Profit, Revenue and Cost

produce q widgets

$$\underbrace{P(q)}_{\text{profit}} = \underbrace{R(q)}_{\text{Revenue}} - \underbrace{C(q)}_{\text{cost}}$$

cost: (i) fixed cost
(ii) costs depend q

Maximize Profit? When

$$0 = \underbrace{P'(q)}_{\text{marginal profit}} = \underbrace{R'(q)}_{\text{marginal revenue}} - \underbrace{C'(q)}_{\text{marginal cost}}$$

marginal cost ($\approx$ cost of producing 1 more item)

or

$$P'(q) = 0 \iff R'(q) = C'(q)$$

Ex: Suppose $R(q) = 5q - 0.003q^2$, $C(q) = 300 + 1.1q$ $\Big\}$ $P(q) = R(q) - C(q)$

what q maximizes the profit?

Find where $R'(q) = C'(q)$!

$R'(q) = 5 - 0.006q$

$C'(q) = 1.1$

$R'(q) = C'(q) \Rightarrow 5 - 0.006q = 1.1$

or $-0.006q = 1.1 - 5 = -3.9$

or $q = \frac{-3.9}{-0.006} = \frac{3.9}{0.006} = 650$

$\boxed{\text{why must this be a max?}}$

So, we should produce 650 widgets to maximize profit!

Ex: The price for a half-day of white water rafting is \$80 when 300 people go. For every \$5 decrease in price 30 additional people go. What price MAXIMIZES REVENUE?

ans: $\boxed{R(p,q) = p \cdot q}$

p - price

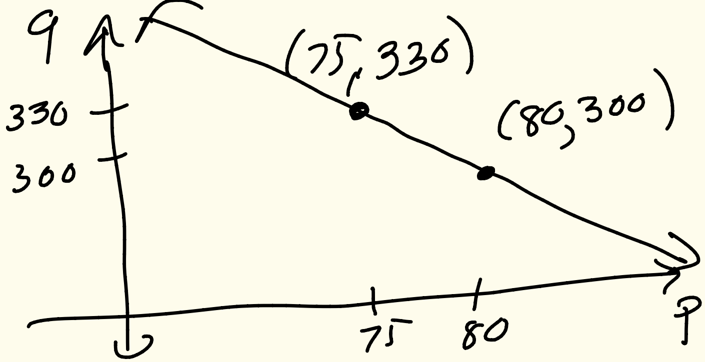
q - # of people

given info: (p,q) $(80, 300)$ and $(75, 330)$

Find a formula for q in terms of p.

Assume q is a LINEAR function of p

$$m = \frac{\Delta q}{\Delta p} = \frac{330-300}{75-80}$$
$$= \frac{30}{-5} = -6$$



$$q - q_0 = m(p - p_0)$$

(p_0, q_0) is a pt on the graph:

$$q - 300 = -6(p-80) \quad \text{or} \quad q - 300 = -6p + 480$$
$$\text{or} \quad \boxed{q = -6p + 780}$$

$$\Rightarrow R(p) = p\cdot(-6p+780) \Rightarrow R'(p) = -12p + 780 = 0$$
$$= -6p^2 + 780p \qquad p = \frac{780}{12} = \frac{390}{6} = \frac{195}{3} = 65$$

If we charge \$65 we maximize revenue

why?

well, $R''(p) = -12 < 0$ so $R'(65) = 0$, $R''(65) < 0$ $\Rightarrow$ 65 is a max!

and derivative test

Exam Review: 2013 exam

4. $F(x) = 3^x + 2\ln x - \frac{1}{x}$ what is $F'(1)$

6. $g(t) = \ln(t^2 - 5)$ Find $g''(t)$

10. Find the equation of the tangent line to $y = x\sin x$ at $x = \pi$.

17. What are the points of inflection of $f(x) = x^4 + x^3 - 3x^2 + 2$?

4. $F(x) = 3^x + 2\ln x - \frac{1}{x}$ what is $f'(1)$?

minus

$\frac{d}{dx}(a^x) = (\ln a)a^x$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$, $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1})$

$= -1\,x^{-1-1} = \frac{-1}{x^2}$

$f'(x) = (\ln 3)3^x + 2\cdot\frac{1}{x} - \left(-\frac{1}{x^2}\right)$

$= (\ln 3)3^x + \frac{2}{x} + \frac{1}{x^2}$

$f'(1) = (\ln 3)3^1 + \frac{2}{1} + \frac{1}{1^2}$

$= 3\ln 3 + 3 = 3(\ln 3 + 1)$

6. $g(t) = \ln(t^2-5)$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln g(x)) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$$

$$g'(t) = \frac{1}{t^2-5} \cdot (t^2-5)'$$

$$= \frac{2t}{t^2-5}$$

quotient rule

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$g''(t) = (g'(t))' = \left(\frac{2t}{t^2-5}\right)'$$

$$= \frac{(t^2-5)(2t)' - 2t(t^2-5)'}{(t^2-5)^2}$$

$$= \frac{(t^2-5)2 - 2t(2t)}{(t^2-5)^2} = \frac{2t^2-10-4t^2}{(t^2-5)^2} = \frac{-2t^2-10}{}$$

$$= \frac{-2t^2-10}{(t^2-5)^2} = \frac{-2(t^2+5)}{(t^2-5)^2}$$

10! $f(x) = x\sin x \quad , x = \pi$

1) $(\pi, f(\pi)) = (\pi, \pi \cdot \sin\pi) = \boxed{(\pi, 0)}$ pt on the tangent line

2) slope: $f'(x) = x\cos x + \sin x \cdot 1$ product rule

$= x\cos x + \sin x$

$(f \cdot g)' = fg' + gf'$

$f'(\pi) = \pi\cos\pi + \sin\pi$

$= \pi \cdot (-1) + 0 = \boxed{-\pi}$

Eqn: $y - 0 = -\pi(x - \pi)$

$$\boxed{y = -\pi x + \pi^2}$$

17. inflection pts of $f(x) = x^4 + x^3 - 3x^2 + 2$

$$f'(x) = 4x^3 + 3x^2 - 6x$$

$$f''(x) = 12x^2 + 6x - 6$$

$$= 6(2x^2 + x - 1) = 0$$

$$= 12\left(x^2 + \frac{x}{2} - \frac{1}{2}\right)$$

$$= 12(x+1)\left(x - \frac{1}{2}\right)$$

$$2x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{4}$$

$$= \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$$

$$= \frac{1}{2} \text{ or } -1$$

chart

$$f''(0) = -6 < 0$$

$$f''(1) = 12 + 6 - 6 = 12 > 0$$

$$f''(-2) = 12 \cdot 4 - 12 - 6 = 30 > 0$$

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$-2 \quad -1 \quad 0 \quad \frac{1}{2} \quad 1$

$\boxed{x = -1 \text{ and } x = \frac{1}{2} \text{ are the pts of inflection}}$