Differentiation Examples

Ex: $s(t) = 2e^t + 2t^{-2} + \frac{t}{e^t}$

Ex: $F(x) = e^{\cos x}$

Ex: $g(x) = \ln(\sqrt{x})$

Ex: $g(x) = \frac{\ln(x^2)}{x^2+1}$

Ex: $m(t) = t\sin(2t)$

Find $m'(t)$ and $m''(t)$

Ex: $s(t) = 2e^t + 2t^{-2} + \frac{t}{e^t}$

$$s'(t) = 2e^t + 2(-2)t^{-2-1} + \left(\frac{t}{e^t}\right)'$$

$$= 2e^t - 4t^{-3} + \left(\frac{t}{e^t}\right)'$$

$$= \boxed{2e^t - 4t^{-3} + \frac{(1-t)}{e^t}}$$

$$\left(\frac{t}{e^t}\right)' = \frac{e^t(t)' - t(e^t)'}{(e^t)^2} = \frac{e^t \cdot 1 - te^t}{e^{2t}}$$

$$= \frac{e^t(1-t)}{e^{2t}}$$

$$= \boxed{\frac{(1-t)}{e^t}}$$

Same

$$(t \cdot e^{-t})' = t(e^{-t})' + e^{-t}(t')$$

$$= t(-e^{-t}) + e^{-t} \cdot 1$$

$$= e^{-t}(1-t) = \boxed{\frac{(1-t)}{e^t}}$$

$(e^{-t})'$?

$\boxed{K = -1}$

$\boxed{F(u) = e^{u}}$ $\quad g(t) = -t$

$e^{-t} = F(\underbrace{g(t)}_{-t})$

$\boxed{F'(u) = e^{u}}$

$g'(t) = -1$

$(e^{-t})' = F'(g(t)) \cdot g'(t)$

$= e^{-t} \cdot -1 = -e^{-t} = Ke^{Kt}$

Rule: $\frac{d}{dt}(e^{Kt}) = Ke^{Kt}$

Ex! $F(x) = e^{\cos x}$

$h(x) = e^x, \quad h'(x) = e^x$

$g(x) = \cos x, \quad g'(x) = -\sin x$

$F(x) = h(g(x))$

CHAIN RULE

$F'(x) = h'(g(x)) \cdot g'(x)$

$= e^{g(x)} \cdot (-\sin x)$

$= \boxed{-\sin x \, e^{\cos x}}$

$\dfrac{d}{dx}\left(e^{u(x)}\right)$

$= e^{u(x)} \cdot u'(x)$

For any function $u(x)$.

Ex: $g(x) = \ln(\sqrt{x})$

$h(x) = \ln x$

$\boxed{h'(x) = 1/x}$

$m(x) = \sqrt{x} = x^{1/2}$

$m'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}}$

$g(x) = h(m(x))$

$\Rightarrow g'(x) = h'(m(x)) \cdot m'(x)$ (chain rule)

$= \frac{1}{x^{1/2}} \cdot \frac{1}{2x^{1/2}}$

$= \frac{1}{2 \cdot x^{1/2} \cdot x^{1/2}} = \boxed{\frac{1}{2x}}$

$\square^{9} \cdot \square^{6} = \square^{9+6}$

Ex: $g(x) = \dfrac{\ln(x^2)}{x^2+1}$ quotient rule

$$g'(x) = \frac{(x^2+1)(\ln x^2)' - \ln x^2 \cdot (x^2+1)'}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\left(\frac{2}{x}\right) - \ln x^2 \cdot (2x+0)}{(x^2+1)^2}$$

$$= \boxed{\frac{\frac{2(x^2+1)}{x} - 2x \ln x^2}{(x^2+1)^2}}$$

$$(\ln x^2)' = \frac{1}{x^2} \cdot (x^2)' = \frac{2x}{x^2} = \frac{2}{x}$$

Ex: $m(t) = t\sin 2t$ product rule

$m'(t) = t(\sin 2t)' + \sin 2t \cdot (t)'$

$\boxed{(\sin 2t)' = (\cos 2t)\cdot(2t)' = 2\cos 2t}$

$= t \cdot 2\cos 2t + (\sin 2t)\cdot 1 = \boxed{2t\cos 2t + \sin 2t}$

$m''(t) = (m'(t))' = (2t\cos 2t + \sin 2t)'$

$\boxed{(\cos 2t)' = (-\sin 2t)\cdot(2t)' = -2\sin 2t}$

$= (2t)(-2\sin 2t) + (\cos 2t)(2) + 2\cos 2t$

$= \boxed{-4t\sin 2t + 4\cos 2t}$

Surge Function: $s(t) = a t e^{-bt}$

a, b positive constants (parameters)

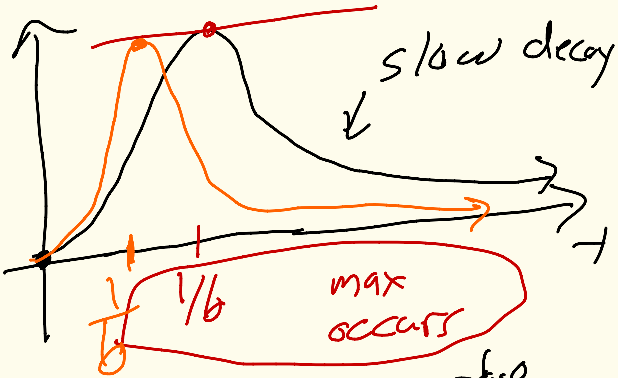
$$s'(t) = (a t e^{-bt})'$$

$$= a(t e^{-bt})'$$ product rule

$$= a\left(t(e^{-bt})' + e^{-bt}(t)'\right)$$

$$= a\left(t(-b)e^{-bt} + e^{-bt}\cdot 1\right)$$

$$= a e^{-bt}\boxed{(1-bt)} = 0$$



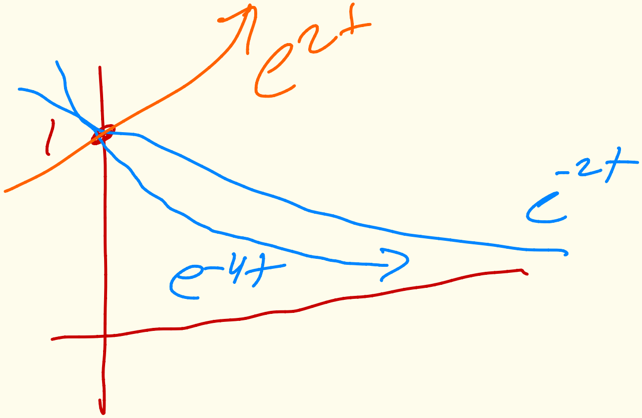
$s'(t) = 0$?

$1 - bt = 0$ or $t = \frac{1}{b}$

$$s(0) = a \cdot 0 \cdot e^{-b\cdot 0}$$

$$= 0$$

$$b > b$$



$$e^{-4t} = \frac{1}{e^{4t}} \quad b=4$$

$$e^{-2t} = \frac{1}{e^{2t}} \quad b=2$$

$$e^{4t} > e^{2t}$$

$$e^{4t} = e^{2t} \cdot \underbrace{e^{2t}}_{\geq 1} \geq e^{2t}$$

$$e^{2t}$$

$$\underline{e^{-6t}}$$

Larger the b
the Faster the decay!