3/6:

- HW 3.3-3.5 due 3/10
- Practice derivative problems
- Read 4.1-4.2

a) $f(x) = \ln(5 - e^{-x})$

b) $P = t^6 \ln(6t)$

c) $y = \ln e^{2x}$

d) $w = \dfrac{3y + y^2}{2 + y}$

e) $y = 3x \sin(2x)$

f) $g(K) = A\cos(BK + C)$

A, B, C constants

c) $y = \ln e^{2x} = 2x$

$$\boxed{\frac{dy}{dx} = 2}$$

$\ln e^{\square} = \square$, $\square$ - any expression

$e^{\ln \square} = \square$, $\square$ - any POSITIVE expression

f) $g(K) = A\cos(BK+c)$

$$g'(K) = A(-\sin(BK+c))\cdot(BK+C)'$$

$$= -A\sin(BK+c)\cdot(B+0)$$

$$= \boxed{-AB\sin(BK+c)}$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cos \underbrace{u(x)}_{\text{any function of } x}) = (-\sin(u(x)))\cdot u'(x)$$

$$= -u'(x)\sin(u(x))$$

CHAIN RULE

a) $f(x) = \ln(5 - e^{-x})$

$$= \frac{u'(x)}{u(x)}$$

$$= \boxed{\frac{e^{-x}}{5 - e^{-x}}}$$

$$\frac{\frac{1}{e^x}}{5 - \frac{1}{e^x}}$$

$u(x) = 5 - e^{-x}$

$u'(x) = 0 - (e^{-x})'$

$= -(e^{-x}) \cdot (-x)'$

$= -(e^{-x})(-1)$

$= e^{-x}$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \xrightarrow[\text{Rule}]{\text{Chain}} \quad \frac{d}{dx}(\ln u(x)) = \frac{1}{u(x)} \cdot u'(x)$$

$$= \frac{u'(x)}{u(x)}$$

w/ $u(x) = x$

$$= \frac{u'(x)}{u(x)} = \frac{1}{x}$$

b) $P = \underbrace{t^6}_{f} \underbrace{\ln(6t)}_{g}$

$$= t^6 \cdot (\ln(6t))' + \ln(6t) \cdot (t^6)'$$

$$= \cancel{t^6}^{5} \cdot \frac{1}{\cancel{6t}} \cdot \cancel{6} + \ln(6t) \cdot (6t^5)$$

$$= t^5 + 6t^5 \ln(6t)$$

$$= \boxed{t^5(1 + 6\ln(6t))} = 0 ?$$

product rule: $(f \cdot g)' = fg' + gf'$

$$1 + 6\ln(6t) = 0$$

$$\ln(6t) = -\frac{1}{6}$$

or $$\underbrace{e^{\ln(6t)}}_{6t} = e^{-1/6}$$

$$6t = e^{-1/6} \Rightarrow \boxed{t = \frac{1}{6e^{1/6}}}$$

$$e^{\ln \square} = \square, \quad \square > 0$$

d) $w = \dfrac{3y + y^2}{2+y}$, not defined when $y = -2$

$$= \frac{(2+y)(3+2y) - (3y+y^2)(1)}{(2+y)^2}$$

$$= \frac{6 + 4y + \cancel{3y} + 2y^2 - \cancel{3y} - y^2}{(2+y)^2} = \frac{y^2 + 4y + 6}{(2+y)^2} \overset{?}{=} 0$$

never 0

Quotient Rule: $\left(\dfrac{f}{g}\right)' = \dfrac{gf' - fg'}{g^2}$

$$y^2 + 4y + 6 = 0$$

$a=1,\ b=4,\ c=6$

$$y = \frac{-4 \pm \sqrt{16-4\cdot1\cdot6}}{2}$$

$$= \frac{-4 \pm \sqrt{-8}}{2}$$

$$= \frac{-4 \pm \sqrt{8(-1)}}{2}$$

$$= -4 \pm \sqrt{8}\cdot\sqrt{-1}$$

i

$$= \frac{-4 \pm 2\sqrt{2}\,i}{2}$$

$$= -2 \pm \sqrt{2}\,i$$

Imaginary

$$ay^2 + by + c = 0$$

$$\Rightarrow \quad y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\sqrt{8} = \sqrt{2\cdot4}$$

$$= \sqrt{2}\cdot\sqrt{4}$$

$$= 2\sqrt{2}$$

e) $y = 3x \sin 2x$

$= (3x)(\sin(2x))' + \sin(2x) \cdot (3x)'$

$= 3x \cos(2x) \cdot 2 + (\sin 2x) \cdot 3$

$= \boxed{3(2x \cos 2x + \sin 2x)}$

$$\frac{d}{dx}(\sin u(x)) = (\cos u(x))\, u'(x)$$

$$= u'(x) \cos u(x)$$

*Chapter 3, Section 3.5, Question 27

The depth of the water, y, in meters, in the Bay of Fundy, Canada, is given as a function of time, t, in hours after midnight, by the function

$$y = 10 + 7.5\cos(0.507t).$$

$\frac{dy}{dt}$ $\frac{\text{meters}}{\text{hr}}$

How quickly is the tide rising or falling (in meters/hour) at each of the following times?

Round your answers to two decimal places.

$t=4$

(a) 4:00 am	The tide is	falling	at	−3.41	*1 meters/hour.
(b) 10:00 am	The tide is		at		*2 meters/hour.
(c) Noon	The tide is		at		*3 meters/hour.
(d) 4:00 pm	The tide is		at		*4 meters/hour.

$t=16$

t hours since midnight

y meters

$$y = 10 + \underline{7.5}\ \underbrace{\cos(\underline{0.507t})}_{\pm 1}$$

$$2.5\text{m} = 10 - 7.5 \le y(t) \le 10 + 7.5 = 17.5\text{ m}$$

$$\frac{dy}{dt} = 0 + 7.5(-\sin(0.507t)) \cdot (0.507)$$
$$= -(7.5)(0.507)\sin(0.507t)$$

$t = 4$: $\left.\frac{dy}{dt}\right|_{t=4} = (-7.5)(0.507) \cdot \sin(0.507 \cdot 4)$
4am

$$\approx -3.4119 \text{ m/hr}$$

graphed with DESMOS (online graphing)