Logistic Function: (4.7) L, C, K positive constants

$$\left(\frac{g}{h}\right)' = \frac{hg' - gh'}{h^2}$$

$$f(t) = \frac{L}{1 + Ce^{-kt}}$$

$$f'(t) = \frac{(1+ce^{-kt})(0) - L(C(-k)e^{-kt})}{(1+ce^{-kt})^2}$$

$$= \frac{LCK\, e^{-kt}}{(1+ce^{-kt})^2} > 0$$

Always increasing

$LCK > 0$

$e^{-kt} > 0$

$(1+ce^{-kt})^2 > 0$

$$f''(t) = LCK\left[\frac{(1+ce^{-kt})^2 \cdot (e^{-kt})' - e^{-kt} \cdot \left[(1+ce^{-kx})^2\right]'}{(1+ce^{-kt})^4}\right]$$

= after some algebra (CHECK!)

$$= \frac{-kLCe^{-kt}(1-ce^{-kt})}{(1+ce^{-kt})^3} \overset{?}{=} 0$$

$f''(t) = 0$ when $1 - ce^{-Kt} = 0$

$\ln \square^{\Delta} = \Delta \ln \square$

or $e^{-Kt} = \frac{1}{c}$ or $\ln(e^{-Kt}) = \ln\left(\frac{1}{c}\right)$

or $-Kt = \ln(1) - \ln(c)$

$= 0 - \ln(c)$

$\ln e^{\square} = \square$

$\ln \frac{\square}{\Delta} = \ln \square - \ln \Delta$

$e^{\ln \square} = \square$

$$\Rightarrow \boxed{t = \frac{-\ln c}{-K} = \frac{\ln c}{K}}$$

$+ + + + + + + \quad f'$

$\frac{\ln c}{K}$

$+ + + \quad 0 \quad - - - - \quad f''$

What is $f\left(\frac{\ln c}{K}\right)$?

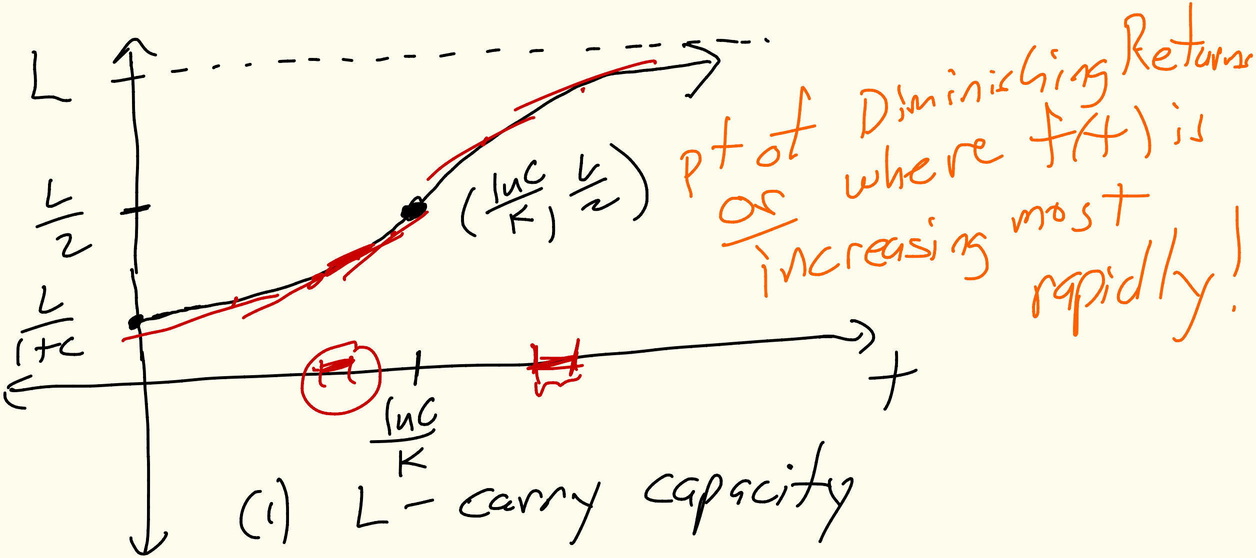
$$f\left(\frac{\ln c}{K}\right) = \frac{L}{1 + ce^{-\cancel{K}\left(\frac{\ln c}{\cancel{K}}\right)}}$$

$$= \frac{L}{1 + ce^{-\ln c}} = \frac{L}{1 + ce^{\ln c^{-1}}}$$

$$= \frac{L}{1 + c \cdot c^{-1}} = \frac{L}{1+1} = \boxed{\frac{L}{2}}$$

$\Rightarrow t = \frac{\ln c}{K}$ is a pt of inflection graph: $F(t) = \dfrac{L}{1+ce^{-kt}}, \quad t \geq 0$

$F(0) = \dfrac{L}{1+ce^{0}} = \dfrac{L}{1+c}, \quad F\left(\dfrac{\ln c}{k}\right) = \dfrac{L}{2}$



(1) L – carry capacity

(2) $\lim_{t \to \infty} F(t) = L$

*Chapter 4, Section 4.7, Question 1

If t is in years since 1990, one model for the population of the world, P, in billions, is

$L = 40$

$C = 11$

$K = 0.08$

$P(0) = \frac{40}{1+11} = \frac{40}{12} = \frac{10}{3} = 3\frac{1}{3}$

$$P = \frac{40}{1 + 11e^{-0.08t}}.$$

(a) What does this model predict for the maximum sustainable population of the world?

carrying capacity

Enter an exact answer.

This model predicts that when t is very large, the population is 40 billion.

(b) According to this model, when will the earth's population reach 25 billion? 39.9 billion?

when is $P(t) = 25$

Round your answers to the nearest year.

The population of the world should be 25 billion in 2016 *1.

The population of the world should be 39.9 billion in ______ *2.

Answer *1: *the absolute tolerance is +/-1*
Answer *2: *the absolute tolerance is +/-1*

$$\frac{40}{1+11e^{-0.08t}} = 25 \quad \text{and solve for } t$$

$$1+11e^{-0.08t} = \frac{40}{25} = \frac{8}{5}$$

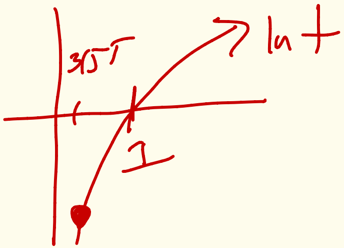
$$11e^{-0.08t} = \frac{8}{5} - 1 = 3/5$$

$$e^{-0.08t} = \frac{3}{55} \Rightarrow \ln\left(e^{-0.08t}\right) = \ln\left(3/55\right)$$

$$-0.08t = \ln\left(3/55\right)$$

$$\text{or} \quad t = \frac{\ln\left(3/55\right)}{-0.08} = 36.359\ldots$$

$$1990 + 36 = 2026$$



*Chapter 4, Section 4.7, Question 8

Find the exact coordinates of the point at which the following curve is steepest:

$$y = \frac{60}{1+6e^{-2t}} \text{ for } t \geq 0.$$

$L = 60$

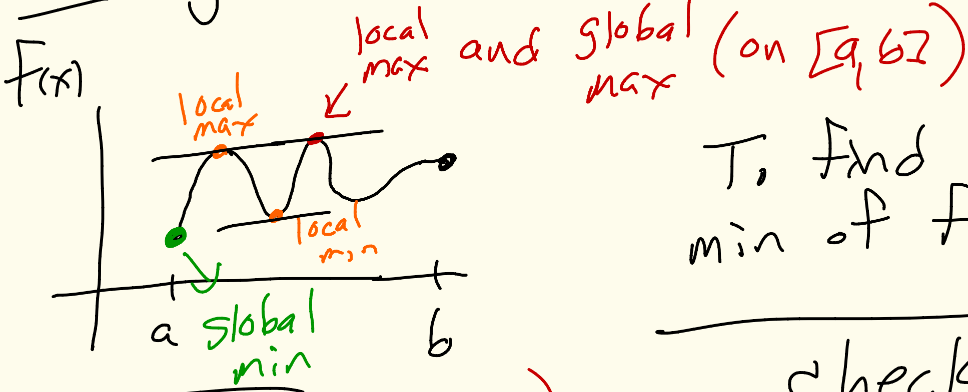
$c = 6$

$k = 2$

ans: the same as the point of diminishing returns

$$\left(\frac{\ln c}{k}, \frac{L}{2}\right) = \left(\frac{\ln 6}{2}, 30\right)$$

4.3: global mins and maxs on [a, b]

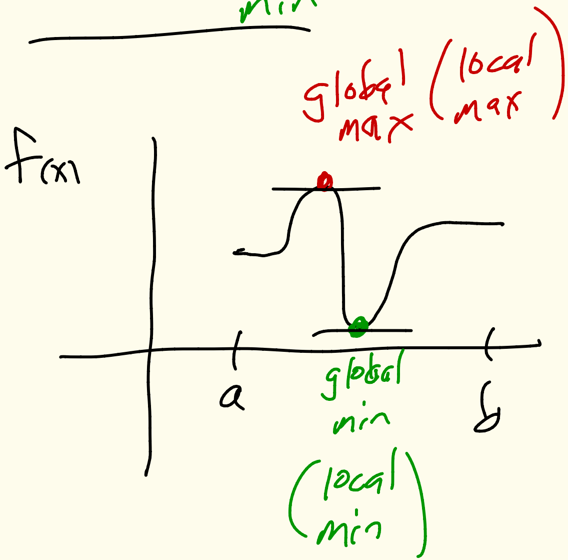


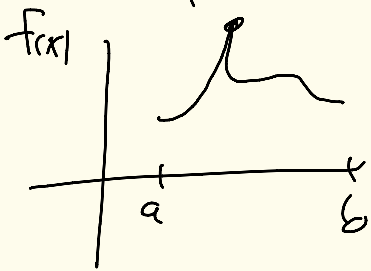
To find GLOBAL max and min of f(x) over [a, b]

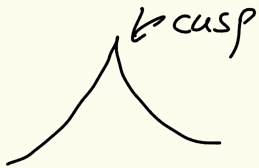
check

(i) critical pts

(ii) endpoints







Chapter 4, Web Quiz, Question 24

$[a,b]=[0,4]$

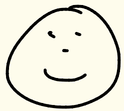
For the following function find the global maximum. $f(x) = x^3 + 2x + 17, 0 \le x \le 4$

- ● 89
- ○ 6
- ○ -6
- ○ There is no global maximum.

(1) Find critical pts

$f'(x) = 3x^2 + 2 \neq 0$

$\Rightarrow$ no critical pts



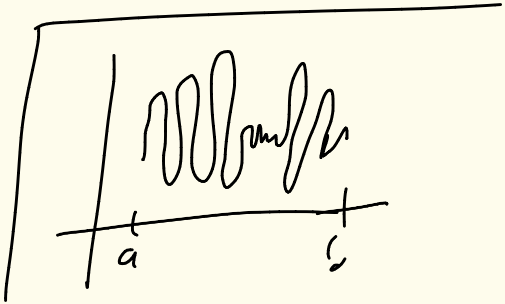
(2) $x=0 \Rightarrow f(0) = 17$

$x=4 \Rightarrow f(4) = 64+8+17$

$= 89$

endpoints

$\max\{17, 89\} = 89$



Chapter 4, Section 4.3, Question 26

Find the exact global maximum and minimum values of the function.

If there is no global minimum or maximum enter *none*.

$$f(x) = x - \ln x \quad \text{for} \quad x > 0$$

$f(x) > 0$ on interval $(0, \infty)$

The global minimum is 1.

The global maximum is NONE.

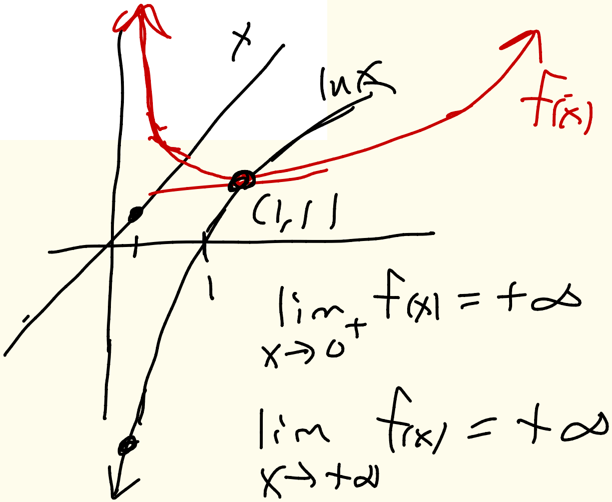
$$f'(x) = 1 - \frac{1}{x} = 0$$

$\Rightarrow x = 1$ only critical pt!

global min: $f(1) = 1 - \ln 1$

$= 1 - 0$

$= 1$



$$\lim_{x \to 0^+} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$