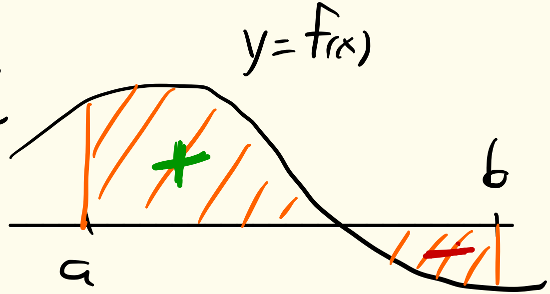
Information:

Chapter 7: only covers 7.1 and 7.2
(7.4 excluded)

Super Makeups: Email me at
johnston@math.umass.edu
and include documentation as to why you
qualify for a super makeup.

Fundamental Thm of Calculus



$$\int_a^b f(x)\,dx \overset{FTC}{=} F(b) - F(a)$$

where $\boxed{F'(x) = f(x)}$

$$F(b) - F(a) = \int_a^b f(x)\,dx = \int_a^b F'(x)\,dx$$

$$\underbrace{F(b)}_{\text{F at b}} = \underbrace{F(a)}_{\text{F at a}} + \underbrace{\int_a^b F'(x)\,dx}_{\text{integral of the rate of change, } F'(x),\text{ of } F(x)}$$

Find $F(x)$ can be HARD!

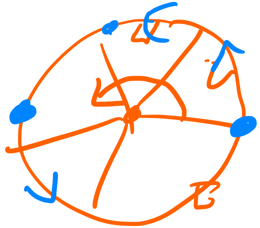
Chapter 5, Web Quiz, Question 38

rate of change

The velocity of a ferris wheel is $r(t) = 10\sin(\pi t)$, where t is measured in minutes. How long does it take the ferris wheel to return to its original position?

- ( ) 4 minutes
- (•) 2 minutes
- ( ) 1.5 minutes
- ( ) 1 minute

$t=1$ (suppose)



$t=0$

$\boxed{A\sin(Bt)+C}$

$r(t) = \underbrace{10}_{A}\sin(\underbrace{\pi}_{B} t)$

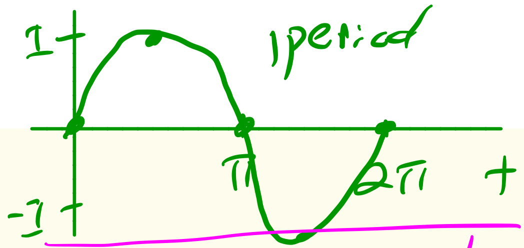
Amplitude $= |A| = \boxed{10}$

period $= \dfrac{2\pi}{|B|} = \dfrac{2\pi}{\pi} = \boxed{2}$

sin(t):

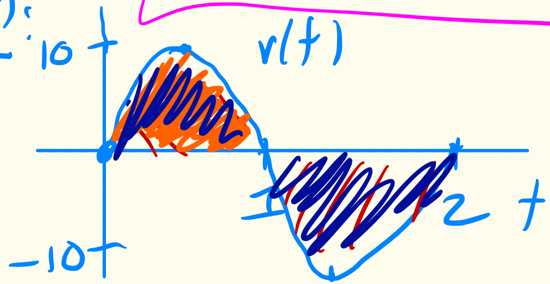
amp $= 1$

period $= 2\pi$



Area $=$ +area + |-area|

10 sin(πt):



$t^* > 0$, some time in the future

$s(t)$ - position $\qquad r(t) = 10\sin(\pi t)$

(or $v(t)$)

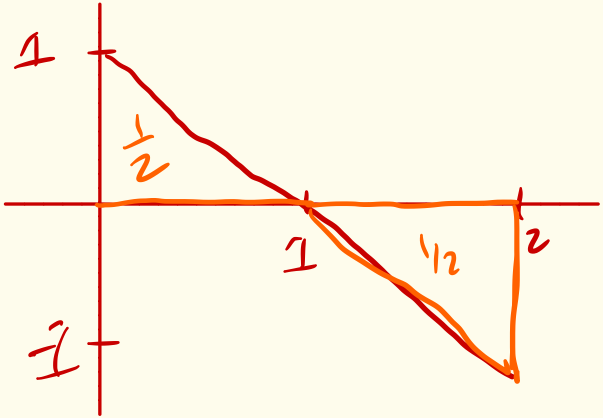
$$\underbrace{s(t^*)}_{\text{position at time } t^*} = \underbrace{s(0)}_{\text{initially}} + \underbrace{\boxed{\int_0^{t^*} r(t)\,dt}}_{\text{change in position over } [0,t^*] \text{ time period}}$$

area under $r(t)$

Ex: $s(1) = s(0) + \boxed{\int_0^1 r(t)\,dt} > 0$

$$\boxed{s(2)} = s(0) + \boxed{\int_0^2 r(t)\,dt} = 0 \quad = s(0) + 0 = \boxed{s(0)}$$

Note: $y = f(x) = -x + 1$



Area between $f(x)$ and the x-axis over $[0,2]$ is $\boxed{1}$

$$\int_0^2 -x+1\,dx = 0 \quad \left(\frac{1}{2} - \frac{1}{2}\right)$$

Chapter 5, Section 5.4, Question 12

The velocity of a car (in miles per hour) is given by $v(t) = 20t - 7t^2$, where t is in hours.

$= t(20 - 7t) = 0$

$\Rightarrow t = 0$ or $20 - 7t = 0$ or $t = \frac{20}{7}$

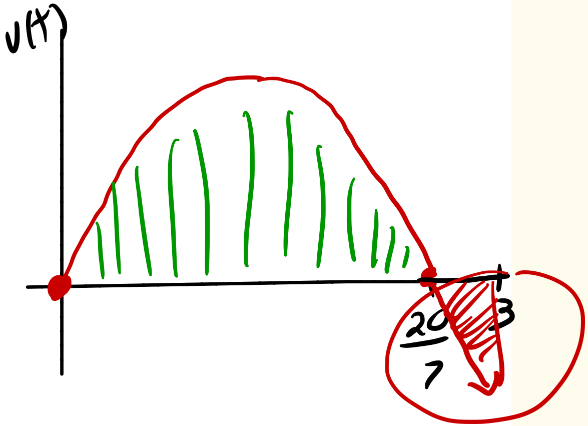
(a) Write a definite integral for the distance the car travels during the first three hours.

$$\underbrace{\int_0^{20/7} v(t)\,dt}_{\text{positive}} - \underbrace{\int_{20/7}^{3} v(t)\,dt}_{\text{negative}}$$

$$= \boxed{\int_0^{20/7} 20t - 7t^2\,dt} - \int_{20/7}^{3} 20t - 7t^2\,dt$$

$$\approx 27.211\ldots - (-0.211\ldots)$$

$$= 27.422\ldots$$



(b) Use a computer or calculator to find the distance traveled during the first three hours.

Enter the exact answer.

Distance traveled is

27.422 miles.

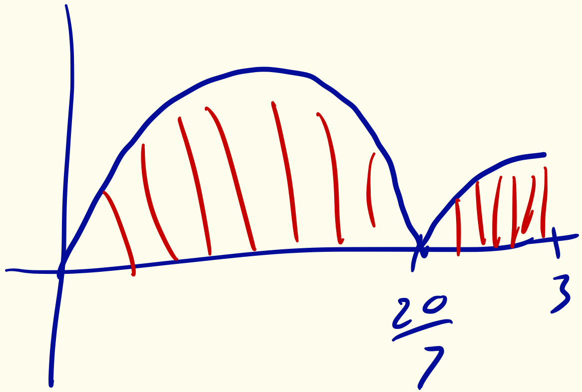
Note:

$$\int_0^{20/7} v(t)\,dt - \int_{20/7}^{3} v(t)\,dt = \int_0^3 |v(t)|\,dt$$

account for $v(t) \le 0$

$$\approx 27.422\ldots$$

$y = |v(t)|$



Chapter 5, Section 5.4, Question 19

A forest fire covers 2002 acres at time $t = 0$. The fire is growing at a rate of $8\sqrt{t}$ acres per hour, where t is in hours. How many acres are covered 24 hours later?

Round your answer to the nearest integer.

Total area covered 24 hours later is 2629 acres

the absolute tolerance is +/-1

$C(t)$ - coverage at time t

$C(0) = 2002$

$$C(24) = C(0) + \int_0^{24} C'(t)\,dt$$

$$= 2002 + \int_0^{24} 8\sqrt{t}\,dt = 2002 + \boxed{8\int_0^{24} t^{1/2}\,dt}$$

$$\approx 2002 + 627.069 = 2629.069\ldots$$

Exact Ans:

$$\int_0^{24} \underbrace{t^{1/2}}_{f(t)}\,dt \overset{FTC}{=} F(24) - F(0) \quad \text{where } F'(t) = f(t) = t^{1/2}$$

what is $F(t)$?

$$f'(t) = (t^{1/2})' = \frac{1}{2}t^{-1/2}$$

$$F(t) = \frac{2}{3}t^{3/2} + C$$

check!

$$F'(t) = \left(\frac{2}{3}\right)\frac{3}{2}t^{3/2-1}$$
$$= 1\,t^{1/2}$$

Yes

$$= 8\cdot\int_0^{24} t^{1/2}\,dt$$

$$= 8\cdot\left[F(24) - F(0)\right]$$

$$= 8\cdot\left[\left(\frac{2}{3}24^{3/2} + \cancel{C}\right) - \left(\frac{2}{3}0^{3/2} + \cancel{C}\right)\right]$$

$$= \boxed{\frac{16}{3}\cdot 24^{3/2}} \approx 627.069\ldots$$

Exact

power rule

$$\frac{d}{dt}(t^r) = r\,t^{r-1}$$

$$\int t^r\,dt = \frac{t^{r+1}}{r+1} + C$$
$r \neq -1$

Ex:

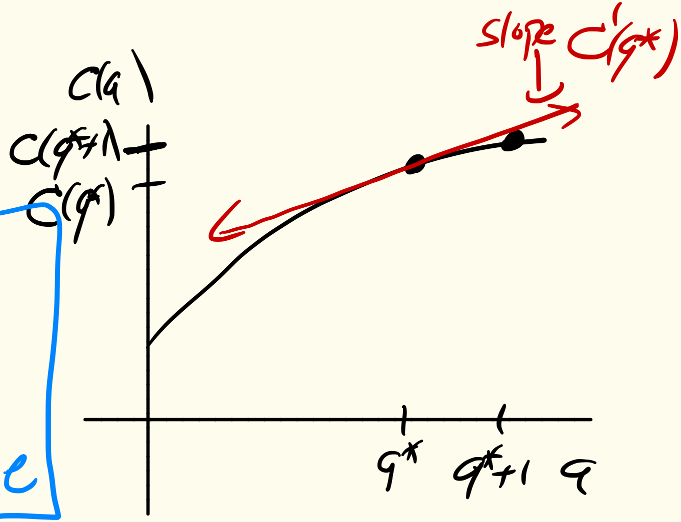
$$\int t^4\,dt = \frac{t^{4+1}}{4+1} + C$$
$r=4$
$$= \frac{1}{5}t^5 + C$$

Profit, Cost and Revenue

q – # widgets we produce

$$\underbrace{P(q)}_{\text{profit}} = \underbrace{R(q)}_{\text{Revenue}} - \boxed{\underbrace{C(q)}_{\text{cost}}}$$

(i) fixed
(ii) variable



Maximize Profit

$$0 = \underbrace{P'(q)}_{\text{marginal profit}} = \underbrace{R'(q)}_{\text{marginal revenue}} - \underbrace{C'(q)}_{\text{marginal revenue}}$$

approx cost of producing one more unit

Chapter 5, Section 5.5, Question 3bcd

The total cost in dollars to produce q units of a product is $C(q)$. Fixed costs are $\$13{,}000$. The marginal cost is

$$C'(q) = 0.008q^2 - q + 49.$$

$C(0) = 13{,}000$

Work to full accuracy but enter your answers to two decimal places.

$C(190) = C(0) + \int_0^{190} C'(q)\,dq$

(a) Estimate $C(190)$, the total cost to produce 190 units.

$= 13{,}000 + \int_0^{190} 0.008q^2 - q + 49\,dq$

The total cost to produce 190 units is $ 22,550 2/3.

$= 13{,}000 + 9550\ 2/3$

$= 22550\ 2/3$

(b) Find the value of $C'(190)$.

$C'(190) = \$$ 147.80

$c'(190) = 147.8$

marginal cost

when $q = 190$

(c) Use parts **(a)** and **(b)** to estimate $C(191)$.

$C(191) = \$$ 22,550 2/3 + 147.80