Integration:

Definite Integral $\rightarrow$ $\int_a^b f(x)\,dx = F(b) - F(a)$

so we need $\underbrace{F(x)}_{\text{anti-derivative}} = \underbrace{\int f(x)\,dx}_{\text{indefinite integral}}$

$$\int K\,dx = Kx + C$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \qquad r \neq -1$$

$$\boxed{\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln\underbrace{|x|}_{\text{absolute value}} + C}$$

since $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int 0\,dx = C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

General Rules

$$\int K f(x)\,dx = K\int f(x)\,dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Ex:

a) $\int \frac{x^{3/2} + 2\sqrt{x} + 4}{x}\,dx$

$$\frac{x^{3/2} + 2x^{1/2} + 4}{x} = \frac{x^{3/2}}{x} + 2\frac{x^{1/2}}{x} + \frac{4}{x}$$

$$= x^{1/2} + 2x^{-1/2} + \frac{4}{x}$$

$$= \int x^{1/2}\,dx + 2\int x^{-1/2}\,dx + 4\int \frac{1}{x}\,dx$$

$$= \frac{x^{1/2+1}}{1/2+1} + 2\frac{x^{-1/2+1}}{-1/2+1} + 4\ln|x| + C$$

$$= \boxed{\tfrac{2}{3}x^{3/2} + 4x^{1/2} + 4\ln|x| + C}$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$$

Integration By Substitution (u substitution) or change of variable

Ex: $\int x e^{x^2} dx = \int \underbrace{e^{x^2}}_{e^u} \underbrace{x\,dx}_{\frac{1}{2}du}$

$u = x^2$

treat as a fraction $\left\{ \frac{du}{dx} = 2x \right.$ $\Rightarrow$ $du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= \int e^u \cdot \frac{1}{2} du = \frac{1}{2} \int e^u du$

$$\boxed{\int e^x dx = e^x + C}$$

$= \frac{1}{2} e^u + C$

$= \boxed{\frac{1}{2} e^{x^2} + C}$

check: $\frac{d}{dx}\left(\frac{1}{2}e^{x^2} + C\right) = \frac{1}{2} \cdot 2x e^{x^2} + 0$

$= x e^{x^2}$ Yes!

Ex:

$$\boxed{\int 3x^2(2x^3+1)^{10}\,dx} = \int \underbrace{(2x^3+1)^{10}}_{u^{10}}\ \underbrace{3x^2dx}_{\frac{1}{2}du}$$

$$\boxed{u = 2x^3+1}$$

$$\frac{du}{dx} = 6x^2 \Rightarrow \quad du = 6x^2dx$$

$$\frac{1}{2}du = 3x^2dx$$

$$= \int u^{10}\tfrac{1}{2}du = \boxed{\frac{1}{2}\int u^{10}du}$$

$$= \frac{1}{2}\frac{u^{10+1}}{10+1} + C = \frac{1}{22}u^{11} + C$$

$$= \boxed{\frac{1}{22}(2x^3+1)^{11} + C}$$

Ex: $\int \frac{\ln t}{t}\, dt$

$\frac{d}{dt}(\ln t) = 1/t$

$\frac{d}{dt}(t) = 1$

$= \int \underbrace{\ln t}_{u} \cdot \underbrace{\frac{1}{t}\, dt}_{du}$

$\boxed{u = \ln t}$

$du = \frac{1}{t}\, dt$

$= \int u^1\, du = \frac{u^{1+1}}{1+1} + C$

$= \frac{1}{2} u^2 + C$

$= \boxed{\frac{1}{2}(\ln t)^2 + C}$

Ex:
$$\int \underbrace{\sqrt{\cos(3t)}}_{\sqrt{u}}\ \underbrace{\sin(3t)\,dt}_{-\frac{1}{3}du}$$

$$\frac{d}{dt}\left(\sin(3t)\right) = 3\cos(3t)$$

$$\frac{d}{dt}\left(\cos(3t)\right) = -3\sin(3t)$$

$$u = \cos(3t)$$

$$du = -3\underbrace{\sin(3t)\,dt}$$

$$\Rightarrow -\frac{1}{3}du = \sin(3t)\,dt$$

$$= \int \sqrt{u}\left(-\frac{1}{3}du\right) = -\frac{1}{3}\int u^{1/2}\,du$$

$$= \boxed{-\frac{1}{3}\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C} = -\frac{1}{3}\frac{u^{3/2}}{3/2} + C$$

$$= -\frac{2}{9}u^{3/2} + C = \boxed{-\frac{2}{9}\left(\cos(3t)\right)^{3/2} + C}$$

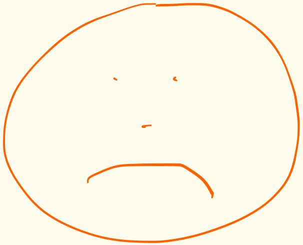
$$\int \sqrt{\cos(3t)}\,\sin 3t\,dt$$

$$u = \sin 3t$$

$$\boxed{du = 3\cos 3t\,dt}$$

$$\sqrt{\frac{1}{3}\frac{du}{dt}} = \sqrt{\cos 3t}$$

$$= \int \underbrace{\sin 3t}_{u}\,(\cos 3t)^{1/2}\,dt$$



NO PAIN!

$$\frac{e^{\ln\left(\sqrt{\cos\left(x^2+e^{\pi x}\right)}\right)}}{x^2+1+B_3(x)}$$

Ex:

$$\int \frac{x^3}{x^4+5}\,dx = \int \underbrace{\frac{1}{x^4+5}}_{\frac{1}{u}} \underbrace{x^3\,dx}_{\frac{1}{4}du}$$

$u = x^4+5$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$$= \int \frac{1}{u}\cdot\frac{1}{4}\,du$$

$$= \frac{1}{4}\int \frac{1}{u}\,du$$

$$= \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln|x^4+5| + C$$

but $x^4+5>0$

$$= \boxed{\frac{1}{4}\ln(x^4+5) + C}$$