Recall: To find GLOBAL mins and Maxs on a interval $[a,b]$, check (i) critical pts

(ii) end points

Ex: Find the GLOBAL min and max of $f(x) = x^3 - 3x^2$ on $[-1, 3]$

ans: crit pts? $f'(x) = 3x^2 - 6x = 3x(x-2) = 0$

$\Rightarrow$ $x = 0$ and $x = 2$

x	$f(x)$
-1	$-1^3 - 3(-1)^2 = -4$
0	0
2	$2^3 - 3\cdot 2^2 = 8 - 12 = -4$
3	$3^3 - 3\cdot 3^2 = 3^3 - 3^3 = 0$

GLOBAL MAX $= 0$ and occurs at $x=0$ and $x=3$

GLOBAL MIN $= -4$ and occurs at $x=-1$ and $x=2$

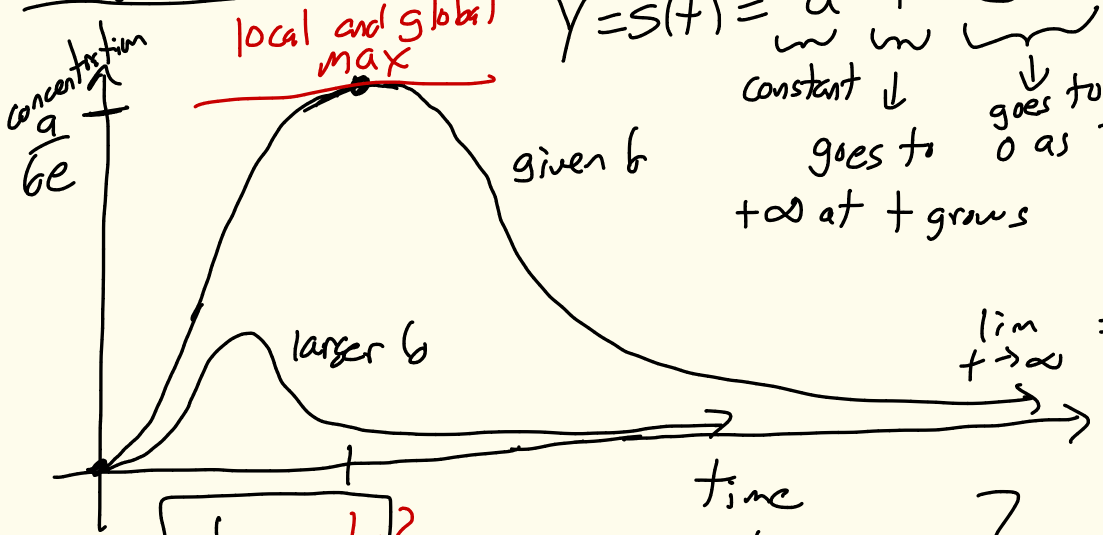
Applications: 4.4 Profit, Revenue, Cost (Th)

4.7 Logistic Function (Done)

$\boxed{\text{4.8 Surge Function}}$

drug injection

$$y = s(t) = \underbrace{a}_{\text{constant}} \cdot \underbrace{t}_{\text{goes to } +\infty \text{ as } t \text{ grows}} \cdot \underbrace{e^{-bt}}_{\text{goes to } 0 \text{ as } t \text{ grows}}, \quad a, b \text{ positive constants}$$



$$\lim_{t \to \infty} = 0$$

$$\boxed{\frac{1}{b} = t}\,?$$

Where is the max?

Max?!

$$s(t) = a t e^{-bt}$$

Product rule

$$\boxed{(f \cdot g)' = f g' + g f'}$$

$$s'(t) = a\left(t \cdot \left(-b e^{-bt}\right) + e^{-bt} \cdot 1\right)$$

$$\left(e^{-bt}\right)' = -b e^{-bt}$$

$$= a\left(-bt e^{-bt} + e^{-bt}\right)$$

$$= \underbrace{a}_{\neq 0} \underbrace{e^{-bt}}_{\neq 0} \underbrace{(-bt + 1)}_{\overset{?}{=} 0!} = 0 \implies \boxed{t = \frac{1}{b}}$$

$$s\left(\frac{1}{b}\right) = a \cdot \frac{1}{b} \cdot e^{-b\left(\frac{1}{b}\right)} = \frac{a}{b} \cdot e^{-1} = \frac{a}{be}$$

So the local max is at $\boxed{\left(\frac{1}{b}, \frac{a}{be}\right)}$
global

Ex: $s(t) = 9.8\,t\,e^{-0.24t}$ $\quad (a = 9.8,\ b = 0.24)$

What is the maximum concentration and when does it occur?

ans: $t_{max} = \frac{1}{b} = \frac{1}{0.24} \approx 4.1666\ldots$

$$s(t_{max}) = s\left(\frac{1}{0.24}\right) = \frac{a}{be} = \frac{9.8}{0.24 \cdot e} \approx 15.0217\ldots$$

or $(\approx 4.17, \approx 15.02)$

Ex! $s(t) = 10 t e^{-0.5t}$ $\quad (a = 10, \ b = 0.5)$

Is $s(t)$ increasing or decreasing when $t = 1$?

ans!

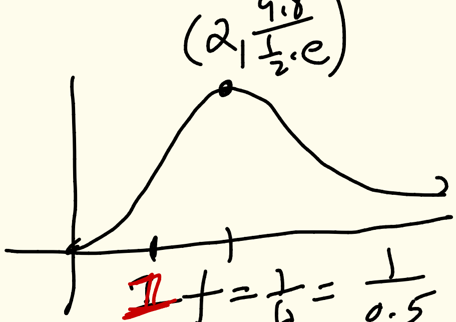
method 1

$$s'(t) = a e^{-bt}(-bt + 1) = 10 e^{-0.5t}(1 - 0.5t)$$

$$s'(1) = \underbrace{10}_{+}\ \underbrace{e^{-0.5(1)}}_{+}\ \underbrace{(1 - 0.5 \cdot 1)}_{+} > 0 \Rightarrow$$ $s(t)$ is INCREASING at $t = 1$

sign $\quad + \quad + \quad +$

method 2



$$t = \frac{1}{b} = \frac{1}{0.5} = 2 > 1$$

Since $1 < 2$ which is the GLOBAL MAX occurs, $s(t)$ is INCREASING at $t = 1$

Profit, Cost and Revenue: (4.4)

Suppose we want to make $\underbrace{q}$ items

$$\underbrace{P(q)}_{\text{profit}} = \underbrace{R(q)}_{\text{revenue}} - \underbrace{C(q)}_{\text{cost}}$$

cost = (i) fixed cost
(ii) cost per unit q

Maximize: $0 = P'(q) = R'(q) - C'(q) = 0$

So $\boxed{P'(q) = 0}$ when $\boxed{R'(q) = C'(q)}$

$P'(q)$: marginal profit

$\underbrace{R'(q)}_{\text{marginal revenue}} = \underbrace{C'(q)}_{\text{marginal cost}}$