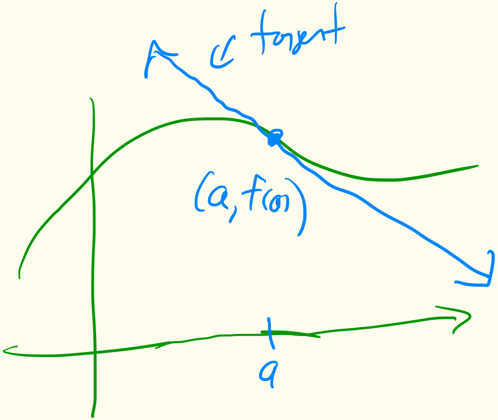
1st Derivative: $y = f(x)$

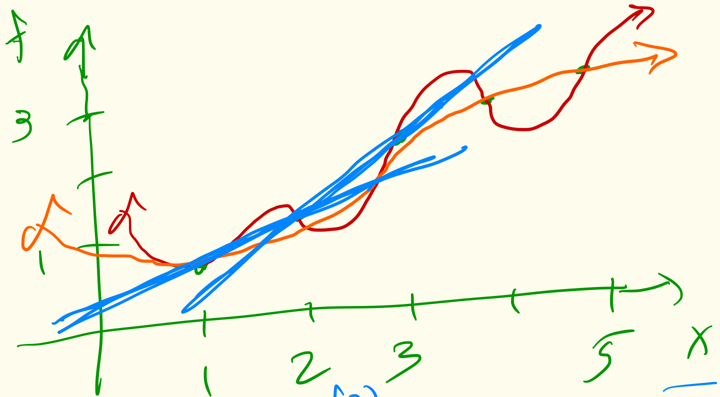
Formula (continuous)



$$f'(a) = \text{slope of the tangent line at } x = a$$

Table (discrete)

x	1	2	3	4	5
f	0.8	1.1	2.2	2.5	3



$$f'(2) \approx \begin{cases} \dfrac{f(3) - f(2)}{3 - 2} \\[1em] \dfrac{f(2) - f(1)}{2 - 1} \end{cases}$$

Ex:

t (sec)	0	0.2	0.4	0.6	0.8	1.0
s (ft)	0	0.5	1.8	3.8	6.5	9.6

Estimate $s'(0.4)$:

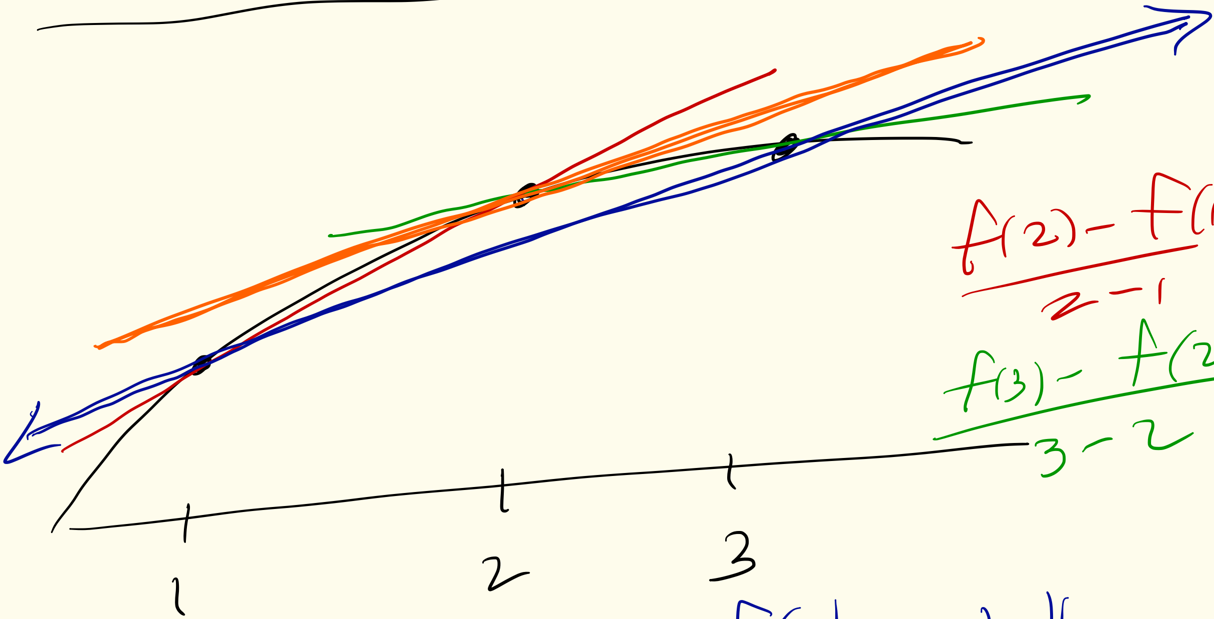
$s'(0.4) \approx$ avg change $[0.4, 0.6]$ $= \frac{\Delta s}{\Delta t} = \frac{3.8 - 1.8}{0.6 - 0.4} = 10$

$s'(0.4) \approx$ avg change $[0.2, 0.4]$ $= \frac{\Delta s}{\Delta t} = \frac{1.8 - 0.5}{0.4 - 0.2} = \frac{1.3}{0.2} = 6.5$

$s'(0.4) \approx$ avg change $[0.2, 0.6]$ $= \frac{\Delta s}{\Delta t} = \frac{3.8 - 0.5}{0.6 - 0.2} = 8.25$

$8.25 = \frac{(10 + 6.5)}{2}$ = avg of the 2 other secant slopes!

Centered Approximation in General is BEST



$$\frac{f(2)-f(1)}{2-1}$$

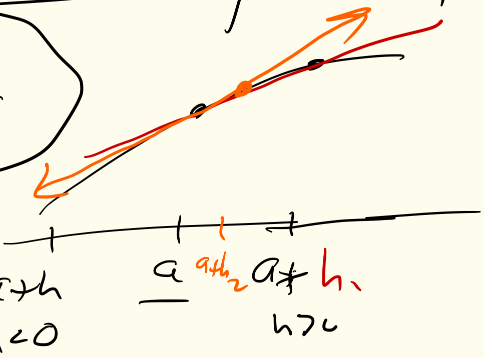
$$\frac{f(3)-f(2)}{3-2}$$

$$f'(2) \approx \frac{f(3)-f(1)}{3-1}$$

Better

1st and 2nd Derivative as a function:

$y = f(x)$



$$f'(a) \overset{\text{defn}}{=} \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{fixed } a$$

We want a FORMULA for $f'(a)$ for an $x = a$.

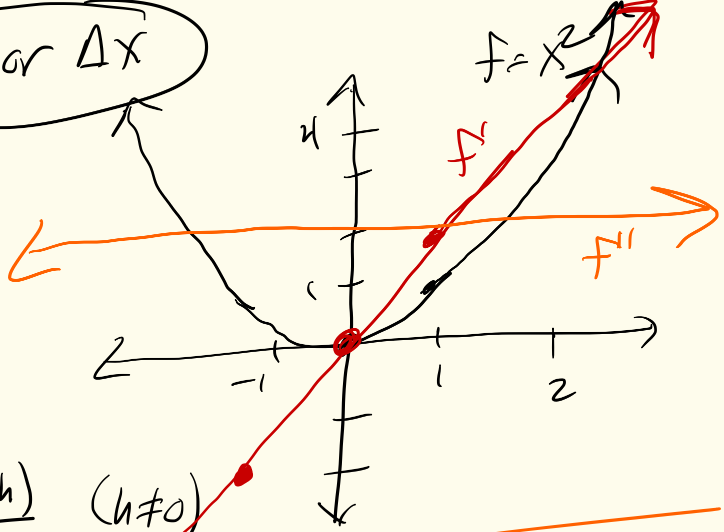
$$\underbrace{f'(x)}_{\text{function of } x} \overset{\text{defn}}{=} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(x+h) - (x) = h$$

Ex: $y = f(x) = x^2$

$$f'(x) \overset{\text{defn}}{=} \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

h or Δx



$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{x^2}+2xh+h^2)-\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h\to 0} 2x+h = 2x+0 = 2x$$

$$\boxed{f'(x) = 2x}$$

$f'(0) = 0$

$f'(-1) = -2$

$f'(1) = 2$

$f'(x) = 2x$

$f''(x) = 2$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$f'(x) = 2x$

$f'(\square) = 2 \cdot \square$

$$= \lim_{h \to 0} \frac{2(x+h) - 2x}{h}$$

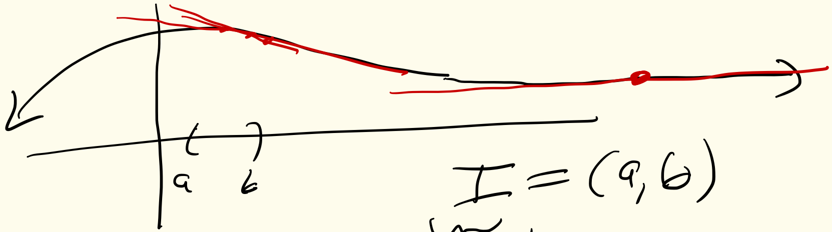
$$= \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{2x}}{h} = \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = \lim_{h \to 0} 2 = 2$$

$F(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$

$y = f(x)$: GEOMETRY



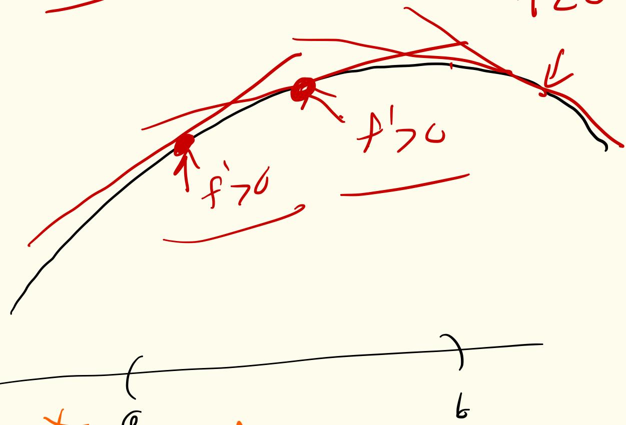
$I = (a, b)$ interval

$f'(x)$ 1st derivative

$f' < 0$ on $I \Rightarrow f$ is DECREASING on I

$f' > 0$ on $I \Rightarrow f$ is INCREASING on I

$f' = 0$ on $I \Rightarrow f$ is CONSTANT on I



$f''(x)$ 2nd derivative

$f'' < 0$ on $I \Rightarrow f$ is CONCAVE DOWN on I

$f'' > 0$ on $I \Rightarrow f$ is CONCAVE UP on I

$f'' = 0$ on $I \Rightarrow f'(x)$ is constant on $I \Rightarrow f(x)$ is a LINE on I

Alternate Notation

$s = f(t)$ distance (meters) of an object moving in 1D at time t (secs)

$t \quad t+\Delta t$

$$f'(t) \overset{\text{defn}}{=} \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta f}{\Delta t} = \frac{df}{dt} \frac{(m)}{(sec)}$$

Leibniz Notation

$f'(t)$ is the same as $\frac{df}{dt}$

$$s'(t) \overset{\text{defn}}{=} \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Leibniz Notation

$s'(t)$ is the same as $\frac{ds}{dt}$