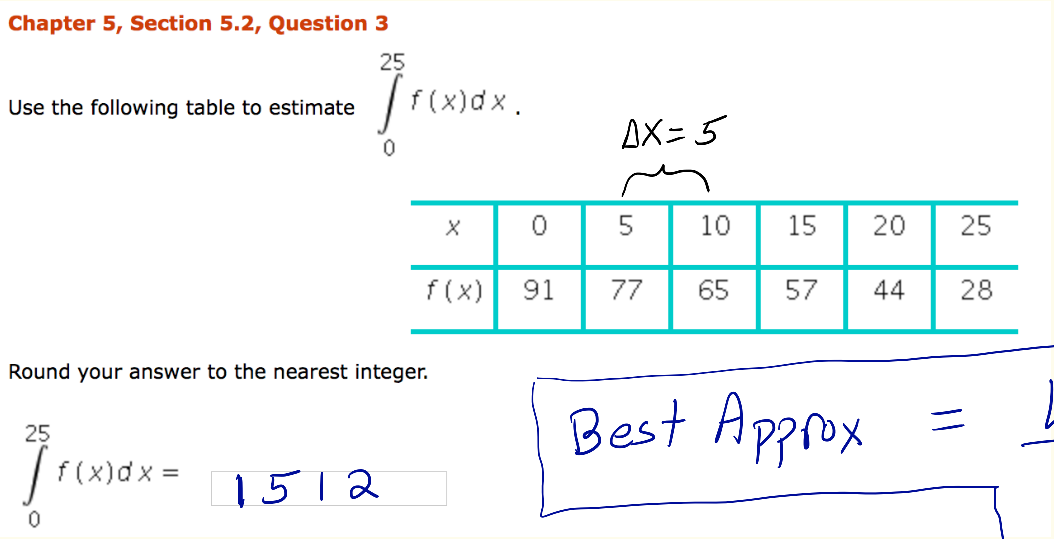


Chapter 5, Section 5.2, Question 3

Use the following table to estimate $\int_{0}^{25} f(x)\,dx$.

$\Delta x = 5$

x	0	5	10	15	20	25
$f(x)$	91	77	65	57	44	28

Round your answer to the nearest integer.

$\int_{0}^{25} f(x)\,dx =$ 1512

$$\boxed{\text{Best Approx} = \frac{LHS + RHS}{2}}$$

$$= \frac{1670 + 1355}{2}$$

$$= \frac{3025}{2}$$

$$= 1512.5$$

$$LHS = [f(0) + f(5) + f(10) + f(15) + f(20)]\cdot \Delta x$$

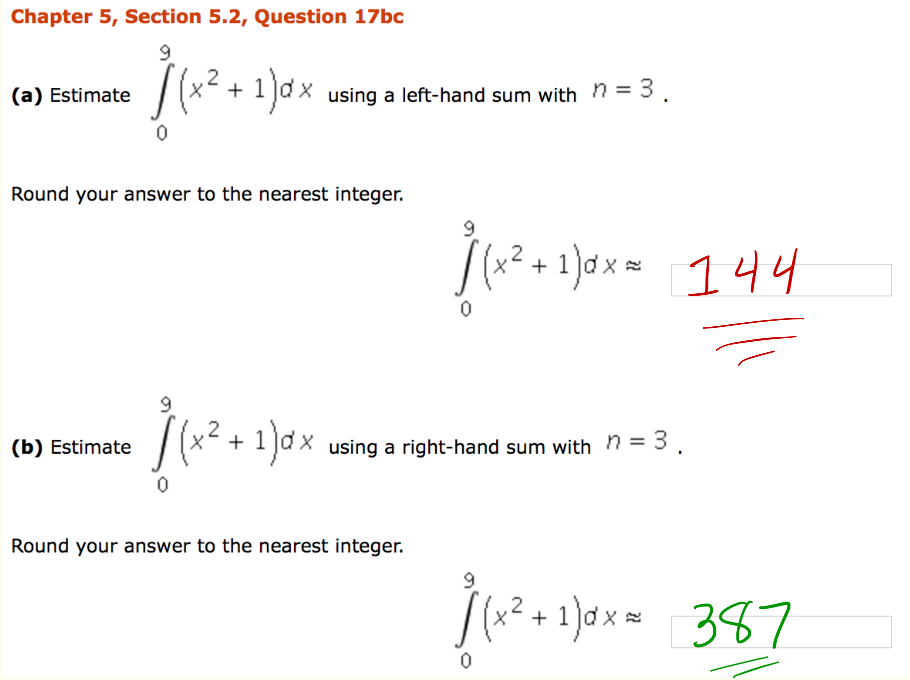
$$= [91 + 77 + 65 + 57 + 44]\cdot 5$$

$$= 334\cdot 5 = \boxed{1670}$$

$$RHS = [f(5) + f(10) + f(15) + f(20) + f(25)]\cdot \Delta x$$

$$= [77 + 65 + 57 + 44 + 28]\cdot 5$$

$$= 271\cdot 5 = \boxed{1355}$$



Chapter 5, Section 5.2, Question 17bc

(a) Estimate $\int_0^9 \left(x^2+1\right)dx$ using a left-hand sum with $n = 3$.

Round your answer to the nearest integer.

$$\int_0^9 \left(x^2+1\right)dx \approx \underline{\underline{144}}$$

(b) Estimate $\int_0^9 \left(x^2+1\right)dx$ using a right-hand sum with $n = 3$.

Round your answer to the nearest integer.

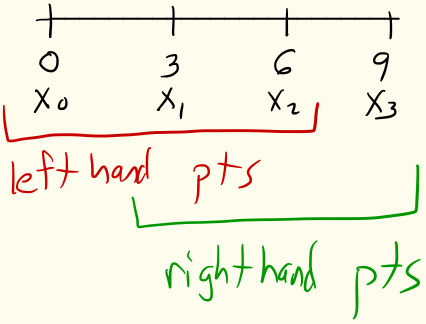
$$\int_0^9 \left(x^2+1\right)dx \approx \underline{\underline{387}}$$

$a = 0$

$b = 9$

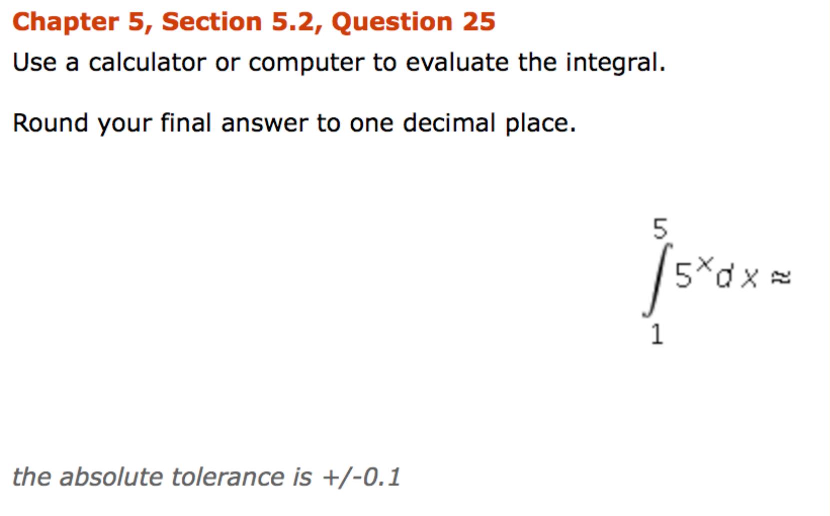
$n = 3$

$$\boxed{\underline{\underline{\Delta x}} = \frac{9-0}{3} = \underline{\underline{3}}}$$



$$\text{(a) } LHS = [f(0) + f(3) + f(6)] \cdot \Delta x$$
$$= [(0^2+1) + (3^2+1) + (6^2+1)] \cdot 3$$
$$= [1 + 10 + 37] \cdot 3 = 144$$

$$\text{(b) } RHS = [f(3) + f(6) + f(9)] \cdot 3$$
$$= [10 + 37 + 82] \cdot 3 = 387$$



Chapter 5, Section 5.2, Question 25

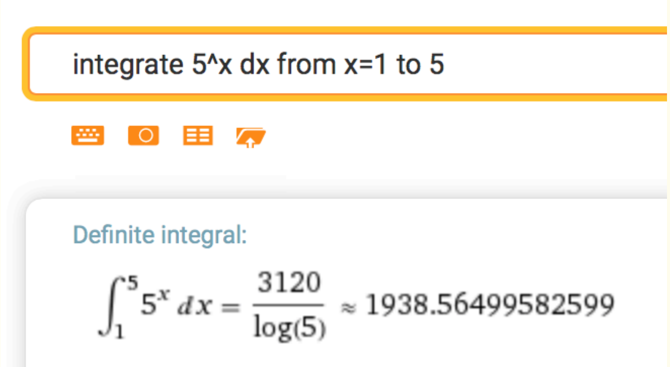
Use a calculator or computer to evaluate the integral.

Round your final answer to one decimal place.

$$\int_1^5 5^x dx \approx$$

the absolute tolerance is +/-0.1

$\underline{\underline{1938.6}}$



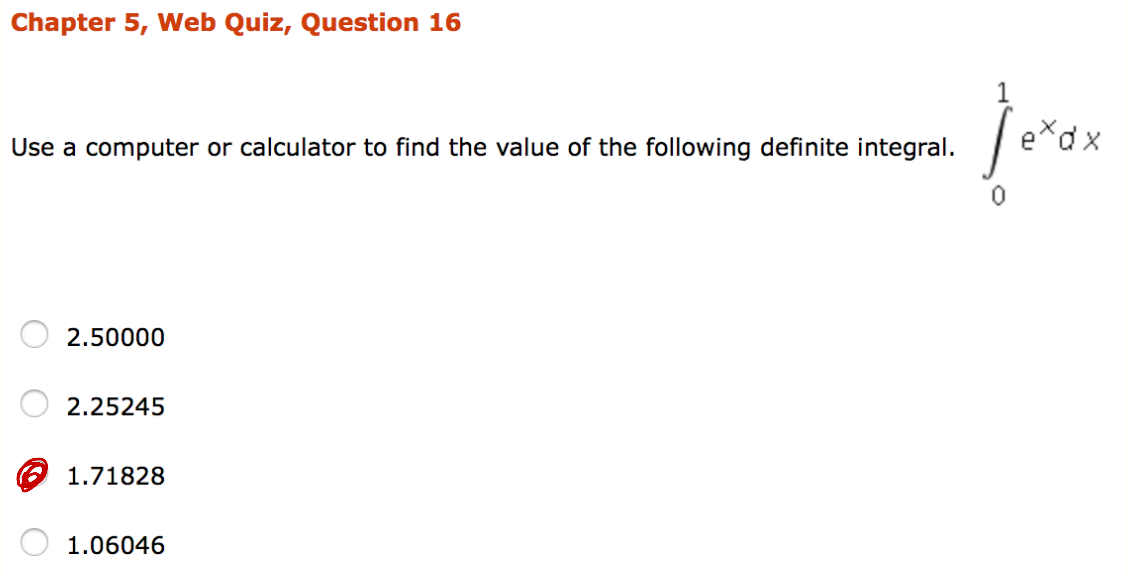
integrate 5^x dx from x=1 to 5

Definite integral:

$$\int_1^5 5^x\, dx = \frac{3120}{\log(5)} \approx 1938.56499582599$$

⇐ Wolfram Alpha

Make sure that you can compute this using <u>your</u> calculator.



Chapter 5, Web Quiz, Question 16

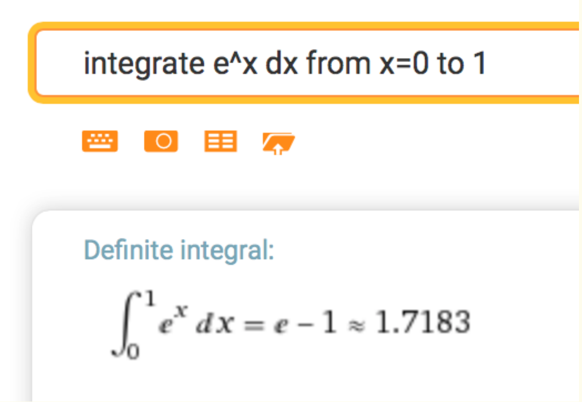
Use a computer or calculator to find the value of the following definite integral. $\int_0^1 e^x dx$

○ 2.50000

○ 2.25245

[✓ 1.71828]

○ 1.06046



integrate e^x dx from x=0 to 1

Definite integral:

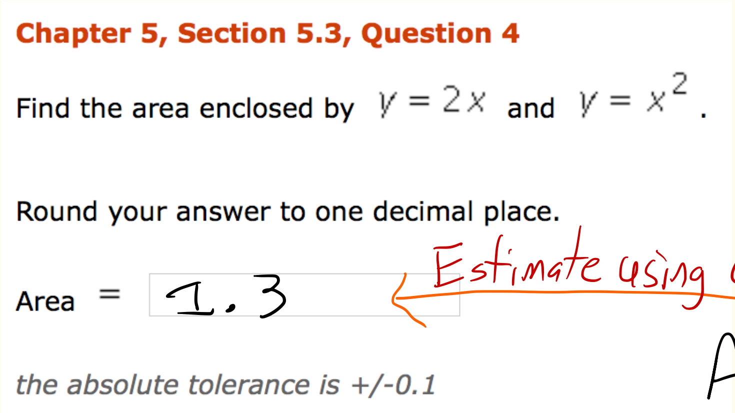
$$\int_0^1 e^x \, dx = e - 1 \approx 1.7183$$

Exact:

$$\int_0^1 e^x dx = e^x \Big|_0^1 = e^1 - e^0$$

$$= e - 1$$

since $\frac{d}{dx}(e^x) = e^x$ which we already know.



Chapter 5, Section 5.3, Question 4

Find the area enclosed by $y = 2x$ and $y = x^2$.

Round your answer to one decimal place.

Area = 1.3

the absolute tolerance is +/-0.1

Estimate using Calculator

Where do $2x$ and x^2 intersect?

$x^2 = 2x \Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0 \Rightarrow x=0$ or $x=2$

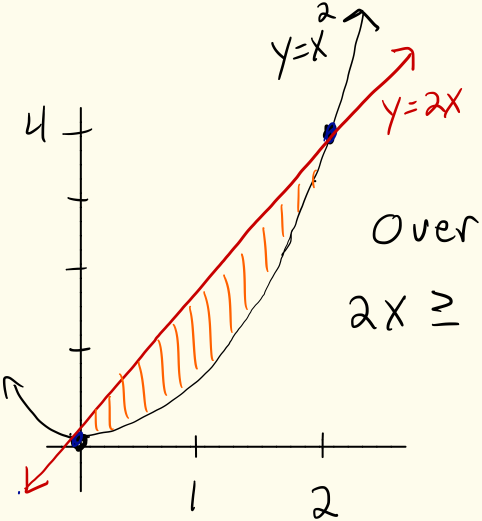
$$\text{Area} = \int_0^2 2x - x^2\,dx$$

EXACT

$$= \left(\frac{2x^2}{2} - \frac{x^3}{3}\right)\Big|_0^2$$

$$= \left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 = \left(4 - \frac{8}{3}\right) - (0-0)$$

$$= \frac{4}{3} = 1.33333\ldots$$

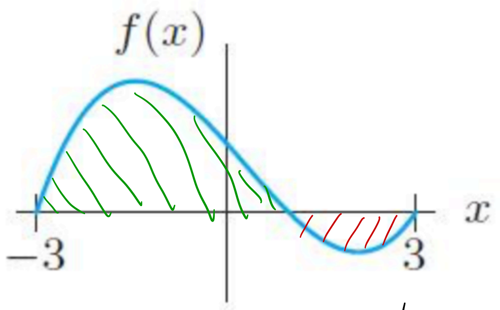


Over $[0,2]$

$2x \geq x^2$

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

Chapter 5, Section 5.3, Question 6



Definite Integral = # = a positive number since there is more + area than − .

Decide whether $\overbrace{\int_{-3}^{3} f(x)dx}^{\text{Definite Integral}}$ is positive, negative, or approximately zero.

○ Approximately zero

○ Negative

● Positive

5.3 #11

Using the figure [redacted], estimate $\int_{-3}^{5} f(x)\,dx$.

○ −18

[✓ −72]

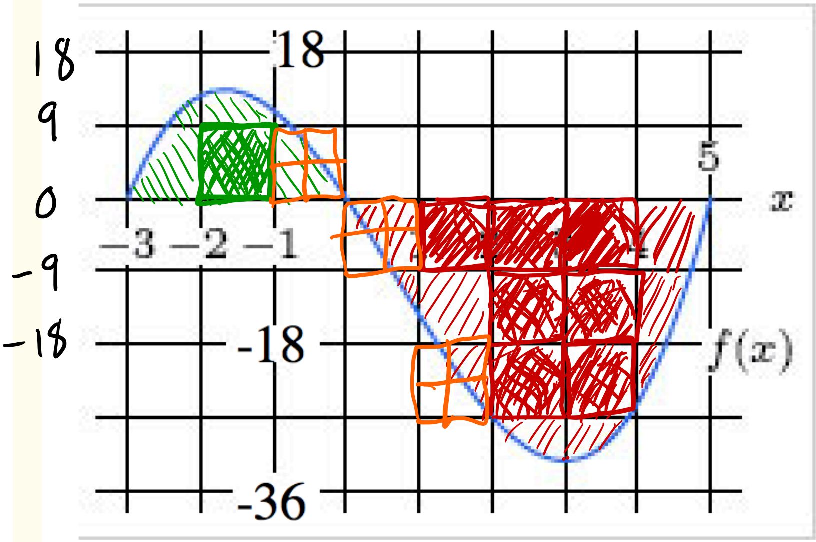
○ 126

○ −126

○ 72

Each box is

$9\{\underbrace{\square}_{1} = 9$



Green boxes: $\underbrace{1}_{\text{whole box}} + \underbrace{2}_{\text{pieces}} = 3$

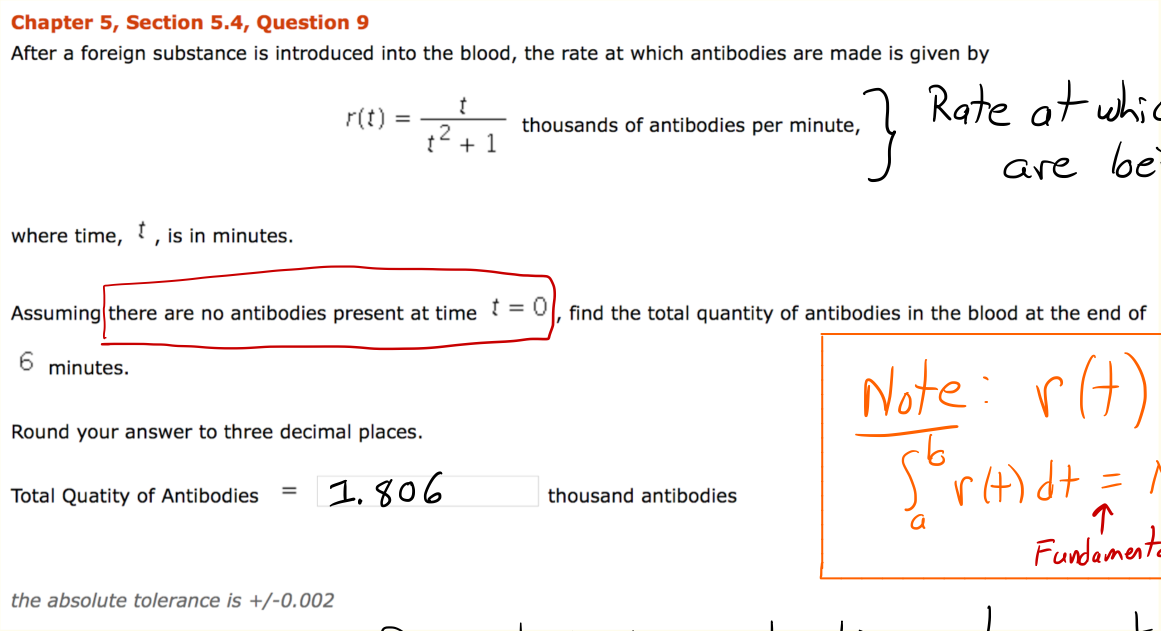
Red boxes: $\underbrace{7}_{\text{whole boxes}} + \underbrace{4}_{\text{pieces}} = 11$

$$\int_{-3}^{5} f(x)dx \approx (3-11)\cdot 9$$

$$= -8\cdot 9$$

$$= \boxed{-72}$$

We could possibly get a better approx by dividing the squares.



Chapter 5, Section 5.4, Question 9

After a foreign substance is introduced into the blood, the rate at which antibodies are made is given by

$$r(t) = \frac{t}{t^2+1}$$ thousands of antibodies per minute,

where time, t , is in minutes.

Assuming there are no antibodies present at time $t = 0$, find the total quantity of antibodies in the blood at the end of 6 minutes.

Round your answer to three decimal places.

Total Quatity of Antibodies = 1.806 thousand antibodies

the absolute tolerance is +/-0.002

} Rate at which antibodies are being produced

Note: $r(t) = N'(t)$ so

$$\int_a^b r(t)\,dt \underset{\uparrow}{=} N(b) - N(a)$$

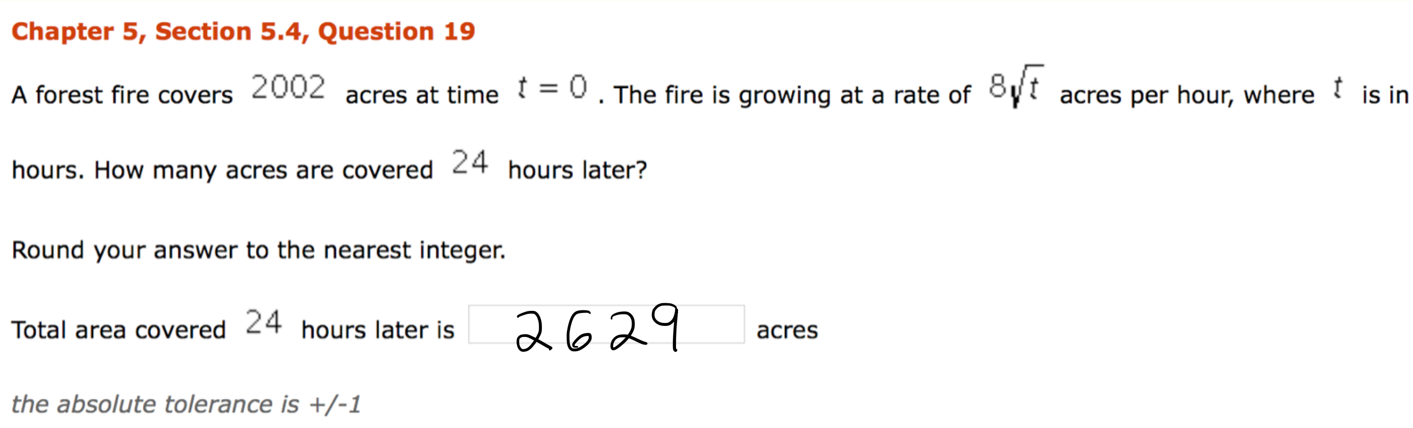
Fundamental Thm of Calc

$N(t)$ = # of antibodies at time t

$$= \boxed{N(0)} + \int_0^t r(t)\,dt = 0 + \int_0^t r(t) = \int_0^t \frac{t}{t^2+1}\,dt$$

$$N(6) = \int_0^6 \frac{t}{t^2+1}\,dt \underbrace{\approx}_{} 1.8055$$

Computer/calculator approx



Chapter 5, Section 5.4, Question 19

A forest fire covers 2002 acres at time $t = 0$. The fire is growing at a rate of $8\sqrt{t}$ acres per hour, where t is in hours. How many acres are covered 24 hours later?

Round your answer to the nearest integer.

Total area covered 24 hours later is 2629 acres

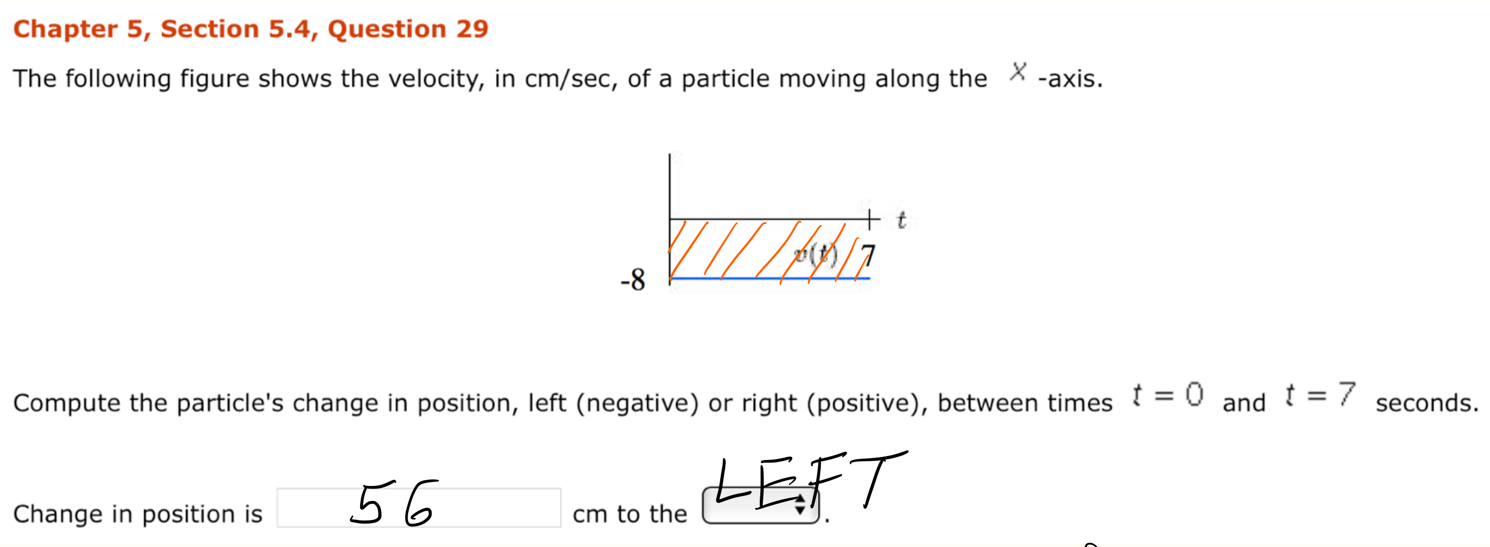
the absolute tolerance is +/-1

$r(t) = 8t^{1/2}$: rate fire is growing

If $A(t)$ = area covered after t hours

$$A(24) = \underbrace{A(0)}_{\text{initial coverage}} + \underbrace{\int_0^{24} 8t^{1/2}\,dt}_{\text{contribution over next 24 hrs}}$$

$$= 2002 + \underbrace{627.07\ldots\ldots}_{\text{calculator}} \approx 2629.07$$



Chapter 5, Section 5.4, Question 29

The following figure shows the velocity, in cm/sec, of a particle moving along the x-axis.

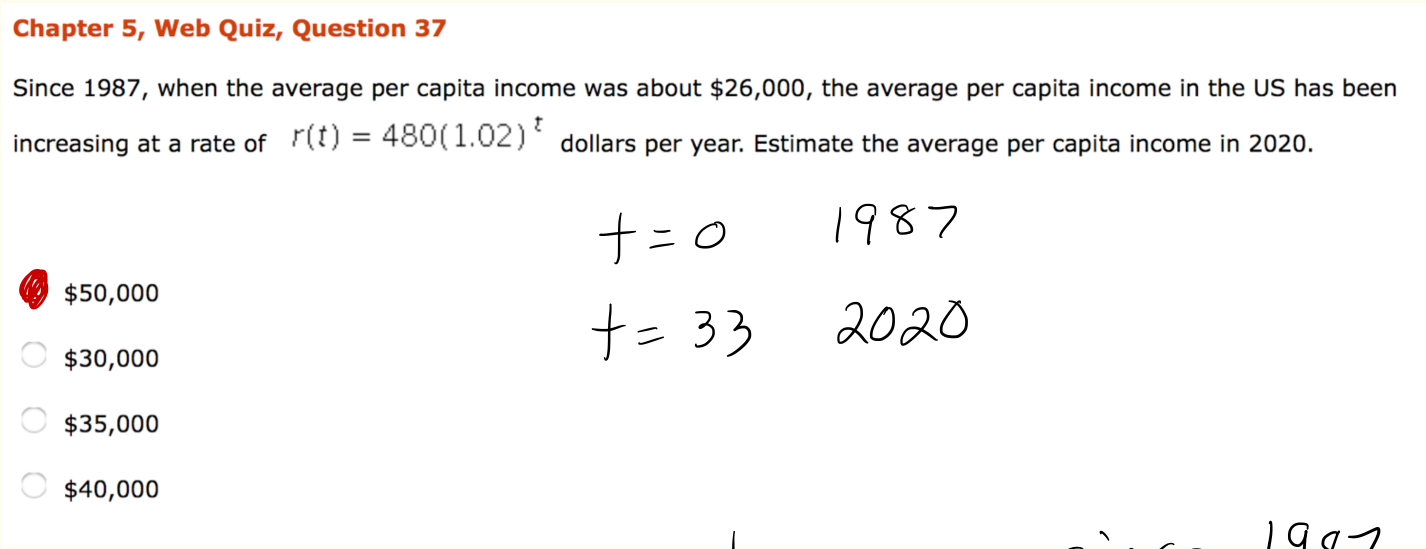
Compute the particle's change in position, left (negative) or right (positive), between times $t = 0$ and $t = 7$ seconds.

Change in position is 56 cm to the LEFT.

$v(t) = s'(t) =$ rate of change of position

change in position over $[0, 7]$ $= \int_0^7 v(t)\,dt = \int_0^7 -8\,dt$

$= -56 =$ orange area



Chapter 5, Web Quiz, Question 37

Since 1987, when the average per capita income was about \$26,000, the average per capita income in the US has been increasing at a rate of $r(t) = 480(1.02)^t$ dollars per year. Estimate the average per capita income in 2020.

● \$50,000

○ \$30,000

○ \$35,000

○ \$40,000

$t = 0$ 1987

$t = 33$ 2020

$A(t)$ = average per capita income since $\underbrace{1987}_{t=0}$

$$A(33) = A(0) + \int_0^{33} r(t)\,dt$$

$$= 26{,}000 + \int_0^{33} 480(1.02)^t\,dt$$

$$= 26{,}000 + \underbrace{22354.2\ldots}_{\text{Calculator}} \approx \boxed{48{,}354.2}$$