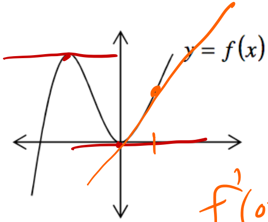
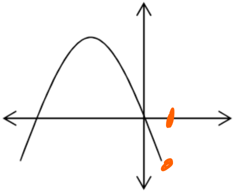
8. Given the function $y = f(x)$ graphed to the right, which of the following represents the graph of $f'(x)$?

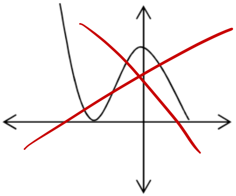


$f'(\text{orange}) > 0$

(A)

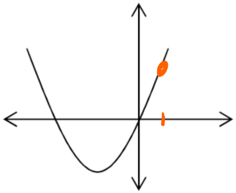


(B)

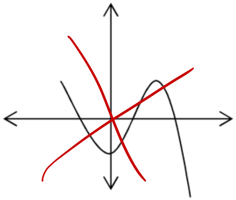


$f'(x)$

(C)



(D)



9. Each year the cost of living in Western Massachusetts goes up. This year the increase was more than the increase last year. Last year the increase was more than the year before. Assuming the cost of living maintains this pattern and the cost of living is modeled by a well-behaved (smooth) function, L, which of the following best describes L?

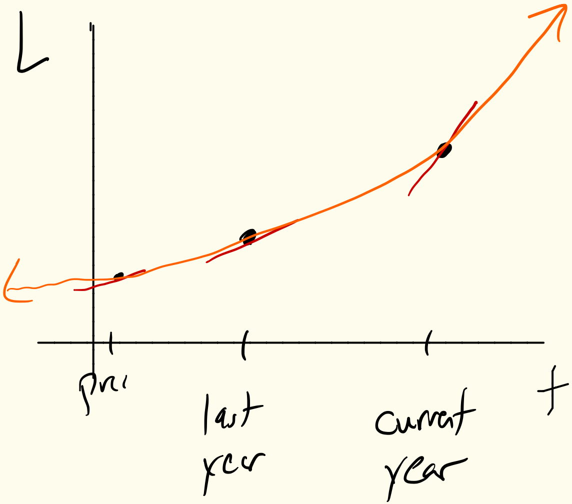
(A) L' is positive and L'' is positive

(B) L is increasing and L' is increasing

(C) L is positive and L' is positive

(D) L is concave up

(E) All of the above



$L' > 0$ increasing

$L'' > 0$ concave up

6. Given that $f(x) = 5x^2 - 3x$, which of the following is equivalent to: $\underbrace{\frac{f(x+h)-f(x)}{h}}$?

(A) $10x - 3$

(B) $10x + 5h - 3$

(C) $10xh - 5h^2 - 3h$

(D) $5x^2h - 3xh$

$f'(x) = 10x - 3$

$\frac{\Delta f}{\Delta h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left(5(x+h)^2 - 3(x+h)\right) - (5x^2 - 3x)}{h}$$

$$= \frac{\left(5(x^2 + 2xh + h^2) - 3x - 3h\right) - (5x^2 - 3x)}{h}$$

$$= \frac{\cancel{5x^2} + 10xh + 5h^2 \cancel{- 3x} - 3h \cancel{- 5x^2} \cancel{+ 3x}}{h}$$

$$= \frac{10xh + 5h^2 - 3h}{h} = \frac{\cancel{h}(10x + 5h - 3)}{\cancel{h}} = \boxed{10x + 5h - 3}$$

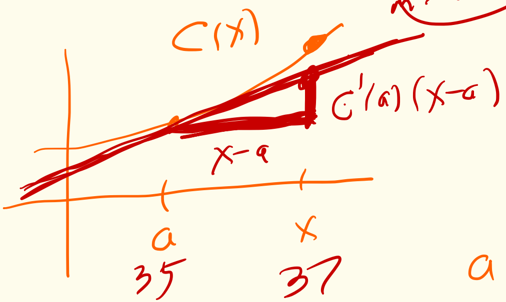
$$f'(x) = \lim_{h \to 0} 10x + 5h - 3 = 10x + 0 - 3$$

10. Given the cost function $C(35) = 458.6$ and $C'(35) = 2.5$ which of the following is the best estimate for $C(37)$?

(A) 453.6
(B) 456.1
(C) 461.1
(D) 463.6
(E) None of these

LOCAL LINEAR APPROX

$$f(x) \approx f(a) + f'(a)(x-a)$$



$$\boxed{C(x) \approx \underbrace{C(a)}_{\text{value at } a} + \underbrace{C'(a)(x-a)}_{\text{change in the tangent line}}}$$

$a = 35$

$x = 37$

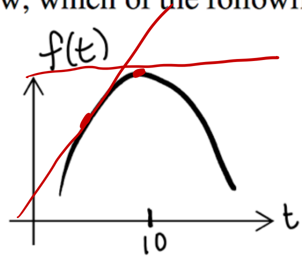
$C(36) \approx 458.6 + (2.5)(1)$

$C(37) \approx C(35) + C'(35)(37-35)$

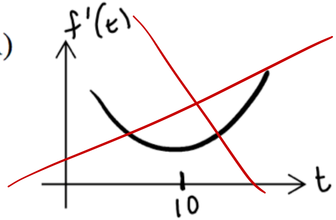
$= 458.6 + 2.5(2) = 458.6 + 5$

$= \boxed{463.6}$

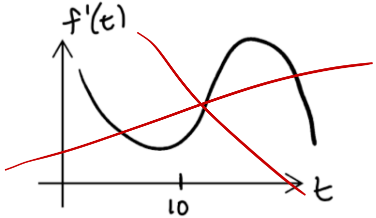
5. Given the graph of $f(t)$ below, which of the following graphs represents $f'(t)$?



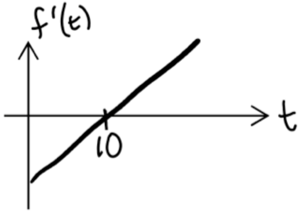
(A)



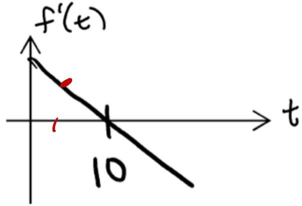
(B)



(C)



(D)



21. Hot soup is spooned into a bowl and immediately starts to cool. The temperature in degrees Celsius of the cooling bowl of soup as a function of time in minutes since it was put in a bowl is given by the function $T(t)$. What are the units of $T''(t)$?

(A) degrees per minute squared
(B) degrees per minute
(C) degrees
(D) minutes per degrees squared

$T(t)$ – temp (C°) at time t (min)

$T'(t) = \frac{\Delta T}{\Delta t}$ $\quad \frac{C^\circ}{\text{min}}$

$T''(t) = \frac{\Delta (T')}{\Delta t}$ $\quad \frac{C^\circ/\text{min}}{\text{min}} = C^\circ/\text{min}^2$

3. Suppose $f(x) = 2x^2 - 3$. Calculate $\dfrac{f(2+h) - f(2)}{h}$.

(A) 8

(B) $\dfrac{2h^2 + 8h + 10}{h}$

(C) $2h + 8$

(D) $\dfrac{2 + 8h}{h}$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2(2+h)^2 - 3) - (2\cdot(2)^2 - 3)}{h}$$

$$= \frac{(2(4 + 4h + h^2) - 3) - (2\cdot 4 - 3)}{h}$$

$$= \frac{\cancel{8} + 8h + 2h^2 \cancel{-3} \cancel{-5}}{h} = \frac{8h + 2h^2}{h}$$

$$= \boxed{8 + 2h}$$

$f'(x) = 4x$

$f'(2) = 8$

11. If $f(x) = e^{x^2}$ and $g(x) = 3x + 2$ then $f(g(x))$ is:

(A) $3e^{x^2} + 2$ (B) $e^{9x^2} e^{12x} e^4$ (C) $3e^{3x^2} + 2$

(D) $10x + e^{i\pi}$

$$f(☺) = e^{☺^2}$$

$$F(g(x)) = F(3x+2)$$

$$= e^{(3x+2)^2} = e^{(9x^2+12x+4)}$$

$$= e^{9x^2} \cdot e^{12x} \cdot e^{4}$$

$$e^{\square + \triangle + \bigcirc} = e^{\square} \cdot e^{\triangle} \cdot e^{\bigcirc}$$

7. Identify how you would move the graph of the function $f(x) = x^2$ so it becomes the graph of $g(x) = (x-1)^2 + 3$.

(A) one unit right and three units up

(B) one unit right and three units down

(C) one unit left and three units up

(D) one unit left and three units down

(E) None of these

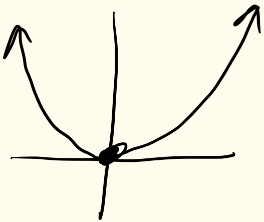
$+3$: moves 3 units up!

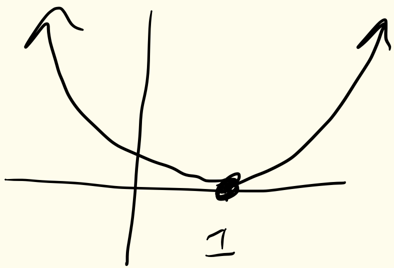
$(x-1)$: moves to right 1 unit

$y = x^2$

$y = (x-1)^2$

right 1 unit





7. Let $f(x) = \sin(2x)$ and $g(x) = \pi\cos(x)$. Which of the following equals 0?

(A) $f(g(0))$

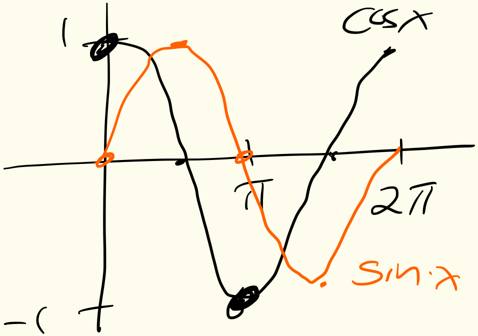
(B) $g(f(0))$

(C) $g(g(0))$

(D) $g\left(f(g(0))\right)$

(A) $f(g(0)) = f(\pi \cdot \cos(0)) = f(\pi \cdot 1) = f(\pi)$
$= \sin(2\pi) = 0$

(C) $g(g(0)) = g(\pi \cdot \cos(0))$
$= g(\pi \cdot 1) = g(\pi)$
$= \pi \cdot \cos(\pi)$
$= \pi(-1) = -\pi$



14. Consider the function: $g(t) = 3e^{0.5t}$. Estimate $g'(2)$ by using an interval size of 0.1 *to the right*. Which of the following is the estimate rounded to 2 decimal places.

(A) 4.07

(B) 4.08

(C) 4.13

(D) 4.18

2 2.1

$$g'(2) \approx \frac{g(2.1) - g(2)}{2.1 - 2} = \frac{3e^{(0.5)(2.1)} - 3e^{(0.5)\cdot 2}}{0.1}$$

$$= \frac{3\left[e^{1.05} - e^{1}\right]}{0.1} = \boxed{4.18}10786\ldots$$

0.1 to left $\quad g'(2) \approx \dfrac{g(2) - g(1.9)}{2 - 1.9}$

Best: $\dfrac{g(2.1) - g(1.9)}{2.1 - 1.9}$