Chapter 7: 7.1 and 7.2 (not 7.4)

integration by parts (HARD!)

Integration: Given $f(x)$

Find $\boxed{F(x) = \int f(x)\,dx \quad \text{or } F'(x) = f(x)}$

indefinite integral or anti-derivative

INTEGRAND

What is it good for?

$$\boxed{\int_a^b f(x)\,dx} = F(b) - F(a) = \#$$

Definite Integral

Fund. Thm of Calc.

How do we find F(x)?

Look at DIFFERENTIATION RULES!

$$\boxed{\frac{d}{dx}(3x) = 3} \Rightarrow \boxed{\int \underbrace{\boxed{3}}_{f(x)} dx = 3x + \underbrace{C}_{\text{arbitrary constant}}} = F(x)$$

check: $\frac{d}{dx}(3x + c) = 3 + 0 = \boxed{3}$

You can always check if you have the correct F(x)! ☺

$$\frac{d}{dx}\left(F(x)\right) = 3 = f(x) \quad \text{Yes}$$

$$\frac{d}{dx}(\pi x) = \pi \quad \text{or} \quad \frac{d}{dx}(kx) = k$$

$$\Rightarrow \boxed{\int k\,dx = kx + C}$$

$r = -1$!

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx$$

$$= \ln|x| + C$$

$$\frac{d}{dx}(x^2) = 2x \qquad \frac{d}{dx}\left(x^{-1/3}\right) = -\frac{1}{3}x^{-1/3-1} = -\frac{1}{3}x^{-4/3}$$

$$= -\frac{1}{3x^{4/3}}$$

$$\frac{d}{dx}(x^r) = r x^{r-1} \quad \text{(power rule)}$$

$$\boxed{\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C, \quad r \neq -1} \text{ (later)}$$

check

$$\int \underline{x}\,dx = \int x^1\,dx = \frac{1}{1+1}x^{1+1} + C$$

$\underline{r=1}$

$$= \boxed{\frac{1}{2}x^2 + C}$$

$$\frac{d}{dx}\left(\frac{1}{2}x^2 + c\right) = 1 \cdot x + 0 = x \quad \text{Yes!}$$

$$\frac{d}{dx}(\sin x) = \cos x \Rightarrow \boxed{\int \cos x\,dx = \sin x + C}$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int -\sin x\,dx = \cos x + C$$

$$-\int \sin x\,dx = \cos x + C$$

$$\boxed{\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C}$$

$$\boxed{\int \sin x\,dx = -\cos x - C = -\cos x + \underbrace{C}_{\text{some constant}}}$$

$$\frac{d}{dx}(\sin(ax)) = \cos(ax)\cdot a \quad \text{so}$$

$$\frac{1}{a}\frac{d}{dx}(\sin(ax)) = \cos ax \quad \text{or}$$

$$\boxed{\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C}$$

$$\frac{d}{dx}\left(e^x\right) = e^x \Rightarrow \boxed{\int e^x\,dx = e^x + C}$$

$$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax} \Rightarrow \boxed{\int e^{ax}\,dx = \frac{1}{a} e^{ax} + C}$$

General Rules: sums and differences and constant multiples

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

$$\Rightarrow \boxed{\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx}$$

$$\frac{d}{dx}\left(K \cdot f(x)\right) = K \frac{d}{dx}\left(f(x)\right) \Rightarrow \boxed{\int K f(x)\,dx = K \int f(x)\,dx}$$

Ex: $\int F(x)\,dx = \int 3x^2 - \sqrt{x} + \cos(2x) + \pi \, dx$

$$= \int 3x^2\,dx - \int \sqrt{x}\,dx + \int \cos(2x)\,dx + \int \pi\,dx$$

$$= 3\int \underbrace{x^2}_{r=2}\,dx - \int \underbrace{x^{1/2}}_{r=1/2}\,dx + \int \cos\underbrace{(2x)}_{a=2}\,dx + \int \underbrace{\pi}_{K}\,dx$$

$$= 3\left(\frac{x^{2+1}}{2+1} + C_1\right) - \left(\frac{x^{1/2+1}}{1/2+1} + C_2\right) + \left(\frac{1}{2}\sin(2x) + C_3\right) + \left(\pi x + C_4\right)$$

$$= 3\frac{x^3}{3} - \frac{x^{3/2}}{3/2} + \frac{1}{2}\sin(2x) + \pi x + (C_1 + C_2 + C_3 + C_4)$$

$$= \boxed{x^3 - \frac{2}{3}x^{3/2} + \frac{1}{2}\sin(2x) + \pi x + C = F(x)}$$

Ex: It can be very hard to find $F(x)$

$$F(x) = \sin(x^2) \Rightarrow F'(x) = \cos(x^2) \cdot 2x$$

$$= 2x\cos(x^2)$$

EASY!

$$\int f(x)\,dx = \int \sin(x^2)\,dx =$$ Let me know when you find a "nice" soln

"nice" soln means CLOSED FORM

Examples

a) $\int x^2 + \pi x^{3/2} - 3 \, dx$

b) $\int e^{2x} - 3\cos(2x) + \frac{1}{x^3} \, dx$

c) $\boxed{\int t^{3/2} - e^{2t} + \sin(4t) \, dt}$

For you!

d) $\int \sin\left(\frac{3}{2}q\right) + \cos(\pi q) \, dq$

e) $\int (x^2+1)(x^3-x) \, dx$

$$\boxed{\frac{t^{3/2+1}}{3/2+1} - \frac{1}{2}e^{2t} - \frac{1}{4}\cos(4t) + C}$$

check

a) $\int \underline{x^2 + \pi x^{3/2} - 3}\, dx$

$= \int \underbrace{x^2}_{r=2} dx + \pi \int \underbrace{x^{3/2}}_{r=3/2} dx - \int 3\, dx$

$$\boxed{\int x^r dx = \frac{x^{r+1}}{r+1} + C \quad r \neq -1}$$

$= \frac{x^{2+1}}{2+1} + \pi \frac{x^{3/2+1}}{3/2+1} - 3x + C$

$= \frac{1}{3} x^3 + \pi \frac{x^{5/2}}{5/2} - 3x + C$

$= \boxed{\frac{1}{3} x^3 + \frac{2\pi}{5} x^{5/2} - 3x + C}$

b) $\int e^{2x} - 3\cos(2x) + \frac{1}{x^3}\,dx$

$$= \int e^{2x}\,dx - 3\int \cos(2x)\,dx + \int \frac{1}{x^3}\,dx$$

$$= \int \underbrace{e^{2x}}_{a=2}\,dx - 3\int \underbrace{\cos(2x)}_{a=2}\,dx + \int \underbrace{x^{-3}}_{r=-3}\,dx$$

$$= \frac{1}{2}e^{2x} - 3\left(\frac{1}{2}\sin(2x)\right) + \frac{x^{-3+1}}{-3+1} + C$$

$$= \boxed{\frac{1}{2}e^{2x} - \frac{3}{2}\sin(2x) - \frac{1}{2}x^{-2} + C}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$$

$r \neq -1$

d) $\int \sin\left(\frac{3}{2}q\right) + \cos(\pi q)\, dq$

$a = 3/2 \qquad a = \pi$

$= -\frac{1}{3/2}\cos\left(\frac{3}{2}q\right) + \frac{1}{\pi}\sin(\pi q) + C$

$= \boxed{-\frac{2}{3}\cos\left(\frac{3}{2}q\right) + \frac{1}{\pi}\sin(\pi q) + C}$

check: $\frac{d}{dx}\left(-\frac{2}{3}\cos\left(\frac{3}{2}q\right) + \frac{1}{\pi}\sin(\pi q) + C\right) = -\frac{2}{3}\cdot\left(-\sin\frac{3}{2}q\right)\cdot\frac{3}{2} + \frac{1}{\pi}\cos(\pi q)\cdot\pi + 0$

$= \sin\left(\frac{3}{2}q\right) + \cos(\pi q)$

Yes!

$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + C$

$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + C$

e) $\int (x^2+1)(x^3-x)\,dx$

$$(x^2+1)(x^3-x) = x^5 - x^3 + x^3 - x = x^5 - x$$

$$= \int x^5 - x^1\,dx = \frac{x^{5+1}}{5+1} - \frac{x^{1+1}}{1+1} + C$$

$$= \boxed{\frac{1}{6}x^6 - \frac{1}{2}x^2 + C}$$