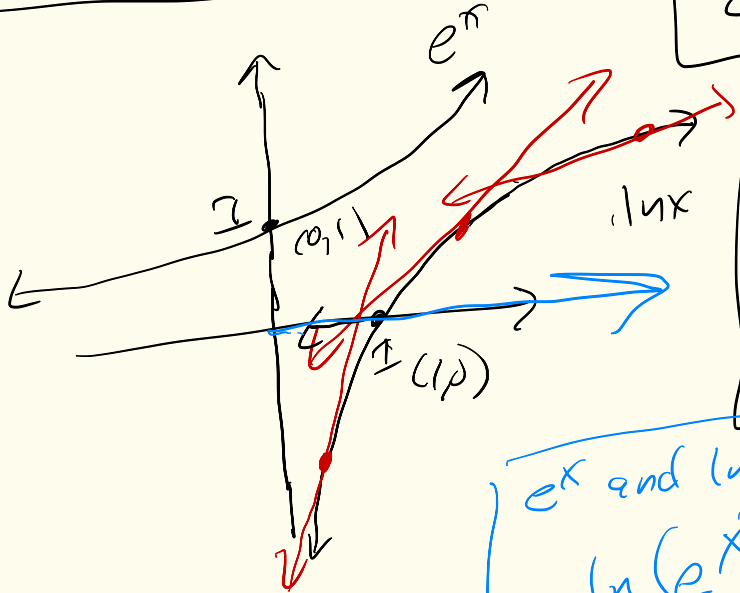
More Derivatives: Given $f(x)$ find $f'(x)$

e^x and $\ln x$



tangent line

$$\boxed{\frac{d}{dx}\left(e^x\right) = e^x}$$ special!

$$\boxed{\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}}$$

$\ln x$: natural log, $\log_e x$

e^x and $\ln x$ are inverses of each other!

$\ln\left(e^x\right) = x$, all x

$e^{(\ln x)} = x$, $x > 0$

Shortcuts:

$$\frac{d}{dx}[c] = 0$$

$$\frac{d}{dx}[mx+b] = m$$

$$\boxed{\frac{d}{dx}[x^n] = nx^{n-1} \quad \text{Power Rule}}$$

$$\left.\begin{array}{l}\frac{d}{dx}[e^x] = e^x \\ \frac{d}{dx}[\ln x] = \frac{1}{x}\end{array}\right\} \text{base } e$$

$$\left.\begin{array}{l}\frac{d}{dx}[a^x] = (\ln a)a^x \\ \frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)\cdot x}\end{array}\right\} \begin{array}{c}\text{any base} \\ a>0\end{array}$$

General Rules:

$$\frac{d}{dx}[c\cdot f(x)] = c\cdot\frac{d}{dx}[f(x)] \quad ((cf)' = cf')$$

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

$$(f \pm g)' = f' \pm g'$$

Ex! $F(x) = (x+1)^2 = (x+1)(x+1) = \underline{x^2} + \underline{2x} + \underline{1}$

$\frac{d}{dx}\left[(x+1)^2\right] = \frac{d}{dx}\left[x^2 + 2x^1 + 1\right] = (2)x^{2-1} + 2(1)x^{1-1} + 0$

$= \boxed{2x + 2}$

Ex: $F(x) = 3e^x + \sqrt{x} - 4x^{1/5} = 3e^x + x^{1/2} - 4x^{1/5}$

$F'(x) = 3 \cdot e^x + \frac{1}{2}x^{1/2 - 1} - (4)\left(\frac{1}{5}\right)x^{1/5 - 1}$

$= 3e^x + \frac{1}{2}x^{-1/2} - \frac{4}{5}x^{-4/5} = \boxed{3e^x + \frac{1}{2x^{1/2}} - \frac{4}{5x^{4/5}}}$

Ex! $F(x) = 16x^{40} - 3x^2 + \pi x - 4$

$F'(x) = 16(40)x^{40-1} - 3 \cdot 2x^1 + \pi - 0$

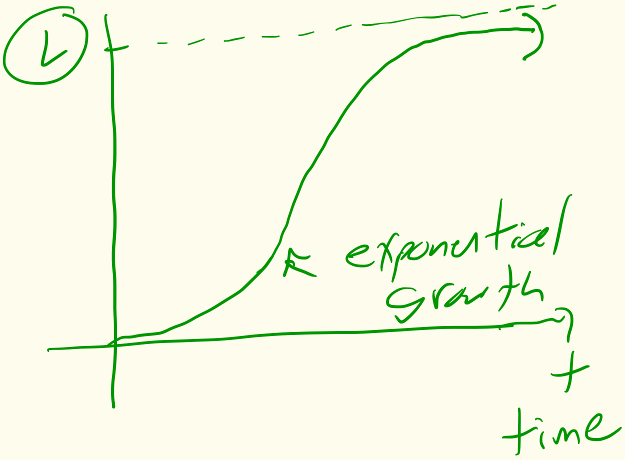
$= 640x^{39} - 6x + \pi$

what about?

Logistic Function

$$F(t) = \frac{L}{1+ce^{-kt}} = L\left[1+ce^{-kt}\right]^{-1}$$

L, c, k positive constants



Surge Function

$$F(t) = a\,t\,e^{-bt}$$

a, b positive constants

what about?

product $(x)\cdot e^x$

$$f(x) = \ln\sqrt{x} + e^{x^{10}} + xe^x + 4$$

$$e^{(x+1)}$$ composition

$$g(x) = e^{x-\ln x} + x^3 \cdot e^{x+1}$$

$$\text{or} \quad h(x) = e^{\cos x} + \left(\sin(x^2)\right)^3$$

We will see!

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

Big 3:

product rule:

$$\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$(f\cdot g)' = fg' + gf'$$

quotient rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

chain Rule:

(compositions)

$$\frac{d}{dx}[f(\underline{g(x)})] = \underline{f'}\,\underline{(g(x))}\cdot \underline{g'(x)}$$

$$(f(g))' = f'(g)\cdot g'$$

Ex: $F(x) = \underline{xe^{x}} + \underline{e^{x+1}} \Rightarrow f'(x) = xe^{x} + e^{x} + e^{(x+1)}$

$$\frac{d}{dx}\left[xe^{x}\right] = x(e^{x})' \underset{\text{add}}{+} e^{x}(x)' = \boxed{x(e^{x}) + e^{x}\cdot 1}$$

product

$h(x) = x+1 \qquad h'(x) = 1$

$g(x) = e^{x} \qquad g'(x) = e^{x}$

$e^{x+1} = g(h(x))$

$$\frac{d}{dx}\left[e^{(x+1)}\right] = \left[e^{x}\right]'\Big|_{x+1} \cdot (x+1)'$$

$$= e^{x}\Big|_{(x+1)} \cdot 1$$

$$= \boxed{e^{(x+1)}}$$

Ex: $F(x) = (x+1)^{2} \qquad F'(x) = 2x+2\,?$

$F(x) = (x+1)(x+1) \Rightarrow f'(x) = (x+1)\cdot(x+1)' + (x+1)(x+1)'$

$$= (x+1)(1) + (x+1)(1)$$

$$= \boxed{2x+2}$$

product

Ex:

$$h(x) = \boxed{\underbrace{e^{\cos x}}_{1}} + \boxed{\underbrace{\ln(\sqrt{x})}_{2}} + \boxed{\underbrace{x\sin(2x)}_{3}}$$

$$e^{\cos x} = f(g(x)) \overset{\text{chain rule}}{\Longrightarrow} \frac{d}{dx}\left[e^{\cos x}\right] = f'(g(x))\cdot g'(x)$$

$f(x) = e^x$, $g(x) = \cos x$

$f'(x) = e^x$, $g'(x) = -\sin x$

$$= f(\cos x)\cdot(-\sin x)$$

$$= e^{\cos x}(-\sin x)$$

$$= -\sin x\cdot e^{\cos x}$$

$$x\cdot\sin(2x) = f(x)\cdot g(x)$$

$f(x) = x$, $f'(x) = 1$

$g(x) = \sin 2x$, $g'(x) = 2\cos 2x$

Aside:

$$\sin 2x = f(g(x))$$

$$f(x) = \sin x \qquad \boxed{f'(x) = \cos x}$$

$$\boxed{g(x) = 2x}, \quad g'(x) = 2$$

$$[\sin 2x]' = f'(g(x)) \cdot g'(x)$$

$$= \cos(2x) \cdot 2$$

$$= 2\cos 2x$$

$x \sin 2x = f(x) \cdot g(x)$

$f(x) = x$

$g(x) = \sin 2x$

$f'(x) = 1$

$g'(x) = 2\cos 2x$

$$\frac{d}{dx}[f \cdot g] = fg' + gf'$$

$$= x \cdot (2\cos 2x) + (\sin 2x)(1)$$

$$= \boxed{2x\cos 2x + \sin 2x}$$