Exam Problems Review

7. Suppose $f(x) = x^5$ and $g(x) = e^{4x} + \sin(6x)$. If $h(x) = \underbrace{f(g(x))}_{\text{composition}}$ find $h'(x)$.

(A) $20x^4e^{4x^5} + 30x^4\cos(6x^5)$

(B) $5(e^{4x} + \sin(6x))^4 * 4e^{4x} + 6\cos(6x)$

(C) $5(e^{4x} + \sin(6x))^4$

(D) $5(e^{4x} + \sin(6x))^4 * (4e^{4x} + 6\cos(6x))$

Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

$f(x) = x^5 \Rightarrow f'(x) = 5x^4$

$g(x) = e^{4x} + \sin 6x \Rightarrow g'(x) = 4e^{4x} + 6\cos 6x$

$h'(x) = f'(g(x)) \cdot g'(x) = 5(g(x))^4 \cdot g'(x)$

$= \boxed{5(e^{4x} + \sin 6x)^4 \cdot (4e^{4x} + 6\cos 6x)}$

21. The demand function for a company's product is $q = 2e^{-0.1p}$ where q is measured in thousands of units and p is measured in dollars. What price should the company charge for each unit in order to maximize their revenue?

(A) \$10
(B) \$35
(C) \$100
(D) \$7358

$P = R - C$

q = # widgets produced (1000s)

p = price (\$)

$R(q,p) = q \cdot p$

$\Downarrow$

$R(p) = 2e^{-0.1p} \cdot p$

$= 2pe^{-0.1p}$

MAXIMIZE!

$R'(p) = 2 \cdot \left[p \cdot (-0.1) e^{-0.1p} + e^{-0.1p} \cdot 1 \right]$

$= \boxed{2e^{-0.1p}(-0.1p+1)} = 0$

$2 \neq 0$, $e^{-0.1p} \neq 0$, $(-0.1p+1) = ?$

$0.1p = 1$

$p = \frac{1}{0.1}$

$= \frac{1}{1/10} = 10$

MAX

$+ + + + + +\ 0\ - - - -\ R'$

$1 \quad 10 \quad p$

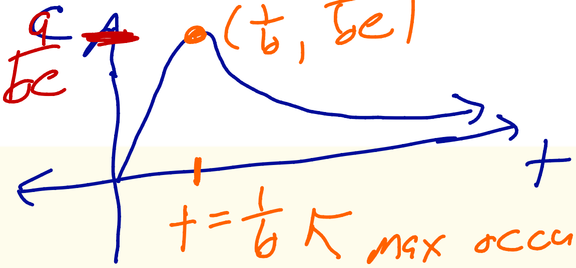
$R'(1) = + \cdot + \cdot + = +$

$R'(100) = + \cdot + \cdot (-)$

$= -$

8. What is the best approximation of the maximum concentration of the surge function $C(t) = 20te^{-0.1t}$?

(A) 2
(B) 10
(C) 74
(D) 543



$t = \frac{1}{b}$ ← max occurs

$c(t) = a t e^{-bt}$

$c\left(\frac{1}{b}\right) = a \frac{1}{b} e^{-b \cdot \frac{1}{b}}$

$= \frac{a}{b} \cdot e^{-1} = \boxed{\frac{a}{be}}$

$e^{\cancel{-b} \cdot \frac{1}{\cancel{b}}} = e^{-1} = \frac{1}{e}$

$\frac{200}{3} = 66\frac{2}{3}$

$a = 20$

$b = \frac{1}{10}$

$t_{max} = \frac{1}{b} = 10$

$c(10) = \frac{20}{\frac{1}{10} \cdot e} = \frac{200}{e} \approx 73.57$

max at $\left(\frac{1}{b}, \frac{a}{be}\right)$

22. A lake is stocked with 1000 fish. The fish population is expected to follow the model: $P(t) = \frac{17{,}000}{1+16e^{-0.7t}}$ where t is the time elapsed, in years. Which of the following is the maximum number of fish this model predicts the lake can hold?

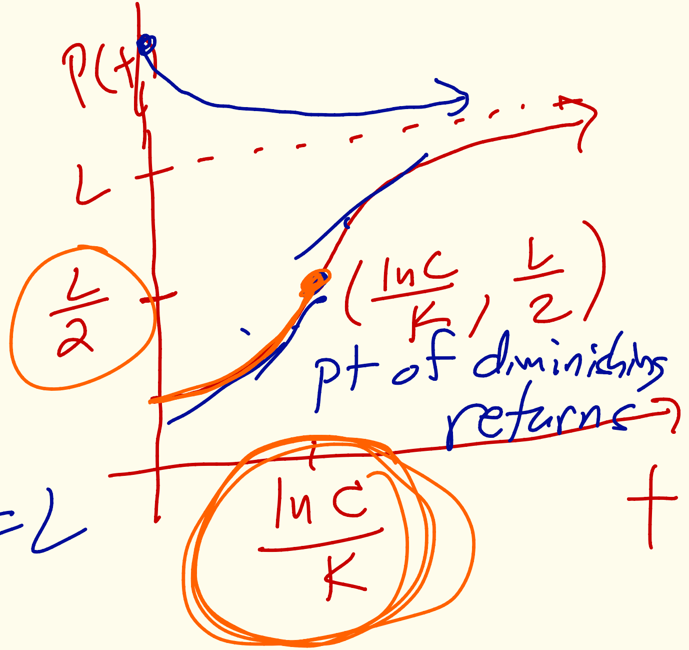
(A) 7,000
(B) 16,000
(C) 17,000
(D) 18,000

Logistic Eqn: $P(t) = \frac{L}{1+Ce^{-kt}}$

$L = 17{,}000$
$C = 16, \quad K = 0.7$

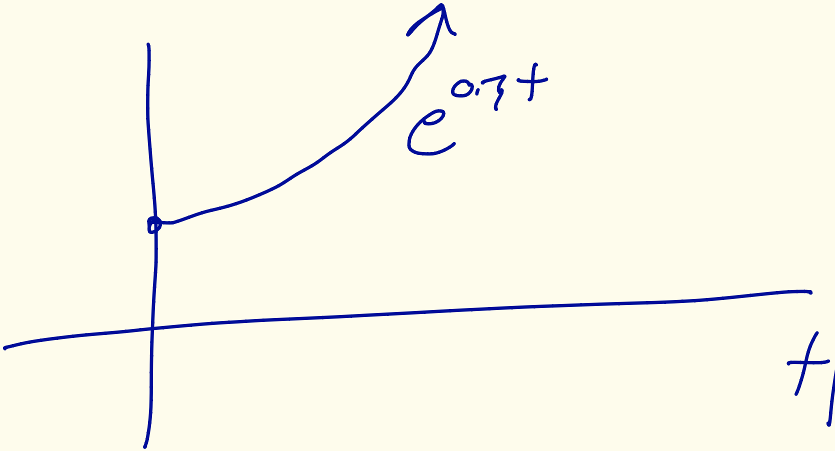
ans: $\boxed{L = 17{,}000}$

$$\lim_{t\to\infty} \frac{L}{1+\underbrace{Ce^{-kt}}_{\to 0}} = \frac{L}{1} = L$$



$$\frac{12000}{1+16\boxed{e^{-0.7t}}}$$

$$F(t) = e^{-0.7t} = \frac{1}{e^{0.7t}} \to \infty \quad = \frac{1}{\text{"}\infty\text{"}} = 0$$



$K > 0$

$$\lim_{t\to\infty} e^{kt} = +\infty$$

$K < 0$

$$\lim_{t\to\infty} e^{kt} = 0$$

23. Suppose a population grows according to a logistic model with a carrying capacity of 8000 and $k = 0.026$. If the initial population is 1400 and time is measured in years, when is the population growing the fastest?

(A) When the population is 8000

(B) During the 4^{th} year

(C) During the 59^{th} year

(D) When the population is 2800 people

$L = 8000, \ K = 0.026$

$1400 = P(0) \Rightarrow C = 33/7$

$$P(t) = \frac{L}{1+Ce^{-kt}} = \frac{8000}{1+\frac{33}{7}e^{-0.026t}}$$

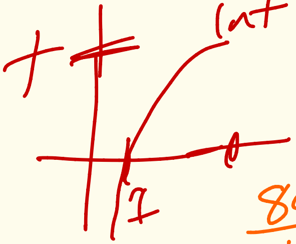
$$= \frac{L}{1+Ce^{0}} = \frac{L}{1+C} = \frac{8000}{1+C}$$

or

$$\frac{8000}{1+C} = 1400$$

$$1+C = \frac{8000}{1400} = \frac{40}{7}$$

$$C = \frac{40}{7} - 1 = \frac{40}{7} - \frac{7}{7} = \boxed{33/7}$$



growing fastest at

$$\left(\frac{\ln C}{K}, \ L/2\right)$$

$$t = \frac{\ln 33/7}{0.026} \approx 59.638$$

$(59.638, 4000)$

14. If $f(x) = \cos(4x)$, then which of the following is equivalent to $f''(\pi)$?

(A) 0

(B) -16

(C) 16

(D) -4

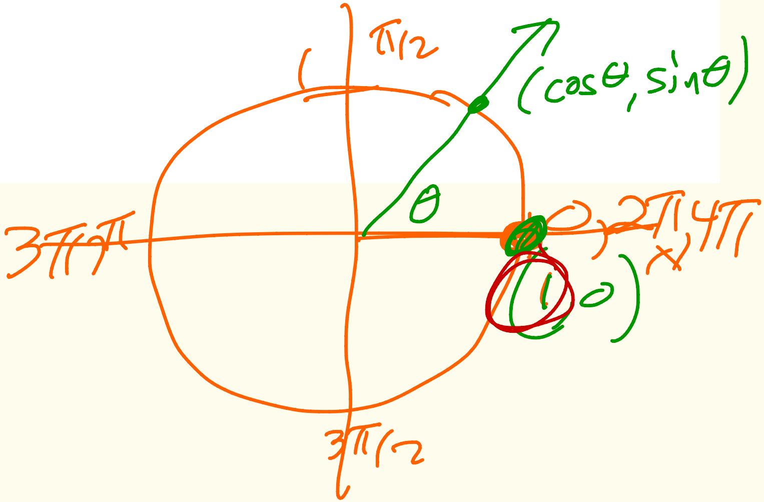
$$f(x) = \cos 4x$$

$$f'(x) = -\underbrace{\sin(4x)\cdot(4x)'}_{\text{Chain Rule}}$$

$$= -4\sin(4x)$$

$$f''(x) = (-4)(\cos 4x)\cdot 4$$

$$= -16\cos(4x)$$

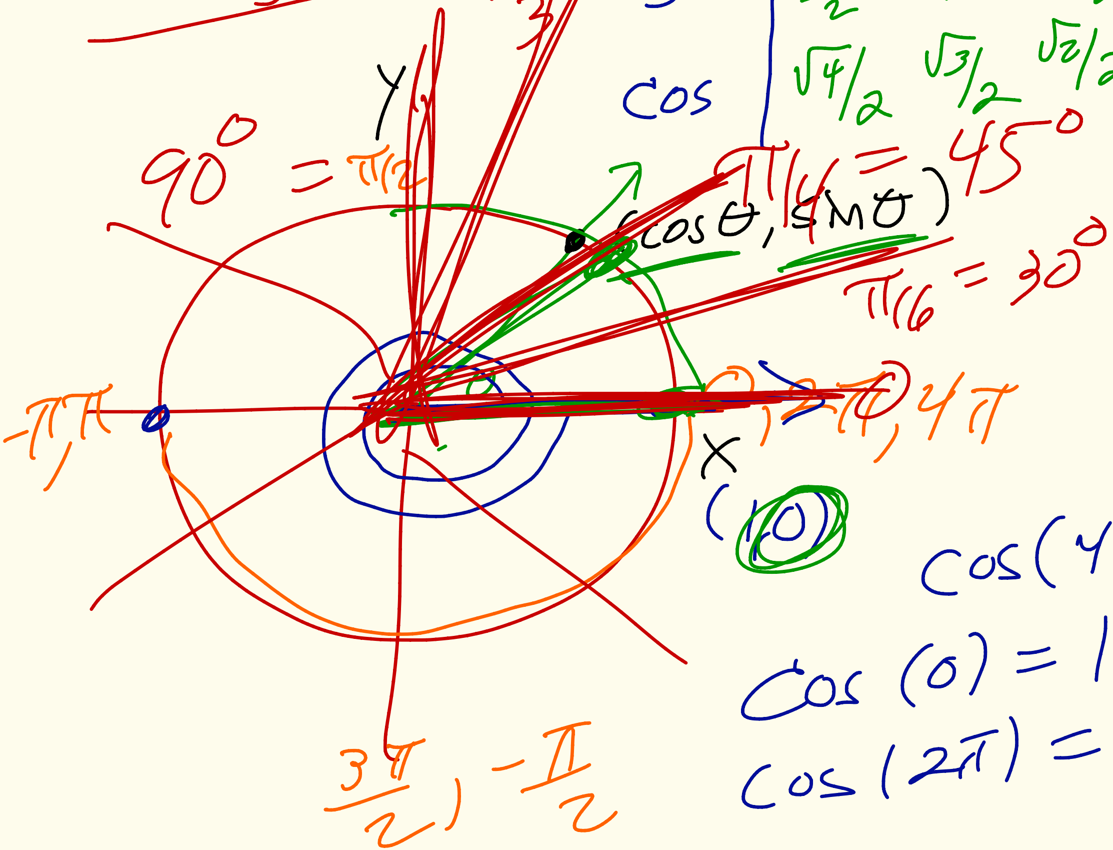


$$\Longrightarrow f''(\pi) = -16\cos(4\cdot\pi)$$

$$= -16.$$

Trig

	0	$\pi/6$	$\pi/2$	$\pi/3$	π
sin	$\frac{\sqrt{0}}{2}$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
cos	$\sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{\sqrt{1}}{2} = 1/2$	$\sqrt{0}/2 = 0$



$\cos(-\pi) = -1$

$\cos(4\pi) = 1$

$\cos(0) = 1$

$\cos(2\pi) = 1$

16. Which of the following is a cubic polynomial, $y = ax^3 + bx^2 + cx + d$ with a critical point at $x = 6$, an inflection point at $(3,7)$, and a leading coefficient of 1?

(A) $y = x^3 - 9x^2 + 61$

(B) $y = 2x^3 - 3x^2 + 73$

(C) $y = x^3 + 8x^2 - 92$

(D) $y = x^3 - 5x^2 + 57$

$y = 1x^3 + bx^2 + 0x + d$

polynomial

$\boxed{a=1}$

$f'(6) = 0 \Rightarrow f'(x) = 3x^2 + 2bx + c$ ①

$0 = 3\cdot 6^2 + 12b + c$ or $\boxed{12b + c = -108}$

(i) $\boxed{f(3) = 7}$ ② (ii) $f''(3) = 0$ ③

$f''(x) = 6x + 2b \Rightarrow 0 = f''(3) = 6\cdot 3 + 2b$

$2b = -18$

$\boxed{b = -9}$

$12\cdot(-9) + c = -108$

$-108 + c = -108$

$c = 0$

$$\boxed{F(x) = x^3 - 9x^2 + d}$$

$$7 = F(3) = 27 - 81 + d$$

$$d = 7 + 81 - 27 = \boxed{61}$$

$$\boxed{F(x) = x^3 - 9x^2 + 61}$$