Final Exam Info:

- Friday 4th, 10:30-12:30 Boyden
- SI Final Review TODAY
  7-9pm HAS 134

25. If time, t, is in hours and concentration, C, is in ng/ml, the drug concentration curve for a drug in the bloodstream is given by the function $C = 8te^{-0.5t}$. Complications can arise whenever the level of the drug is above 4.3 ng/ml. How long must a patient wait before being safe from complications?

(A) Approximately 1 hour after taking the drug.
$\boxed{\text{(B)}}$ Approximately 4 hours after taking the drug.
(C) Approximately 6 hours after taking the drug.
(D) Approximately 40 hours after taking the drug.

$\left(\frac{1}{b}, 8\frac{1}{b} e^{-0.5\frac{1}{b}}\right)$

Surge Function

$c(t) = 8te^{-0.5t}$

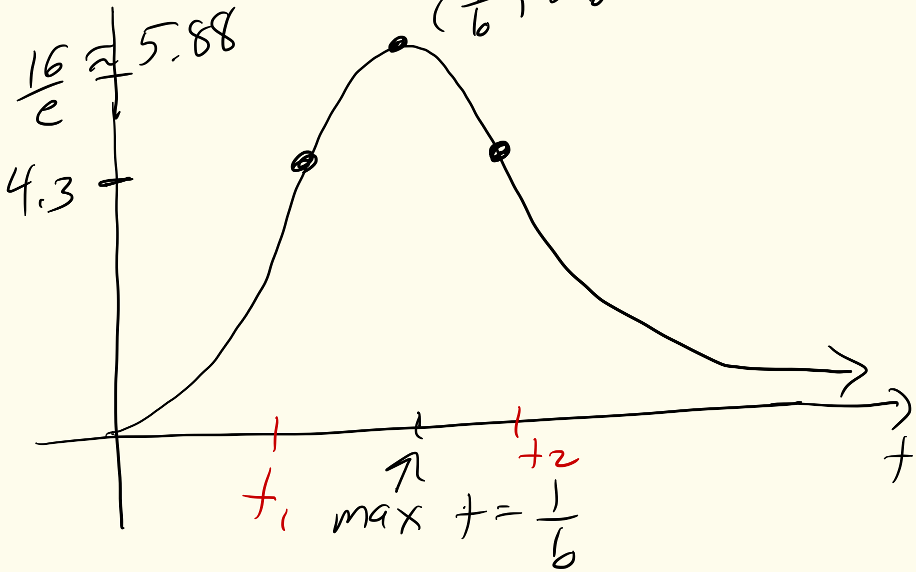
$a = 8$

$b = 0.5$

$\frac{8}{\frac{1}{2}} e^{-\frac{1}{2}\cdot\frac{1}{1/2}}$

$= 4e^{-1}$

$= 4/e = 1.4$

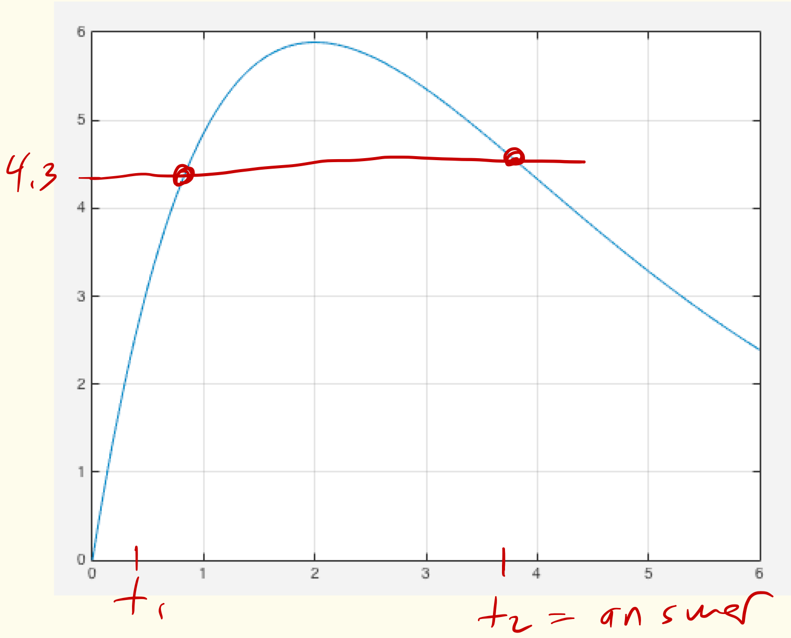


$$c(t) = 8t\,e^{-\frac{1}{2}t}$$

$a = 8 \quad b = 1/2$

$$t_{max} = \frac{1}{b} = \frac{1}{1/2} = 2$$

$$c(2) = 8 \cdot 2 \cdot e^{-\frac{1}{2}\cdot 2} = \frac{16}{e} = 5.886\ldots$$



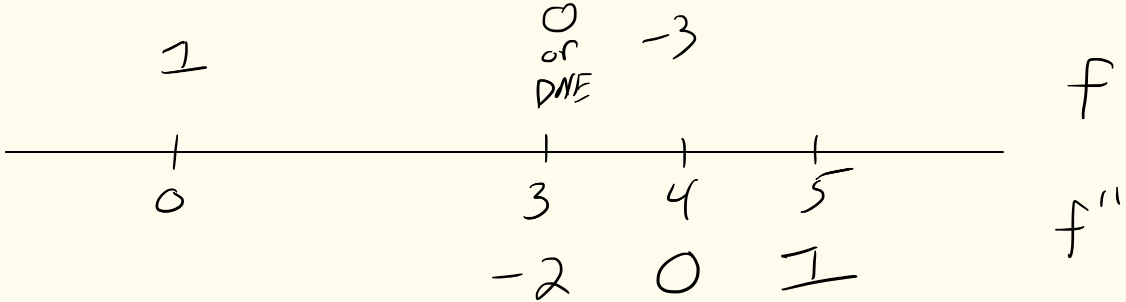
15. A student finds the critical number of a function, $\boxed{f(x) \text{ to be } x = 3.}$ She then calculates the following values: $f'(0) = 1$, $f'(4) = -3$, $f''(3) = -2$, $f''(4) = 0$, $f''(5) = 1$. Which of the following is *true*?

(A) ~~$f(x)$ is increasing at $x = 4$~~

(B) ~~The point at $x = 3$ is a local minimum~~

(C) The point at $x = 0$ is an inflection point

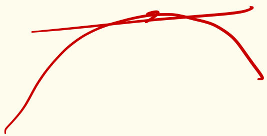
(D) $f(x)$ is concave up at $x = 5$ Yes since $f''(5) = 1 > 0$



$\underline{x = 3!}$

$f'(3) = 0$ or $f'(3) = DNE$

$f''(3) -2 < 0 \Rightarrow$ concave down



Chapter 7, Section 7.2, Question 21

Find the integral.

$$\int \sqrt{\cos(3t)}\sin(3t)\,dt$$

Check your answer by differentiation.

$$\int \sqrt{\cos(3t)}\sin(3t)\,dt = -\frac{2}{9}(\cos(3t))^{3/2} + C$$

$$\int (\cos 3t)^{1/2}\, \underbrace{\sin(3t)\,dt}_{-\frac{1}{3}du} = \int u^{1/2} \cdot \left(-\frac{1}{3}du\right)$$

$$\boxed{u = \cos 3t}$$

$$du = -3\sin 3t\,dt$$

or

$$-\frac{1}{3}du = \sin 3t\,dt$$

$$= -\frac{1}{3}\int u^{1/2}\,du$$

$$= -\frac{1}{3}\, u^{3/2}/_{3/2} + C$$

$$= -\frac{2}{9}u^{3/2} + C = \boxed{-\frac{2}{9}(\cos 3t)^{3/2} + C}$$

Chapter 7, Section 7.2, Question 23

Find the integral.

$$\int \sin^7\theta \cos\theta \, d\theta$$

$$= \int \underline{(\sin\theta)^7} \, \underline{\cos\theta \, d\theta}$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

Check your answer by differentiation.

$$\int \sin^7\theta \cos\theta \, d\theta = \qquad + C$$

$$= \int u^7 \, du = \frac{1}{8}u^8 + C$$

$$= \boxed{\frac{1}{8}\sin^8\theta + C}$$

Chapter 7, Section 7.2, Question 35

Find the integral.

$$\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$$

Check your answer by differentiation.

$$\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy = \qquad + C$$

$$\int e^{y^{1/2}} \cdot \underbrace{y^{-\frac{1}{2}}dy} = \int e^{u}\, 2du = 2e^{u} + C$$

$$= \boxed{2e^{y^{1/2}} + C}$$

$u = y^{1/2}$

$du = \frac{1}{2} y^{-1/2} dy \Rightarrow 2du = y^{-1/2} dy$

21. Approximate the area under the curve $y = x^2 + 2x$ from $x = 0$ to $x = 4$ using $\Delta x = 2$.

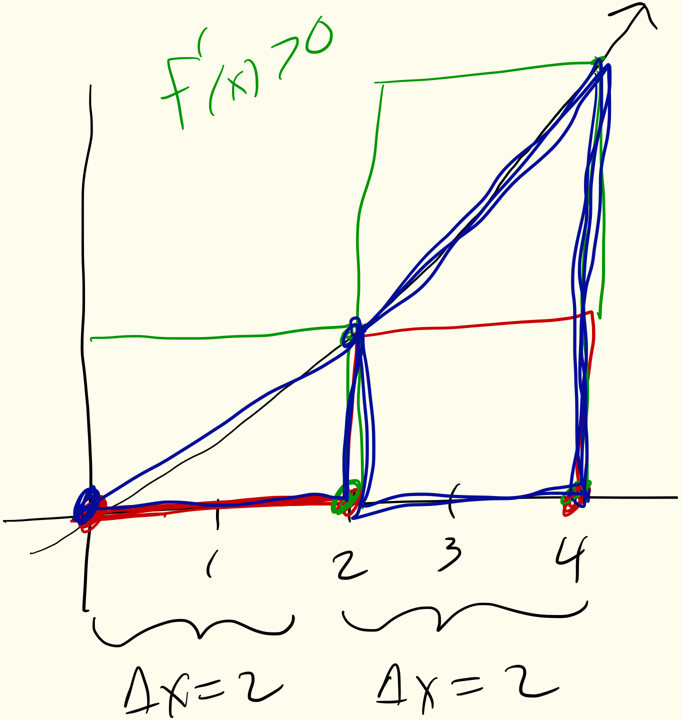
$= x(x+2)$

(A) 40
(B) 76
(C) 16
(D) 37

$y = f(x)$

$y(0) = 0 \quad y(2) = 8 \quad y(4) = 24$

$y(1) = 3 \quad y(3) = 15$



$x_0 = 0,\ x_1 = 2,\ x_2 = 4$

$LHS = (f(0) + f(2)) \cdot \Delta x$

$= (0 + 8) \cdot 2 = 16$

$RHS = (f(2) + f(4)) \cdot \Delta x$

$= (8 + 24) \cdot 2 = 64$

$LHS \le \int_0^4 f(x)\,dx \le RHS$

$\text{Approx} = \frac{1}{2}(LHS + RHS)$

$= \frac{1}{2}(16 + 64) = \boxed{40}$

23. The velocity of a particle moving along the x-axis is given by $f(t) = 6 - 2t$ cm/sec. Use a graph of $f(t)$ to find the exact change in position of the particle from time $t = 0$ to $t = 3$ seconds

(A) 0 cm
(B) 3 cm
(C) 6 cm
(D) 9 cm

Let's call $f(t) = v(t)$

$v(t) = 6 - 2t$ velocity

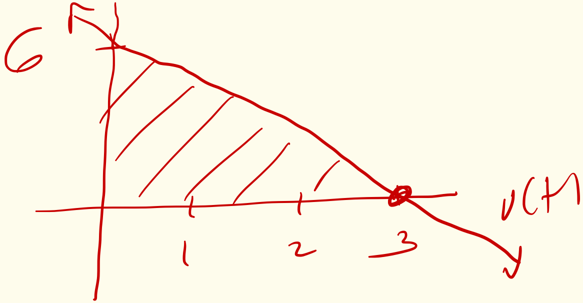
$s(t)$ - position

$\int_a^b v(t)\,dt = s(b) - s(a)$ since $s'(t) = v(t)$

$$s(b) = s(a) + \underbrace{\int_a^b v(t)\,dt}_{\text{total displacement}}$$

$$s(3) = s(0) + \boxed{\int_0^3 6 - 2t\,dt}$$

$$\int_0^3 6-2t\,dt$$

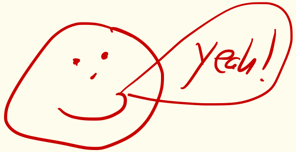


$$=\left(\frac{1}{2}\right)(3)(6)=9$$

or

$$\left(6t-\frac{2t^2}{2}\right)\Big|_0^3 = \left(6t-t^2\right)\Big|_0^3$$

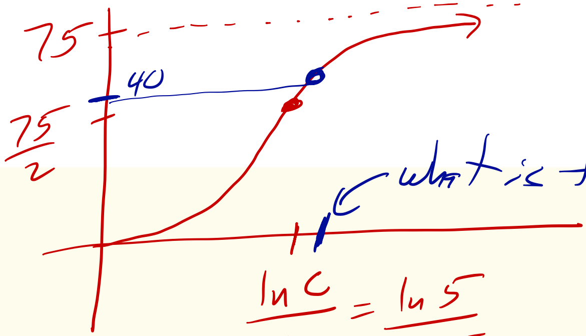
$$=(18-9)+(0-0)=\boxed{9}$$



19. Suppose a population is given by $P(t) = \dfrac{75}{1+5e^{-0.05t}}$, where P is in thousands and t is in years. When will this population reach $\boxed{40{,}000?}$

$\boxed{\text{(A)}}$ 34.86 years
(B) 22.45 years
(C) 15.46 years
(D) 35.21 years

$L = 75$
$C = 5$
$K = 0.05$



what is t?

$$\frac{\ln C}{K} = \frac{\ln 5}{0.05}$$

$$\frac{75}{1+5e^{-0.05t}} = 40$$

or $$1+5e^{-0.05t} = \frac{75}{40} = \frac{15}{8}$$

$$5e^{-0.05t} = \frac{15}{8} - 1 = \frac{15}{8} - \frac{8}{8} = \frac{7}{8}$$

$$e^{-0.05t} = \frac{7}{40} \implies t = \frac{\ln(7/40)}{-0.05} = 34.85\ldots$$

25. Find the area bounded between $y = 3x$ and $y = x^2$.

(A) 3

(B) 4

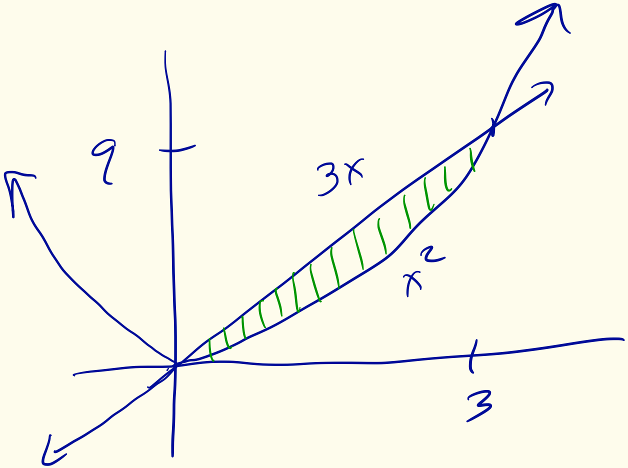
(C) 4.5

(D) 5

Limits of integration:

$x^2 = 3x$ or $x^2 - 3x = 0$

or $x(x-3) = 0 \Rightarrow x = 0$ or $x = 3$



$$\text{Area} = \int_0^3 3x - x^2\,dx$$

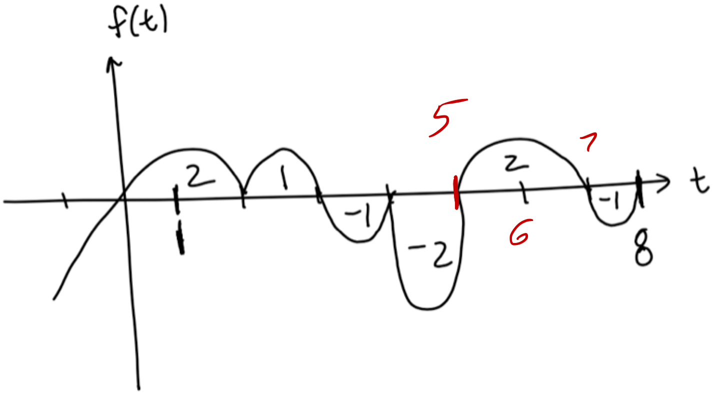
$$= \left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^3$$

$$= \left(\left(\frac{3}{2}\right)9 - (1\cdot 9)\right) - (0 - 0)$$

$$= \frac{1}{2}\cdot 9 = 4.5$$

26. Consider the graph to the right with areas given. Calculate $\int_5^8 f(t)\,dt$

(A) 3
(B) – 3
(C) 15
(D) 1



$$\int_5^8 f(t)\,dt = 2 + (-1) = 1$$

Area between f(t) and the t axis from 5 to 8? ans: $2 + |-1| = \boxed{3}$