= \int_0^x p(t)\, dt \quad \text{since } p(t) = 0 \text{ for } t < 0$$

$$= \int_0^x 0.04 e^{-0.04t}\, dt = 0.04 \cdot \frac{1}{-0.04} e^{-0.04t} \Big|_0^x$$

$$= -\left(e^{-0.04 \cdot x} - e^{-0.04 \cdot 0}\right) = \boxed{1 - e^{-0.04x}}$$

13.) Suppose that $p(x)$ is a probability density function and $P(x)$ is a cumulative distribution function for the same distribution. Which of the following represents the fraction of the population between $x = 3$ and $x = 6$?

I. $\int_3^6 p(x)dx$ II. $p(6) - p(3)$ III. $\int_3^6 P(x)dx$ IV. $P(6) - P(3)$

(A) I and II

(B) II and III

(C) II and IV

(D) I and IV

$$P(3 \le x \le 6) = \boxed{\int_3^6 p(x)\,dx}$$

$$\overset{FTC}{=} P(x)\Big|_3^6$$

$$= \boxed{P(6) - P(3)}$$

14.) The temperature adjusted for wind-chill is a temperature which tells you how cold it feels, as a result of the combination of wind and temperature. If the temperature is 10°F and the wind speed is 20 mph, how cold does it feel?

Temperature (°F)

Wind Speed (mph)	35	30	25	20	15	10	5	0
5	31	25	19	13	7	1	−5	−11
10	27	21	15	9	3	−4	−10	−16
15	25	19	13	6	0	−7	−13	−19
20	24	17	11	4	−2	−9	−15	−22
25	23	16	9	3	−4	−11	−17	−24

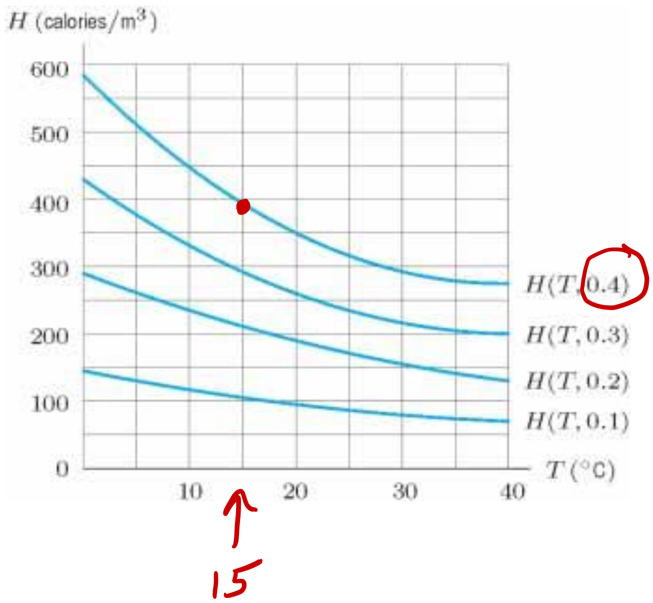
(A) −2

(B) 0

(C) − 9

(D) 3

15.) An airport can be cleared of fog by heating the air. The amount of heat required, $H(T,w)$ (in calories per cubic meter of fog), depends on the temperature of the air, T $(in\ °C)$, and the wetness of the fog, w. What is the best estimate $H(15,0.4)$?



(A) 95

(B) 400

(C) 150

(D) 200

16.) Suppose that the monthly mortgage payment M of a home (in dollars) is a function of P, its original price of the home (in dollars), and the interest rate r. So, $M = f(P,r)$. Which of the following is true?

(A) $\frac{\partial M}{\partial P}$ is positive and $\frac{\partial M}{\partial r}$ is positive.

(B) $\frac{\partial M}{\partial P}$ is positive and $\frac{\partial M}{\partial r}$ is negative.

(C) $\frac{\partial M}{\partial P}$ is negative and $\frac{\partial M}{\partial r}$ is negative.

(D) $\frac{\partial M}{\partial P}$ is negative and $\frac{\partial M}{\partial r}$ is positive.

$\frac{\partial M}{\partial P} > 0$ since if P increases then so does M

$\frac{\partial M}{\partial r} > 0$ since if the interest rate increases then so does M.

$F(8.5, 12) = F(8+\frac{1}{2}, 10+2) \approx F(8,10) + F_x(8,10)\cdot\frac{1}{2} + F_y(8,10)\cdot 2$

$= 520 + F_x(8,10)\cdot\frac{1}{2} + F_y(8,10)\cdot 2$

? ?

17.) The table below represents $f(x, y)$.

$F_x(8,10) \approx \frac{F(10,10) - F(8,10)}{10-8}$

$= \frac{575-520}{2}$

$= \frac{55}{2} = \boxed{27.5}$

so it must be A

$y \backslash x$	0	2	4	6	8	10
0	500	510	525	560	590	640
5	440	450	470	500	540	610
10	410	420	445	480	$\boxed{520}$	575
15	390	405	430	460	490	525
20	375	385	410	435	475	500

Which of the following would be used to best estimate $f(8.5,12)$? Use only positive values for h (only positive values for Δx and Δy) to estimate the derivatives.

$F_y(8,10) \approx \frac{F(8,15) - F(8,10)}{15-10}$

$= \frac{490-520}{5}$

$= \frac{-30}{5} = \boxed{-6}$

(A) $f(8.5,12) \approx 520 + 27.5(8.5 - 8) - 6(\underbrace{12 - 10}_{2})$ ½

(B) $f(8.5,12) \approx 490 + 17.5(8.5 - 8) - 3(12 - 15)$

(C) $f(8.5,12) \approx 480 + 20(8.5 - 8) - 4(12 - 10)$

(D) $f(8.5,12) \approx 445 + 17.5(8.5 - 4) - 3(12 - 10)$

18.) A two-variable function which gives an airline's revenue, $R = f(x, y) = 350x + 200y$, as a function of the number of full price tickets, x, and the number of discount tickets, y, sold. Find the rate of change of revenue, R, as x increases with y fixed at $y = 10$.

(A) 550

(B) $350x$

(C) 200

(D) 350

$R(x) = F(x, 10)$

$= 350x + 2000$

$\Rightarrow R'(x) = 350$

19.) Find $f_y(9, -3)$ when $f(x, y) = 7xy - 9y^2$.

(A) 153

(B) 117

(C) -270

(D) 99

$F_y = 7x - 18y$

$F_y(9, -3) = 7 \cdot 9 - 18(-3)$

$= 63 + 54$

$= 117$

20.) Suppose $f(x, y) = xe^y + y\cos(x)$ Find f_x.

(A) $f_x = xe^y - \cos(x)$

(B) $f_x = e^y - \sin(x)$

(C) $f_x = xe^y + \cos(x)$

(D) $f_x = e^y - y\sin(x)$

$F_x = e^y - y\sin(x)$

21.) Let $f(x, y) = 7x + 6x^2y^2 - 5y^2$. Find $\frac{\partial f}{\partial y}$.

$$F_y = 0 + 6x^2 \cdot 2y - 5 \cdot 2 \cdot y$$

$$= 12x^2y - 10y$$

(A) $7 + 12x^2y$

(B) $12xy - 10$

(C) $12x^2y - 10y$

(D) $12xy - 10y$

22.) Given $f(x, y) = x\ln(2x + 4y)$, find $f_x(x, y)$.

(A) $\frac{2x}{2x+4y} + \ln(2x + 4y)$

(B) $2\ln(2x + 4y)$

(C) $\ln(2x + 4y) + \frac{x}{2x+4y}$

(D) $\frac{2x}{2x+4y}$

$$F_x = (x \ln(2x+4y))_x$$

product rule

$$= x(\ln(2x+4y))_x + \ln(2x+4y)(x)_x$$

$$= x \cdot \frac{1}{2x+4y} \cdot 2 + \ln(2x+4y) \cdot 1$$

$$= \frac{2x}{2x+4y} + \ln(2x+4y)$$

23.) Let $f(x,y) = x^4y^6 - \sqrt{2}x^7y^8 + 17x - y$. Find $f_{yy}(x,y)$.

(A) $30x^4y^4 - 56\sqrt{2}x^7y^7$

(B) $30x^4y^5 - 56\sqrt{2}x^7y^7$

(C) $30x^4y^4 - 56\sqrt{2}x^7y^6$

(D) $4x^4y^4 - 6\sqrt{2}\,x^7y^6$

$$F_y = x^4 \cdot 6y^5 - \sqrt{2}x^7 \cdot 8y^7 + 0 - 1$$
$$= 6x^4y^5 - 8\sqrt{2}x^7y^7 - 1$$
$$F_{yy} = (F_y)_y$$
$$= 30x^4y^4 - 56\sqrt{2}x^7y^6$$

24.) Let $f(x,y) = \frac{y}{x+y}$. Find $\frac{\partial^2 f}{\partial y \partial x}$.

(A) $\frac{x-y}{(x+y)^3}$

(B) $\frac{y-x}{(x+y)^3}$

(C) $\frac{2y}{(x+y)^6}$

(D) $\frac{x}{(x+y)^2}$

$$F(x,y) = y.(x+y)^{-1}$$
$$f_x = y \cdot -1\,(x+y)^{-2} \cdot (x)_x$$
$$= \frac{-y}{(x+y)^2}$$

$$F_{xy} = (F_x)_y = -\left(\frac{y}{(x+y)^2}\right)_y$$
$$= -\left[\frac{(x+y)^2 \cdot 1 - y \cdot 2(x+y)^1}{(x+y)^4}\right]$$
$$= \frac{(x+y)\left[2y - (x+y)\right]}{(x+y)^4}$$
$$= \boxed{\frac{y-x}{(x+y)^3}}$$

25.) Given a continuous function $f(x, y)$ whose first and second partial derivatives all exist and are continuous, which of the following is always true?

(A) $f_{xx} = f_{yy}$

(B) $f_{xy} = -f_{yx}$

(C) $f_{xx} < 0$ and $f_{yy} < 0$

(D) $f_{xy} = f_{yx}$