Info:

8.3 due TODAY

9.1 11/5

9.2 11/7

9.3 11/9

9.4 11/9

Start Today

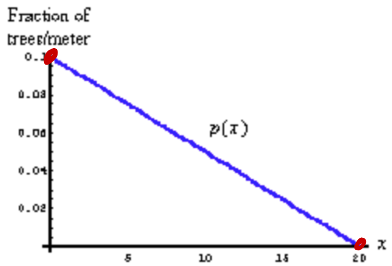
9.5A 11/11

Exam 2 Th 11/15

- makeup forms by 11/7
- practice exams and info posted ~~this weekend~~
- office hour 2:30 - 3:30 today

Chapter 8, Web Quiz, Question 36

The distribution of the heights, *x*, in meters, of trees is represented by the density function $p(x)$.



Find the mean height of the trees.

$(0, \frac{1}{10})$ $(20, 0)$

$$p(x) = mx + b$$
$$= mx + \frac{1}{10}$$
$$m = \frac{\Delta y}{\Delta x} = \frac{\frac{1}{10} - 0}{0 - 20} = \frac{-\frac{1}{10}}{20}$$
$$= -\frac{1}{200}$$

$$p(x) = \begin{cases} 0 & x < 0 \\ -\frac{1}{200}x + \frac{1}{10} & 0 \le x \le 20 \\ 0 & x > 20 \end{cases}$$

$$\text{mean} = \mu = \int_{-\infty}^{\infty} x\,p(x)\,dx = \int_0^{20} x\left(-\frac{1}{200}x + \frac{1}{10}\right)dx$$
$$= \int_0^{20} -\frac{1}{200}x^2 + \frac{1}{10}x\,dx = \left(-\frac{1}{200}\frac{x^3}{3} + \frac{1}{10}\frac{x^2}{2}\right)\Big|_0^{20}$$
$$= \left(-\frac{1}{600}x^3 + \frac{1}{20}x^2\right)\Big|_0^{20} = \left(-\frac{1}{600}20^3 + \frac{1}{20}\cdot 20^2\right) - (0+0)$$
$$= -\frac{20\cdot 400}{600} + 20 = -\frac{20\cdot 4}{6} + 20 = -\frac{20\cdot 2}{3} + 20 = -\frac{40}{3} + 20$$
$$= -\frac{40}{3} + \frac{60}{3} = \frac{20}{3} = 6\tfrac{2}{3}$$

$\boxed{\text{Wiley: } 6.67}$

Functions of 2 variables

$$z = f(x, y)$$

independent variables

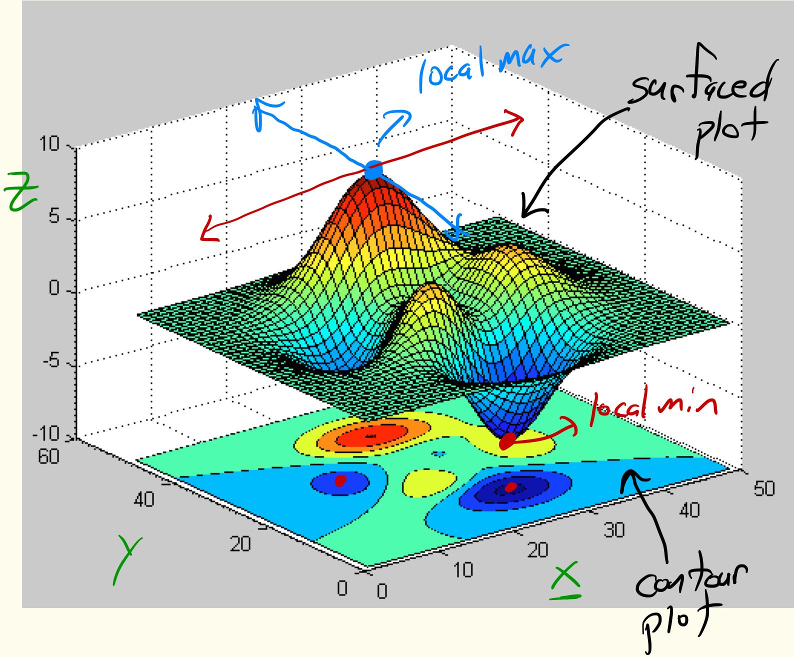
Where are the LOCAL/GLOBAL mins and maxs?

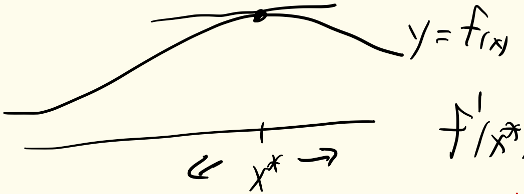
Slope red TANGENT line?

$$\frac{\partial f}{\partial x}$$

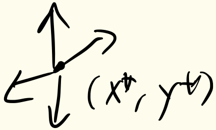
Slope blue TANGENT line?

$$\frac{\partial f}{\partial y}$$





$$f'(x^*) = 0$$



$+ + +\ 0\ - - -$

$-$

f' 1st Derivative Test

f'' 2nd Derivative Test

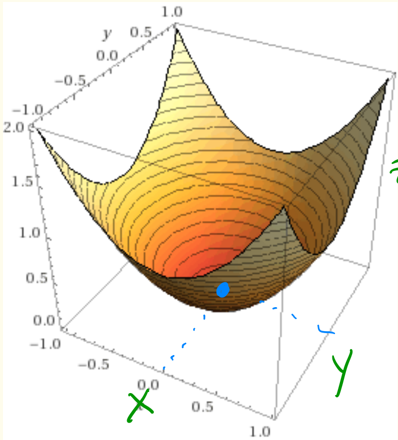
Ex: $z = f(x,y) = x^2 + y^2$

$$\frac{\partial f}{\partial x} = 2x + 0 = 2x$$

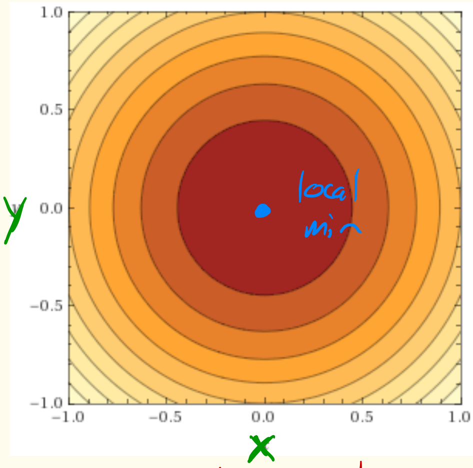
$$\frac{\partial f}{\partial y} = 0 + 2y = 2y$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \text{ at } \boxed{(0,0) \text{ critical pt.}}$$

local min



surface



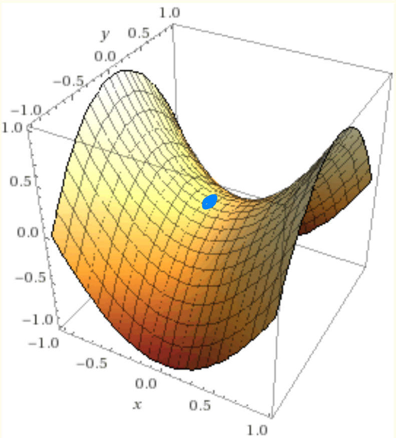
contour plot

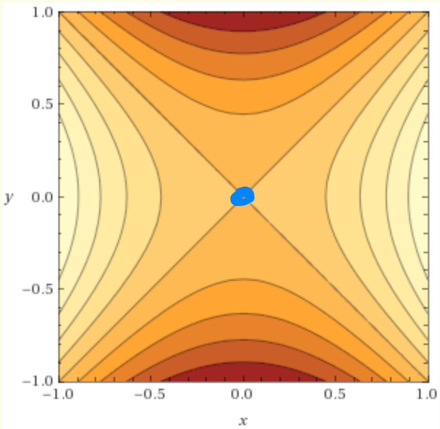
Ex: $z = f(x,y) = x^2 - y^2$

$$\frac{\partial f}{\partial x} = 2x - 0 = 2x$$

$$\frac{\partial f}{\partial y} = 0 - 2y = -2y$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \text{ at } \boxed{(0,0) \text{ crit pt}}$$

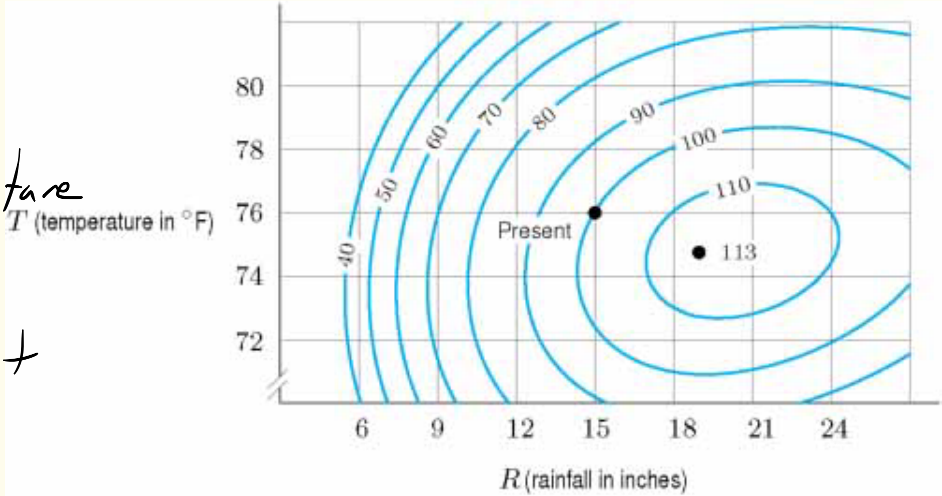




Corn Production

$$C = f(R, T)$$

\# bushels   rain   temperature



what is f at the point labeled Present?

$f(15, 76) = 100$

where is the max? at $\approx (18.25, 75)$

Roller Blading

$$C = f(w, s)$$

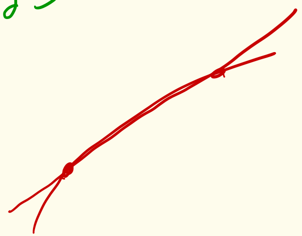
$$f(140, 9) = 6.7$$

Calories burned per minute

$w \backslash s$	8 mph	9 mph	10 mph	11 mph
120 lbs	4.2	5.8	7.4	8.9
140 lbs	5.1	6.7	8.3	9.9
160 lbs	6.1	7.7	9.2	10.8
180 lbs	7.0	8.6	10.2	11.7
200 lbs	7.9	9.5	11.1	12.6

$\frac{\partial f}{\partial w} > 0$, positive

$\frac{\partial f}{\partial s} > 0$, positive



Estimate $\frac{\partial f}{\partial w}(160, 8)$

$$\underbrace{\frac{\partial f}{\partial w}}_{\frac{\Delta f}{\Delta w}}(160,8) \approx \frac{f(180,8) - f(160,8)}{180-160}$$

$$= \frac{7.0 - 6.1}{20} = \frac{0.9}{20} = \boxed{0.045}$$

$$\underbrace{\frac{\partial f}{\partial s}}_{\frac{\Delta f}{\Delta s}}(160,8) \approx \frac{f(160,9) - f(160,8)}{9-8} = \frac{7.7 - 6.1}{1}$$

$$= \boxed{1.6}$$

Partial Derivatives

$z = f(x,y)$ $\qquad$ $g(x)$

$$\frac{\partial f}{\partial x} = f_x \qquad \frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$\frac{\partial f}{\partial y} = f_y \qquad \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

$$\underbrace{g'(x)}_{\frac{dg}{dx}} \qquad \underbrace{g''(x)}_{\frac{d^2 g}{dx^2}} \quad \ldots$$

mixed partials

$$\left.\begin{aligned} \frac{\partial^2 f}{\partial y \partial x} &= f_{xy} \\ \frac{\partial^2 f}{\partial x \partial y} &= f_{yx} \end{aligned}\right\} \text{For us these are always the same}$$

Compute $f_x, f_y, f_{xx}, f_{yy}, f_{xy}$ and f_{yx}

Ex: $f(x,y) = 3x^2 y$

$f_x = 6xy$

$f_{xx} = (f_x)_x = (6xy)_x = 6y$

$f_y = 3x^2$

$f_{yy} = (f_y)_y = (3x^2)_y = 0$

$$\boxed{f_{xy} = (f_x)_y = (6xy)_y = 6x \qquad f_{yx} = (f_y)_x = (3x^2)_x = 6x}$$

EQUAL

Ex: $f(x,y) = \sin(xy) + xe^y$

$f_x = \cos(xy)\cdot y + e^y$

$= y\cos(xy) + e^y$

$f_{xx} = (f_x)_x = (y\cos(xy) + e^y)_x$

$= y(-\sin(xy))\,y + 0$

$= -y^2\sin(xy)$

$f_y = \cos(xy)\cdot x + xe^y$

$= x\cos(xy) + xe^y$

$f_{yy} = (f_y)_y = (x\cos(xy) + xe^y)_y$

$= x(-\sin(\underline{xy}))\,x + xe^y$

$= -x^2\sin(xy) + xe^y$