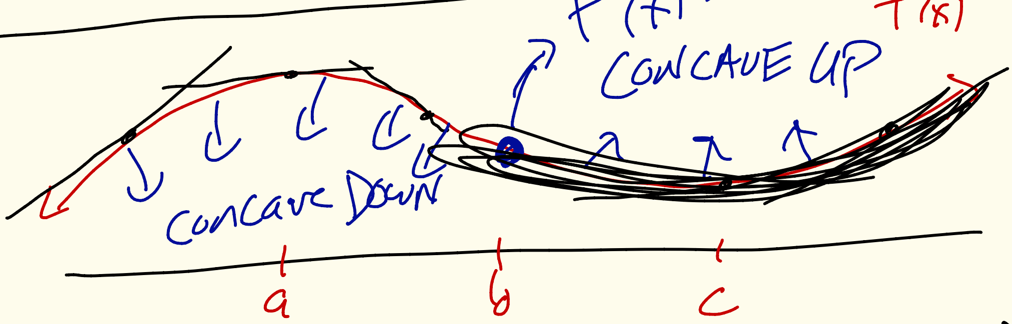
Local Mins and Maxs



$+ \ + \ + \ + \ 0 \ - \ - \ - \ - \ - \ 0 \ + \ + \ +$ $f'(x)$

1st Derivative Test: Suppose p is a critical pt
$\left(\begin{array}{c} f'(p) = 0 \\ \text{or} \\ f'(p) = DNE \end{array} \right)$

(a) If $\boxed{f'(x)}$ changes sign from $\boxed{+ \text{ to } -}$ at p $\Rightarrow$ $x = p$ is a $\boxed{\text{LOCAL MAX}}$

(b) If $f'(x)$ change sign from $\boxed{- \text{ to } +}$ at p $\Rightarrow$ $x = p$ is a $\boxed{\text{LOCAL MIN}}$

$\underline{Ex}$: $F(x) = x^3 - 9x^2 - 48x + 52$

$f'(x) = 3x^2 - 18x - 48$

$= 3(x^2 - 6x - 16) = \boxed{3(x+2)(x-8)} = 0$

$\Rightarrow$ $x = -2$ and $x = 8$ are the critical pts

$+ + + 0 - - - - - - 0 + + + +$ $\quad f'$

$-2 \qquad 3 \qquad 8$

$- - - - \quad 0 \quad + + + + + + +$ $\quad f''$

1st Derivative Test

$x = -2$ is a LOCAL MAX

$x = 8$ is a LOCAL MIN

$f'(0) = -48$

$f'(10) = 3(10+2)(10-8) > 0$

$f'(-10) = 3(-10+2)(-10-8)$

$= (+)\cdot(-)(-) = + > 0$

$f''(x) = 6x - 18 = 0$

$x = 3$

$f''(-2) < 0$

$f''(4) = 6 > 0$

2^{nd} Derivative Test

Suppose $f'(p) = 0 \Rightarrow p$ is a critical pt

(a) If $f''(p) > 0 \Rightarrow x = p$ is a LOCAL MIN

(b) If $f''(p) < 0 \Rightarrow x = p$ is a LOCAL MAX

(c) If $f''(p) = 0 \Rightarrow$ test fails

Note! The 2^{nd} Derivative test can not be applied when $f'(p) = DNE$ since the $f''(p) = DNE$

Back to $f(x) = x^3 - 9x^2 - 48x + \underline{52}$ cubic order 3

$f'(x) = 3(x+2)(x-8) \implies x = -2, 8$ are critical pts

quadratic order 2

$f''(x) = 6x - 18$ linear order 1

$\underline{x = -2}$! $f'(-2) = 0$

$f''(-2) = -30 < 0$ $\implies$ $x = -2$ is a LOCAL MAX

concave down

$x = 8$ is a LOCAL MIN

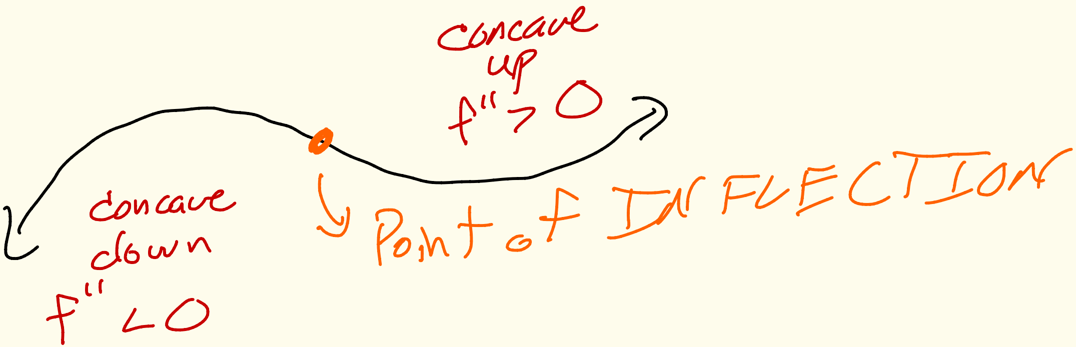
$\underline{x = 8}$! $f'(8) = 0$

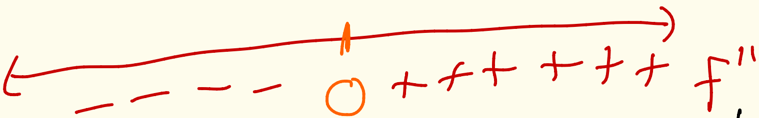
$f''(8) = 30 > 0$ $\implies$

concave up

	-2	8	
	0	0	f'
	$-$	$+$	f''

Concavity and Points of INFLECTION

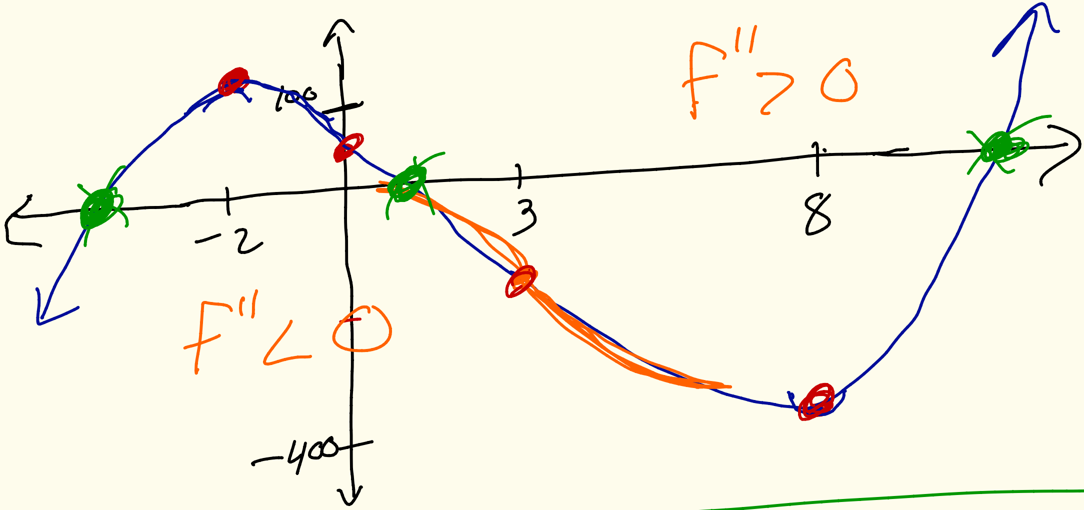




<u>Defn</u>: A point P where the concavity of the graph of $f(x)$ (measured by $f''(x)$) changes sign as you pass through P.

graph $f(x) = x^3 - 9x^2 - 48x + 52$

x	$f(x)$
-2	104
0	52
3	-146
8	-396



x-intercepts! $f(x) = x^3 - 9x^2 - 48x + 52 = 0$

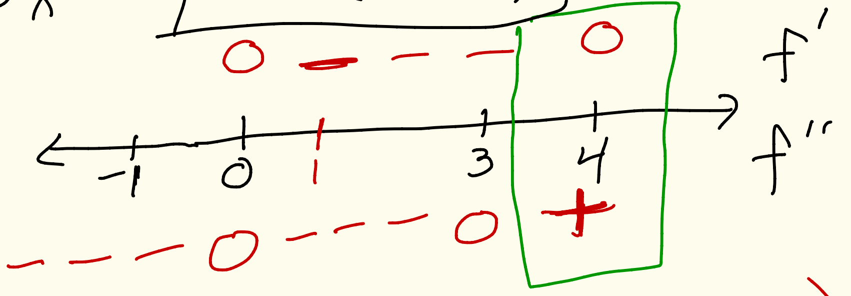
ans! $-4.44\ldots$, $0.93\ldots$, $12.5057\ldots$

Ex: Find the local mins, maxs, and inflection pts

for $f(x) = x^5 - 5x^4 + 30$

ans: $f'(x) = 5x^4 - 20x^3 = \boxed{5x^3(x-4)} = 0$

$\Rightarrow$ $x=0$, $x=4$ crit pts



$f''(x) = 20x^3 - 60x^2$
$= 20x^2(x-3)$
$= 0$

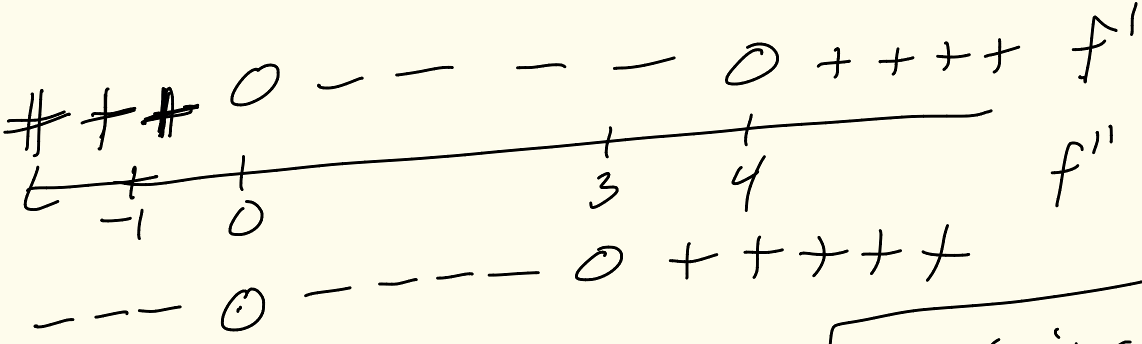
when $x=0$
$x=3$

$f'(4)=0$ and $f''(4) > 0$ $(\Rightarrow)$

$\Rightarrow x=4$ LOCAL MIN

$f''(4) = (+)(+)(+) > 0$, $f''(1) = (+)(+)(-) < 0$, f''

$f''(-1) = (+)(+)(-) < 0$



(i) $f'(4)=0$ and $f''(4)>0 \Longrightarrow \boxed{x=4 \text{ is a LOCAL MIN}}$

(ii) $f''(x)$ changes sign at $x=3 \Longrightarrow \boxed{x=3 \text{ is a pt of inflection}}$

(iii) $x=0$?

$$f'(-1) = 5(-1)^3(-1-4)$$
$$= (+)(-)(-)$$
$$= + \Longrightarrow \boxed{x=0 \text{ is a LOCAL MAX}}$$

Logistic Equation:

$$f(t) = \frac{L}{1+Ce^{-Kt}}$$

L, K, and C are **positive** constants (parameters)

$$= L\left(1+Ce^{-Kt}\right)^{-1}$$

$$f'(t) = L(-1)\left(1+Ce^{-Kt}\right)^{-2}\cdot\left(1+Ce^{-Kt}\right)'$$

general power rule + chain rule

You should check this!!

$$= -L\left(1+Ce^{-Kt}\right)^{-2}\left(0+C(-K)e^{-Kt}\right)$$

$$= \frac{LKCe^{-Kt}}{\left(1+Ce^{-Kt}\right)^2} = 0 \, ?$$

NO! All terms are positive $\Rightarrow f'(t) > 0$

so $f(t)$ is always increasing.

Chapter 10, Section 10.2, Question 7

Use the fact that the derivative gives the slope of a curve to decide which of the graphs (A)-(F) in the figure below could represent a solution to the differential equation.

$$\frac{dy}{dx} = 5x$$