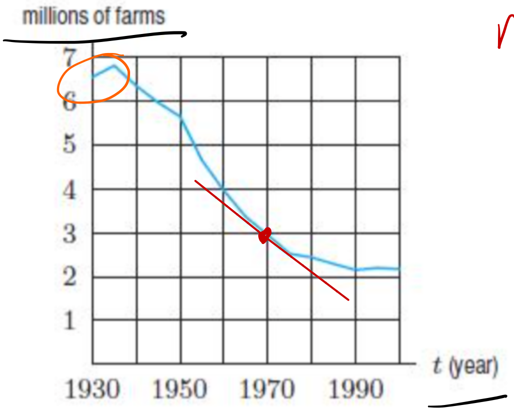
Feb 8th : Problems

Chapter 2, Section 2.1, Question 2

N(t)

The figure below shows $N = f(t)$, the number of farms in the US[1] between 1930 and 2000 as a function of year, t.



red line
tangent line
at t=1970

(a) Is $f'(1970)$ positive or negative? What does this tell you about the number of farms?

$f'(1970)$ is [ ]. negative

DECREASING

That means that the number of farms in the US was [ ] in 1970.

Chapter 2, Section 2.1, Question 5

The distance (in feet) of an object from a point is given by $s(t) = t^2$, where time t is in seconds.

(a) What is the average velocity of the object between $t = 8$ and $t = 10$?

The average velocity between $t = 8$ and $t = 10$ is 18 ft/sec.

(b) By using smaller and smaller intervals around 8, estimate the instantaneous velocity at time $t = 8$.

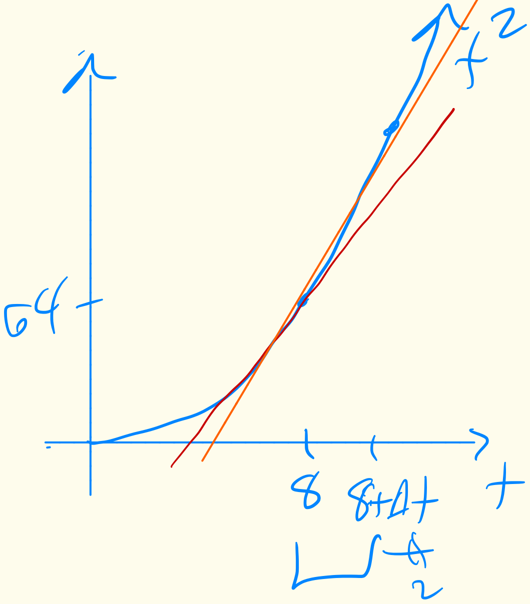
Round your answer to the nearest integer.

The instantaneous velocity at $t = 8$ is about 16 ft/sec.

(a) avg velocity $[8, 10]$ $= \frac{S(10) - S(8)}{10 - 8} = \frac{100 - 64}{2} = \frac{36}{2} = 18$

(b) next page

$s(t) = t^2$



Future: EXACT $s'(t) = 2t$

$s'(8) = 2 \cdot 8 = 16$

$$s'(8) \approx \frac{\Delta s}{\Delta t} = \frac{s(8+\Delta t) - s(8)}{\Delta t}$$

Δt	$\frac{s(8+\Delta t) - s(8)}{\Delta t}$
2	$\frac{s(10) - s(8)}{2} = 18$
0.5	$\frac{s(8.5) - s(8)}{0.5} = 16.5$
0.1	$\frac{s(8.1) - s(8)}{0.1} = 16.1$
0.01	$\frac{s(8.01) - s(8)}{0.01} = 16.01$

as $\Delta t \to 0$

$\frac{\Delta s}{\Delta t} \to 16$

Chapter 2, Section 2.2, Question 10

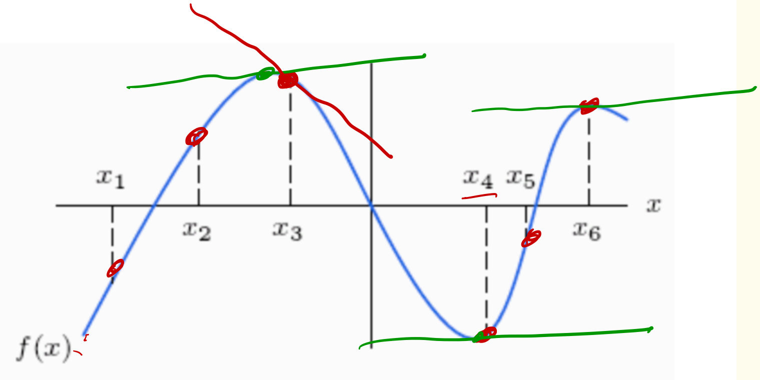
In the graph of f in the figure below, at which of the labeled x -values is

(a) $f(x)$ greatest? X_3

(b) $f(x)$ least? X_4

(c) $f'(x)$ greatest? X_5

(d) $f'(x)$ least? X_3



Chapter 2, Section 2.1, Question 17

Estimate $P'(0)$ if $P(t) = 200(1.08)^t$.

$a = 1.08 > 1$

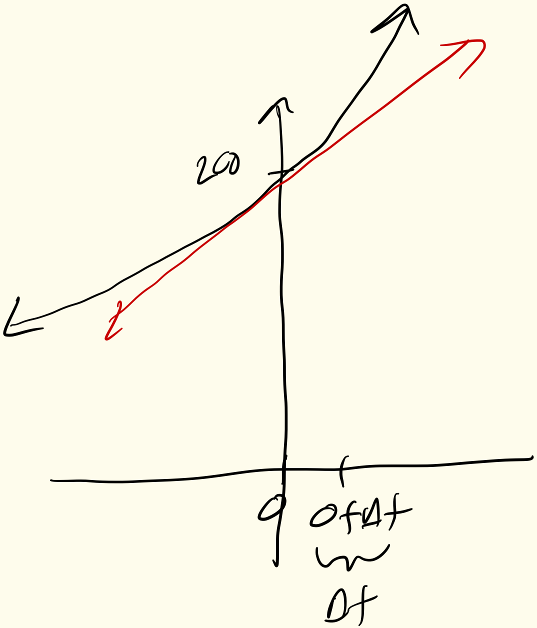
Round your answer to the nearest integer.

$P'(0) =$ 15

$P(t)$ is an exponential ca^t, $0 < a < \infty$

$P(0) = 200(1.08)^0 = 200 \cdot 1 = 200$

Δt	$\Delta P / \Delta t$
0.1	$\frac{P(0.1) - P(0)}{0.1} = 15.45159\ldots$
0.01	$\frac{P(0.01) - P(0)}{0.01} = 15.39813\ldots$
0.001	$\frac{P(0.001) - P(0)}{0.001} = 15.3928\ldots$

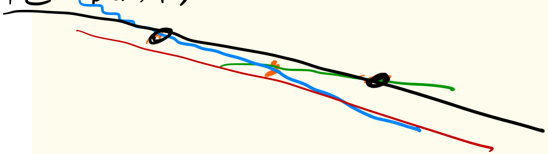


Exact: $P'(t) = 200 \cdot \ln(1.08)\,(1.08)^t$

$P'(0) = 200 \cdot \ln(1.08) \cdot 1 = 15.392208\ldots$

Chapter 2, Web Quiz, Question 19

(Discrete Data)



Estimate $f'(x)$ at $x = 3$ using the table below.

x	0	1	2	3	4	5	6	7
$f(x)$	12	20	26	23	22	20	17	15

- -1
- 1
- -3
- -2

2 3 4

avg

$$\frac{\Delta f}{\Delta x} = \frac{f(4)-f(3)}{4-3} = \frac{-1}{1} = -1$$

$$\frac{\Delta f}{\Delta x} = \frac{f(3)-f(2)}{3-2} = \frac{-3}{1} = -3$$

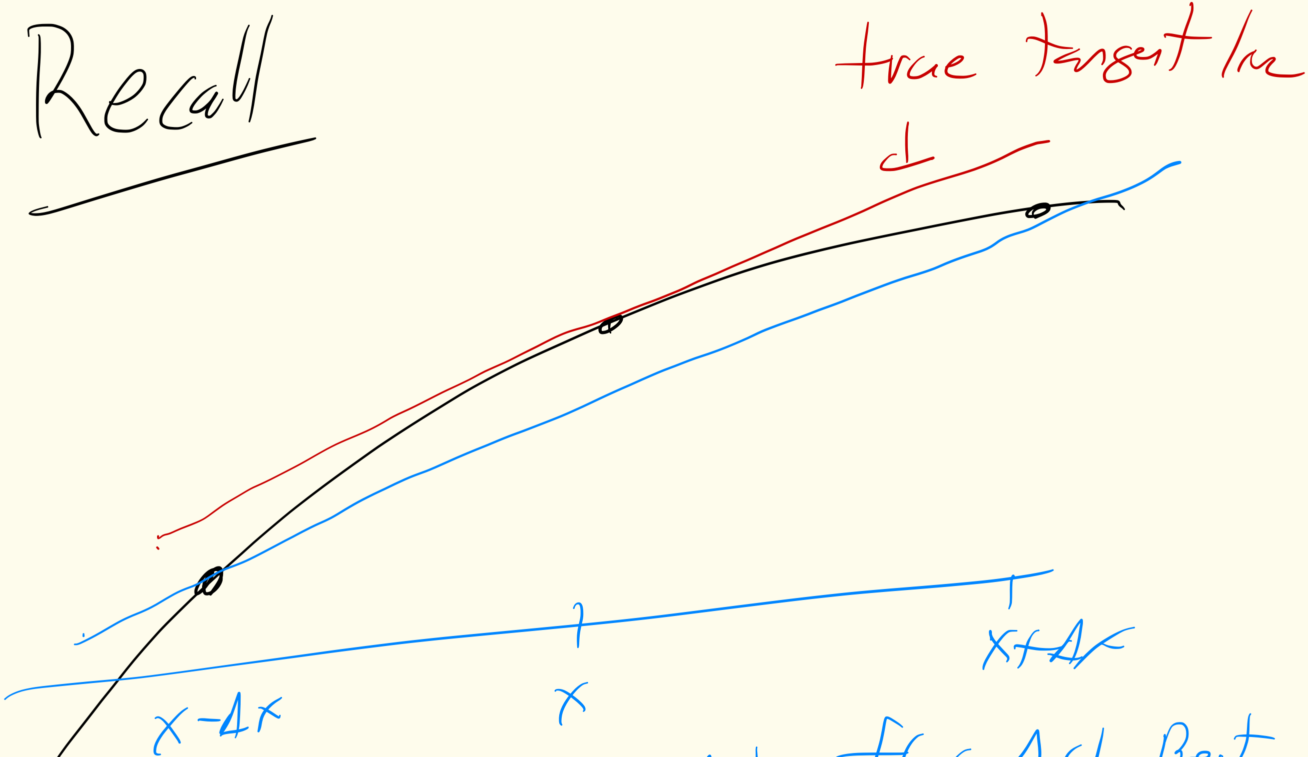
$$\frac{\Delta f}{\Delta x} = \underbrace{\frac{f(4)-f(2)}{4-2}}_{\text{"centered approx"}} = \frac{-4}{2} = -2$$

$$\frac{-1+(-3)}{2}$$

2 3 3.5

$$\frac{f(3.5)-f(2)}{3.5}$$

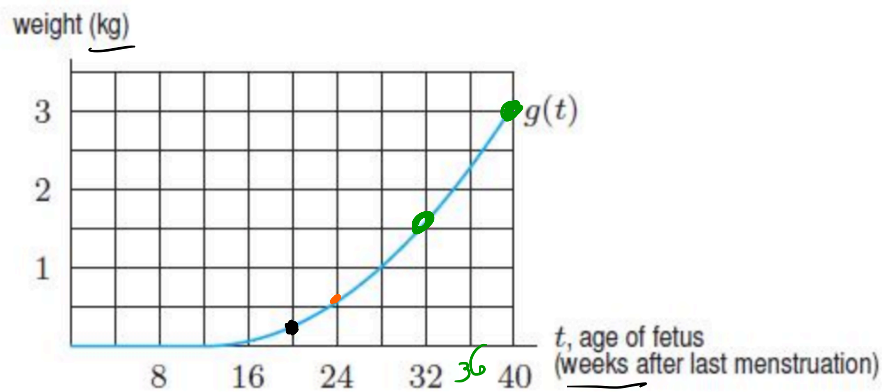
Recall



$$f'(x) \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$ Best approx.

Chapter 2, Section 2.3, Question 24

Consider $g(t)$ in the figure below, which gives the weight of a human fetus as a function of its age.



(a) Estimate $g'(20)$. Round your answer to three decimal places.

$$g'(20) \approx \frac{g(24) - g(16)}{24 - 16} = \frac{0.6 - 0.1}{8} = \frac{0.5}{8} = 0.0625$$

$g'(20) =$ 0.0625 kg/week

Have fun! :)

(b) Estimate $g'(36)$. Round your answer to three decimal places.

Limits and Continuity:

$x \to a \leftarrow x$

$$\boxed{\lim_{x \to a} f(x) = L}$$

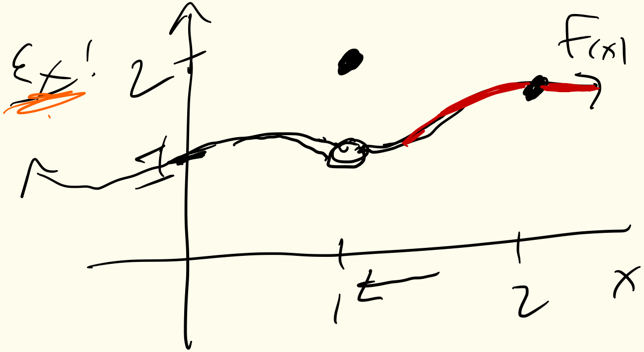
Does $f(x)$ approach a fixed value L as $x \to a$.

The limit DOES NOT DEPEND on what happens at a!

Continuity: $F(x)$ is CONTINUOUS at a if

$$\underbrace{\lim_{x \to a} F(x)}_{(1)} \underbrace{=}_{(3)} \underbrace{F(a)}_{(2)}$$

Ex!



$f(1) = 2$

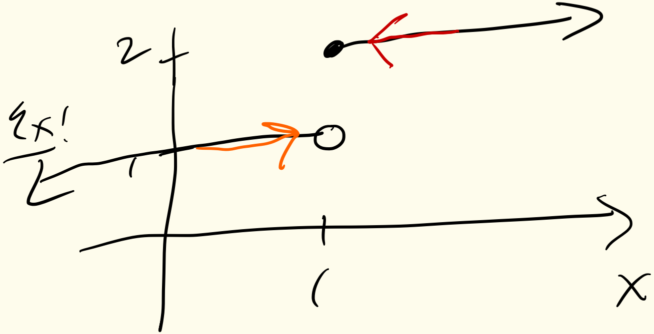
$\lim_{x \to 1} f(x) = 1$

$2 \neq 1$

$f(x)$ is NOT continuous at $x = 1$.

Q: Is $\lim_{x \to 2} f(x) = f(2)$? Yes

Ex!



Is $f(x)$ continuous at $x = 1$?

$$\boxed{\lim_{x \to 1} f(x) = \text{Does Not Exist!}}$$

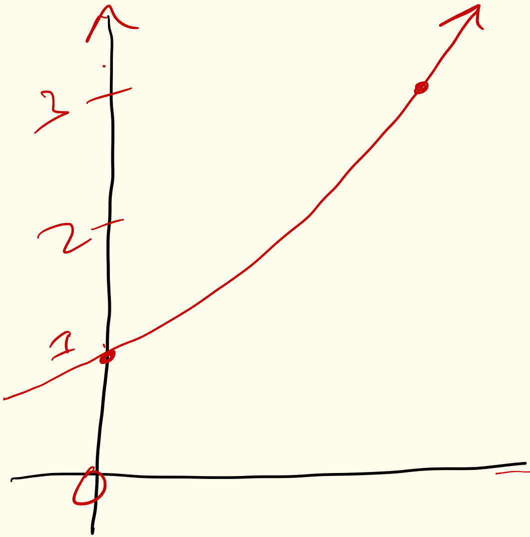
Chapter 2, Focus On Theory, Question 14

$f(1) = 7^1 = 7$

Is the function $\underbrace{f(x) = 7^x}_{\text{exponential}}$ continuous on the interval $0 \le x \le 10$?

Yes

exponential base $a = 7 > 1 \Rightarrow$ grows



We can determine continuity from the graph. How?

If we can draw the graph over the given interval, then f(x) is continuous on the interval