Math 128 **Exam #2** **Fall 2015** **SPECIAL CODE: 111901**

Name _____

Signature: _____

~~~~~~

Academic Honesty Statement: By signing my name above, I acknowledge that I understand each of the following behaviors:

> *using a calculator on a cell phone (or any other communication technology); referring to a piece of paper or object with helpful information on it (cheat sheet, crib sheet, bill of a baseball cap, etc...); looking at a test or answer sheet that is not my own; allowing another student to look at my test or answer sheet; communicating with other students (verbally or nonverbally); taking the test for another student; taking my bubble sheet of answers with me when I've finished; talking while waiting to hand in my test materials to the proctors*

to be a form of academic dishonesty (cheating). I am also pledging not to engage in any of these behaviors. I understand that if I do engage in these behaviors, the consequences will be failure of the exam and a formal charge of academic dishonesty to the Ombuds Office.

~~~~~~

Please shut off all cell phones, ear phones, computers, beepers, etc…

Please put everything away except a #2 pencil and a calculator that is NOT your cell phone. You may write on the test. There are twenty five multiple choice questions and each question is worth four points.

1. On the bubble sheet, where it says "Name," please print your last name, leave a space, and then print your first name in the rectangles. Then fill in the bubbles underneath.

2. On the bubble sheet, where it says "Identification Number," please write your entire Student ID number in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in your ID # correctly.

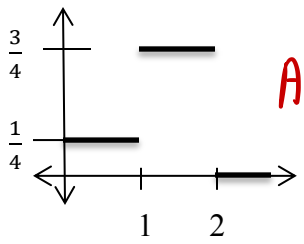
4. On the bubble sheet, where it says "Special Codes," please write the numbers: 111901 in the rectangles and fill in the bubbles underneath. Please double check to make sure you bubbled in the special code correctly.

5. Lastly, on the bubble sheet, in the margin above your name, please neatly print "Exam #2 Fall 2015", your section number (01 or 02), and sign your name.

Please make sure you bubble in your answers carefully on the bubble sheet and ***circle your answers on your test booklet****.*

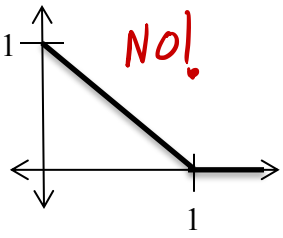
1.) Which of the following graphs does not represent a PDF? All are ≥ 0

(A)



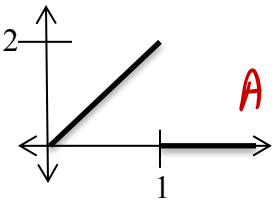
$A = (2-1)\left(\frac{3}{4}-\frac{1}{4}\right) = 1$

(B)



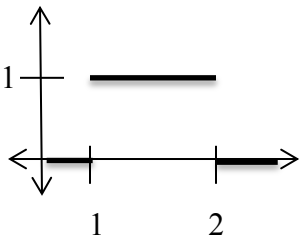
No! $A = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2} \neq 1$

(C)



$A = \frac{1}{2}(1-0)(2-0) = 1$

(D)



$A = (2-1) \cdot 1 = 1$

2.) Which of the following is true if $F(x)$ represents a CDF and $p(x)$ represents a PDF?

(A) $\int_{-\infty}^{\infty} F(x)dx = 1$

(B) $\int_{-\infty}^{x} F(x)dx = p(x)$

(C) $\int_{-\infty}^{\infty} xp(x)dx = F(x)$

(D) As $x \to \infty, F(x) \to 1$

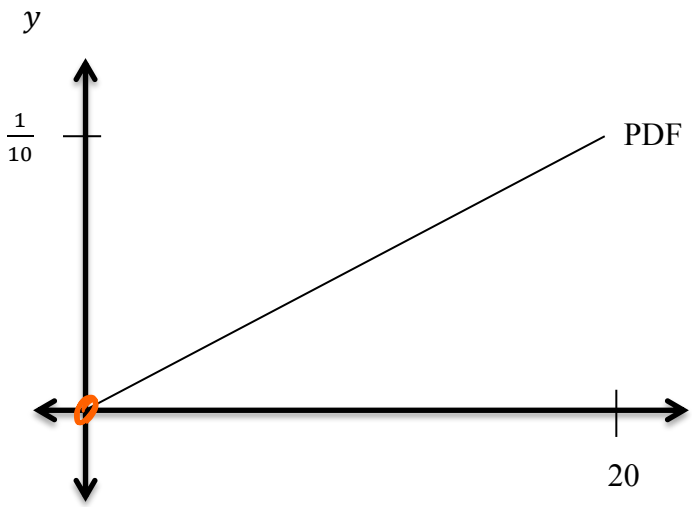
$\lim_{x \to \infty} F(x) = \lim_{x \to \infty} \int_{-\infty}^{x} p(t)dt$

$F(x)$, the CDF

$=$ (neglecting some ∞/lim details)

$= \int_{-\infty}^{\infty} p(t)dt = 1$

3.) Find the CDF given the graph of the PDF below:



$P(x) = mx + b$

$m = \frac{\Delta y}{\Delta x} = \frac{1/10}{20} = 1/200\,x$

(A) $\begin{cases} 0 & x < 0 \\ 0.005x + 0.1 & 0 \le x \le 20 \\ 1 & x > 20 \end{cases}$

(B) $\begin{cases} 0 & x < 0 \\ 0.005x & 0 \le x \le 20 \\ 1 & x > 20 \end{cases}$

(C) $\begin{cases} 0 & x < 0 \\ 0.0025x^2 + 0.1x & 0 \le x \le 20 \\ 1 & x > 20 \end{cases}$

(D) $\begin{cases} 0 & x < 0 \\ 0.0025x^2 & 0 \le x \le 20 \\ 1 & x > 20 \end{cases}$

$P(x) = \int_{-\infty}^{x} p(t)\,dt = \int_{0}^{x} \frac{1}{200}\,t\,dt$

$= \frac{1}{400}\left(t^2 \Big|_0^x\right)$

$= \frac{x^2 - 0^2}{400} = \frac{1}{400}x^2$

4.) The heights of humans are normally distributed with a mean height of 65 inches and a standard deviation of 5 inches. What is the probability that a randomly selected person is more than 67 inches?

$\mu = 65$

$\sigma = 5$

(A) 34.46%
(B) 27.42%
(C) 21.19%
(D) 15.87%

$Pr(x \ge 67) = \int_{67}^{\infty} p(x)\,dx$

$= \int_{67}^{\infty} \frac{1}{5\sqrt{2\pi}}\, e^{-\frac{(x-65)^2}{2(5)^2}}\,dx$

Since $e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ goes to 0 rapidly, "replace" ∞ with say, $67 + 46 = 97$

$\approx \int_{67}^{97} p(x)\,dx \approx$ **0.344578**

ASSUMING that the highest possible grade is 100

5.) Exam scores in Math 128 are normally distributed with a mean of 71 with a standard deviation of 6 points. Which of the following would not represent the fraction of people who scored higher than 80?

(A) $\int_{80}^{100} \frac{1}{6\sqrt{2\pi}} e^{-\frac{(x-71)^2}{2(6)^2}} dx$ $= P(80 \le X \le 100)$ $= P(X<80)$ AND $P(X>100)$

(B) $1 - \int_{80}^{100} \frac{1}{6\sqrt{2\pi}} e^{-\frac{(x-71)^2}{2(6)^2}} dx$ $= 1 - P(80 \le X \le 100) = P(80 \le X < \infty) = P(X \ge 80)$

(C) $\int_{80}^{\infty} \frac{1}{6\sqrt{2\pi}} e^{-\frac{(x-71)^2}{2(6)^2}} dx$

(D) $1 - \int_{0}^{80} \frac{1}{6\sqrt{2\pi}} e^{-\frac{(x-71)^2}{2(6)^2}} dx$ $= 1 - P(0 \le X \le 80) = P(X \ge 80)$

6.) Given the pdf $p(x) = -18x + 6$ on the interval $0 \le x \le \frac{1}{3}$, which of the following lists contains the median?

(A) 0.068; 0.20; 0.25; 0.569; 0.239; 0.5; 0.289

(B) 0.222; 0.098; 0.09; 0.146; 0.162; 0.427

(C) 0; 0.854; 0.073; 0.166; 0.143; 0.33; 0.675

(D) 0.047; 0.308; 0.667; 0.75; 0.783; 0.983

$\frac{1}{2} = \int_{-\infty}^{T} p(x)\,dx = \int_{0}^{T} -18x + 6\,dx$

$= (-9x^2 + 6x)\Big|_0^T = -9T^2 + 6T$

$\Rightarrow -18T^2 + 12T - 1 = 0$

Quadratic Formula $\Rightarrow T \approx 0.569$ or $T \approx 0.0976$ must be $\le 1/3$

7.) Given the following CDF $F(x) = \frac{1}{100}x^2$ on the interval $0 \le x \le 10$. Which of the following values represents the mean?

(A) 25
(B) 3.33
(C) 6.67
(D) 41.67

$\Rightarrow F'(x) = \frac{1}{50}x = PDF$

$\mu = \int_0^{10} x \cdot \left(\frac{1}{50}x\right) dx = \frac{1}{150}x^3\Big|_0^{10} = \frac{10^3 - 0^3}{150}$

$= \frac{1000}{150} = \frac{20}{3} = 6\frac{2}{3}$

8.) Given the exponentially distributed PDF $p(x) = 0.458e^{-0.458x}$, which of the following represents the mean?

$= \lambda e^{-\lambda x}$ so $\lambda = 0.458$

(A) 0.458
(B) −.0458
(C) 1.513
(D) 2.183

$\Rightarrow \mu = \frac{1}{\lambda} = \frac{1}{0.458} \approx 2.183$

9.) If X is a random variable distributed uniformly on $1 \le x \le 7$, find the CDF.

(A) $F(x) = \frac{1}{6}$

(B) $F(x) = \frac{1}{6}x$

(C) $F(x) = \frac{1}{6}x - \frac{1}{6}$

(D) $F(x) = \frac{1}{7}x - \frac{1}{7}$

$$pdf = \begin{cases} 0 & x < 1 \\ \frac{1}{7-1} & 1 \le x \le 7 \\ 0 & x > 7 \end{cases} = \begin{cases} 0 & x < 1 \\ 1/6 & 1 \le x \le 7 \\ 0 & x > 7 \end{cases}$$

$$cdf = F(x) = \int_{-\infty}^{x} p(t)\,dt = \int_{1}^{x} \frac{1}{6}\,dt = \frac{1}{6}t\Big|_1^x = \frac{1}{6}x - \frac{1}{6}$$

10.) The time needed to complete an oil change on a car has an exponential distribution with a mean of 31 minutes. What is the probability that the next oil change will take 7 minutes or less?

(A) 0.1617
(B) 0.2426
(C) 0.2023
(D) 0.1516

$$p(x) = \lambda e^{-\lambda x} \Rightarrow \mu = 1/\lambda = 31 \Rightarrow \lambda = \frac{1}{31}$$

$$\int_0^7 p(x)\,dx = \int_0^7 \frac{1}{31} e^{-1/31 x}\,dx \approx 0.20213 \quad \text{calculator}$$

11.) The wait times at the ER are exponentially distributed with a mean wait time of 40 minutes. What is the median wait time at the ER?

$$p(x) = \lambda e^{-\lambda x}$$

(A) 40 minutes
(B) 28 minutes
(C) 20 minutes
(D) 44 minutes

$$\mu = 40 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{40}, \quad T = \frac{\ln 2}{1/40} = 40 \ln 2 \approx 27.72588$$

12.) Assume that X is a random variable with a normal distribution. If the mean is 17 with a standard deviation of 6, find $P(2 \le X \le 11)$.

$\mu = 17$
$6 = 6$

(A) 0.1524
(B) 0.1818
(C) 0.8476
(D) 0.1986

$$P(2 \le X \le 11) = \int_2^{11} \frac{1}{6\sqrt{2\pi}} e^{\frac{-(x-17)^2}{2 \cdot 6^2}}\,dx \approx 0.152446$$

13.) Assume T is a random variable representing time, with an exponential distribution and a mean of 21. Find $P(T \le 21)$.

(A) 0.50
(B) 0.0333
(C) 0.6319
(D) 0.0476

$$P(T \le 21) = \int_0^{21} \lambda e^{-\lambda t}\,dt \qquad \mu = 21 = \frac{1}{\lambda}$$

$$\Rightarrow \lambda = \frac{1}{21}$$

$$= \int_0^{21} \frac{1}{21} e^{-t/\mu}\,dt$$

$$\approx 0.63212$$

14.) Assume X is a random variable with a uniform distribution on $3 \le x \le 12$. Find $P(3 \le X \le 5)$.

$P(x) = \begin{cases} 0 & x<3 \\ 1/9 & 3 \le x \le 12 \\ 0 & x>12 \end{cases}$

$P(3 \le X \le 5) = \int_3^5 P(x)\,dx = \int_3^5 \frac{1}{9}\,dx$

$= \frac{5-3}{9} = 2/9 = 0.2222\ldots$

(A) 0.2222
(B) 0.50
(C) 0.1667
(D) 0.60

15.) Assume X is a normally distributed random variable with $\mu = 48$ and $\sigma = 14$. What is the median?

P(x) Normal Distribution

$\Rightarrow \mu = T$ so $T = \mu = 48$

(A) 0.5
(B) 14
(C) 48
(D) 62

16.) The following table represents the function $f(x, y)$. Find $f_x(9,5)$.

		x			
		1	**5**	**9**	**13**
y	**1**	6	10	13	23
	3	12	20	28	33
	5	18	19	31	43

$f_x(9,5) \approx \frac{f(13,5) - f(9,5)}{13-9} = \frac{43-31}{4}$

$= 12/4 = \boxed{3}$

(A) 3
(B) 4
(C) 9
(D) 12

17.) The profit that a company makes, measured in thousands of dollars, is a function of revenue (in dollars) and cost (in dollars), $P = f(R, C)$. Which of the following is true?

(A) $\frac{\partial f}{\partial C}$ is positive and $\frac{\partial f}{\partial R}$ is negative.

(B) $\frac{\partial f}{\partial C}$ is negative and $\frac{\partial f}{\partial R}$ is positive.

(C) Both $\frac{\partial f}{\partial C}$ and $\frac{\partial f}{\partial R}$ are negative.

(D) Both $\frac{\partial f}{\partial C}$ and $\frac{\partial f}{\partial R}$ are positive.

$\frac{\partial f}{\partial R} > 0$ since profit goes up if revenue goes up

$\frac{\partial f}{\partial C} < 0$ since profit goes down if cost goes up

18.) Given $f(x,y) = \ln\left(\frac{y^4}{x^4}\right)$, find $f_x(x,y)$.

$\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$

(A) $-\ln\left(\frac{4y^4}{x^5}\right)$

(B) $-\frac{4}{x}$

(C) $-4x$

(D) $\frac{4}{y}$

$f_x = \frac{1}{y^4/x^4} \cdot (y^4/x^4)_x = \frac{x^4}{y^4} \cdot -4x^{-5} \cdot y^4$

$= -\frac{4x^4y^4}{y^4x^5} = -4/x$

19.) Given $f(x,y) = \frac{e^{-y}}{x^2+y^2}$. Find $f_y(x,y)$

Quotient Rule

(A) $-\frac{2ye^{-y}}{(x^2+y^2)^2}$

(B) $\frac{2ye^{-y}}{(x^2+y^2)^2}$

(C) $-\frac{e^{-y}(x^2+y^2+2y)}{(x^2+y^2)^2}$

(D) $\frac{e^{-y}(x^2+y^2+2y)}{(x^2+y^2)^2}$

$f_y = \frac{(x^2+y^2)\cdot(e^{-y})_y - e^{-y}\cdot(x^2+y^2)_y}{(x^2+y^2)^2}$

$= \frac{-(x^2+y^2)e^{-y} - 2ye^{-y}}{(x^2+y^2)^2} = -\frac{e^{-y}(x^2+y^2+2y)}{(x^2+y^2)^2}$

20.) Given $f(x,y) = 2x + 5x^3y^3 - 4y^2$. Find $\frac{\partial f}{\partial x}$.

(A) $15x^3y^2 - 8y$

(B) $2 + 45x^2y^2 - 8y$

(C) $2 + 15x^3y^2$

(D) $2 + 15x^2y^3$

$f_x = 2 + 15x^2y^3$

21.) Given $f(x,y) = \cos(xy^2)$, find $f_{yy}(x,y)$.

(A) $-y^4\cos(xy^2)$

(B) $-4x^2y^2\cos(xy^2) - 2x\sin(xy^2)$

(C) $-2xy^3\cos(xy^2) - 2y\sin(xy^2)$

(D) $-4xy^3\cos(xy^2)$

$f_y = -\sin(xy^2)\cdot(xy^2)_y$

$= -2xy\sin(xy^2)$

$f_{yy} = (f_y)_y = -2x(y\sin(xy^2))_y$

$= -2x(y\cdot\cos(xy^2)\cdot 2xy + \sin(xy^2))$

$= -4x^2y^2\cos(xy^2) - 2x\sin(xy^2)$

22.) Given $f(x,y) = xy^2 + y^2e^{x^2} + 5y^4$, find $f_{xy}(x,y)$.

(A) $2y + 4x^2ye^{x^2}$

(B) $2y + 4xye^{x^2}$

(C) $2x + 60y^3$

(D) $2y + 2ye^{x^2}$

$F_x = y^2 + 2xy^2e^{x^2}$

$F_{xy} = (F_x)_y = 2y + 4xye^{x^2}$

23.) Find all critical value(s) of $f(x,y) = x^2 + 8x + y^2 + 16y - 10$

(A) $(-4,-8)$

(B) $(4,-8)$

(C) $(4,8)$

(D) $(-4,8)$

$F_x = 2x + 8 = 0 \Rightarrow x = -8/2 = -4$

$F_y = 2y + 16 = 0 \Rightarrow y = -16/2 = -8$

$(-4,-8)$

24.) Find all critical value(s) of $f(x,y) = x^3 + y^3 - 300x - 243y - 1$

(A) $(10,9)$

(B) $(-10,-9)$

(C) $(10,9),(-10,-9)$

(D) $(10,9),(-10,-9),(-10,9),(10,-9)$

$F_x = 3x^2 - 300 = 0 \Rightarrow x^2 = 100 \Rightarrow x = \pm 10$

$F_y = 3y^2 - 243 = 0 \Rightarrow y^2 = 81 \Rightarrow y = \pm 9$

25.) Find all critical value(s) of $f(x,y) = 2x^3 - 3xy + 2y^3$.

(A) $(0,0),\left(\frac{1}{2},\frac{1}{2}\right)$

(B) $(-3,6),(6,-3),(0,0),(1,8)$

(C) $(0,0),\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{8},\frac{1}{32}\right),\left(\frac{1}{8},\frac{1}{8}\right)$

(D) $\left(\frac{1}{2},\frac{1}{2}\right)$

$F_x = 6x^2 - 3y = 0 \Rightarrow \boxed{y = 2x^2}$

$F_y = 6y^2 - 3x = 0 \Rightarrow 6(2x^2)^2 - 3x = 0$

$\Rightarrow 24x^4 - 3x = 3x(8x^3 - 1) = 0$

$\Rightarrow x = 0$ or $x^3 = 1/8 \Rightarrow x = 1/2$

$x = 0$: $y = 2 \cdot 0^2 = 0$ $(0,0)$

$x = \frac{1}{2}$: $y = 2 \cdot (\frac{1}{2})^2 = 1/2$ $(1/2, 1/2)$

This page is left blank for additional work.