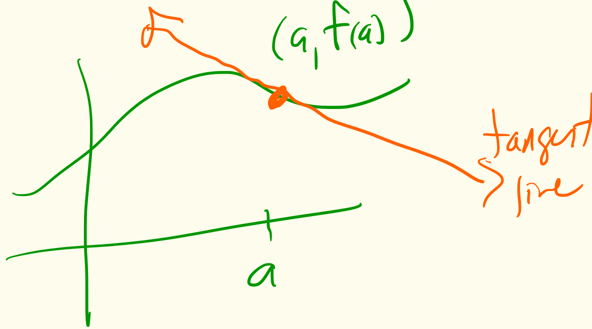
Chapter 3: Rules of Differentiation

$y = f(x)$



slope $\boxed{m = f'(a)}$

$\boxed{\text{we need } f'(x)}$

$m = f'(a)$ pt: $(a, f(a))$

tangent line:

pt-slope form: $y - \underbrace{f(a)}_{\#} = \underbrace{f'(a)}_{\#}(x - \underbrace{a}_{\#})$

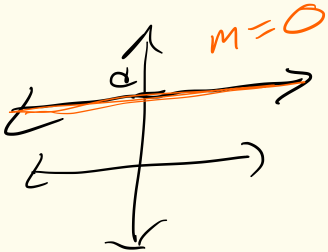
$y = \underbrace{f'(a)}_{m}\, x + \underbrace{(f(a) - a f'(a))}_{b}$

slope int form:

Given $y = f(x)$ find a formula for $f'(x)$

$f(x+h) = c$ $\quad f(☺) = c$

<u>Constant!</u> $y = f(x) = \underline{c}$ $\quad f(x) = c$



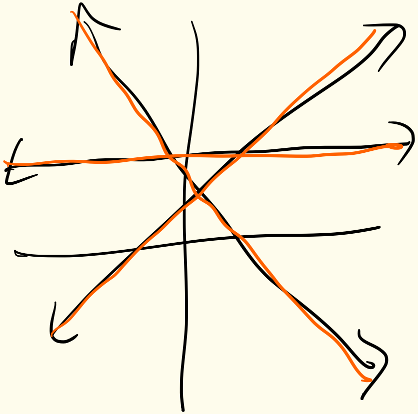
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

Yes! $\frac{d}{dx}(f(x)) = \boxed{\frac{d}{dx}(c) = 0}$

Linear function?

$y = f(x) = \underbrace{m}_{\text{slope}} x + \underbrace{b}_{\substack{\text{y-int} \\ (0,\, b)}}$



$$\boxed{\frac{d}{dx}(mx+b) = m}$$

Formulas:

$y = f_{(x)}$

$\frac{d}{dx}(3x) = 3$

$\frac{d}{dx}(x) = 1$

$\underbrace{\frac{d}{dx}(e^x) = e^x}_{\text{assume}}$

$\frac{d}{dx}[c] = 0$

$\frac{d}{dx}[mx+b] = m$

$y = \underbrace{3x}+0 \Rightarrow f'_{(x)} = \boxed{3}$

Ex: $\frac{d}{dx}[3x] = 3 \cdot \frac{d}{dx}[x]$

$= 3 \cdot 1 = \boxed{3}$

$\frac{d}{dx}[c\,f_{(x)}] = c \cdot \frac{d}{dx}[f_{(x)}]$

$\frac{d}{dx}[f_{(x)} \pm g_{(x)}] = \frac{d}{dx}[f_{(x)}] \pm \frac{d}{dx}[g_{(x)}]$

Ex: $\frac{d}{dx}(6x+4e^x+4) = \frac{d}{dx}(6x) + \frac{d}{dx}(4e^x) + \frac{d}{dx}(4)$

$= 6 \cdot \frac{d}{dx}(x) + 4\frac{d}{dx}(e^x) + \frac{d}{dx}(4) = 6 \cdot 1 + 4 \cdot e^x + 0$

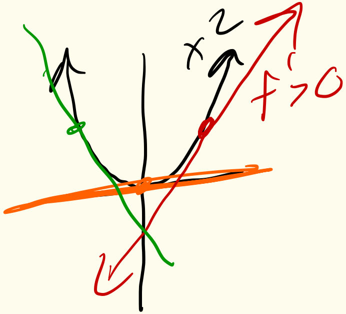
$= \boxed{4e^x + 6}$

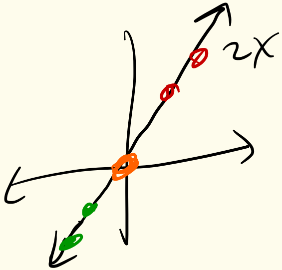
Power Rule

$y = f(x) = x^n$

$$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$$

Ex: $\frac{d}{dx}\left(x^2\right) = 2 \cdot x^{2-1} = 2 \cdot x$

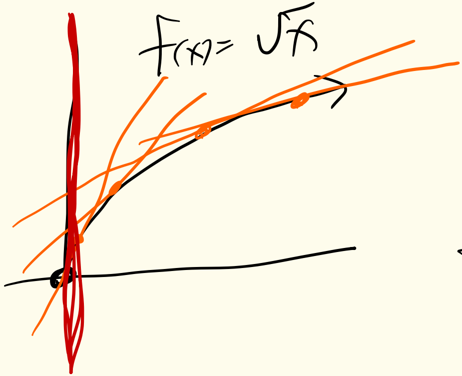




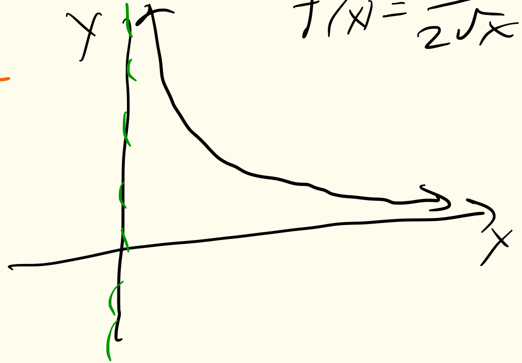
Ex: $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}\left(x^{1/2}\right)$

$= \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$



$f'(x) = \frac{1}{2\sqrt{x}}$



Polynomials: $y = a_n x^n + a_{n-1} x^{n-1} + \underbrace{\cdots}_{\text{pattern continue}} + a_2 x^2 + a_1 x + a_0$

$a_n \neq 0$

nth degree poly

Ex: $y = f(x) = \underline{6}x^5 - x^3 + 1$ 5^{th} degree poly

$\frac{d}{dx}[f(x)] = \frac{d}{dx}[6x^5 \underset{\text{minus}}{-} x^3 + 1] = 6\frac{d}{dx}[x^5] - \frac{d}{dx}[x^3] + \frac{d}{dx}[1]$

$= 6 \cdot 5 x^{5-1} - 3x^{3-1} + 0$

$\boxed{f'(x) = 30x^4 - 3x^2}$ or $\boxed{\frac{dy}{dx} = 30x^4 - 3x^2}$

Note: Looking forward

$f'(x) = 0 \implies 30x^4 - 3x^2 = 0$ Algebra

$3x^2(10x^2 - 1) = 0$

Ex! Find the eqn of the tangent line to

$y = f(x) = 3x^2 + 4\sqrt{x} + 4$ at $a = 4$

$= 3x^2 + 4x^{1/2} + 4$

$f'(x) = 6x + 2x^{-1/2} + 0$

$= 6x + \frac{2}{\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = f'(x) = 6x + \frac{2}{\sqrt{x}}$

slope tangent line: $f'(4) = \left.\frac{dy}{dx}\right|_{x=4} = 6 \cdot 4 + \frac{2}{\sqrt{4}} = 25$

$a=4$

pt: $(a, f(a)) = (4, f(4)) = (4, 60)$

$f(4) = 3 \cdot 4^2 + 4 \cdot \sqrt{4} + 4$
$= 48 + 8 + 4$
$= 60$

Eqn!

$y - 60 = 25(x-4)$

or

$y = 25x - 40$

Exponentials and Logarithms

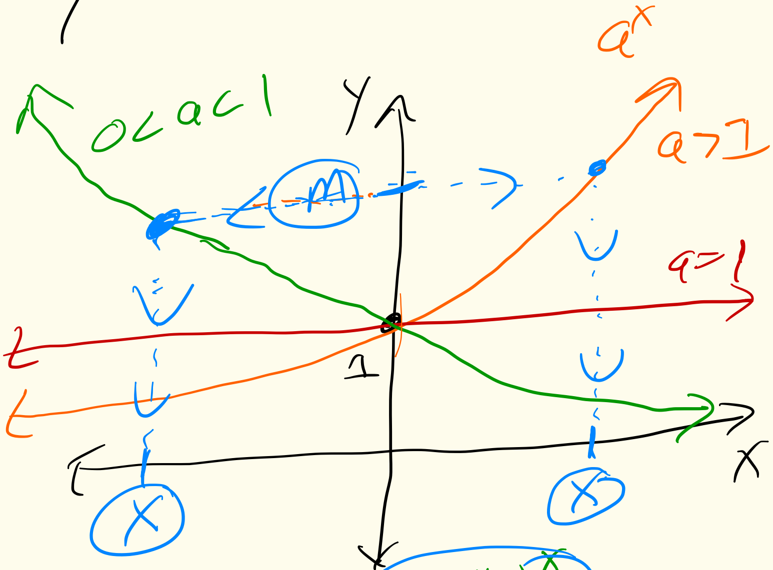
$y = f(x) = a^x$

$0 < a < \infty$

↑ base

$f(0) = a^0 = 1$

All pass through $(0, 1)$



Ex: $a = 2$

$y = f(x) = 2^x$

$2^{-1} = \frac{1}{2}, \quad 2^2 = 4, \quad 2^{10} = 1024$

ex: $a = 1$

$y = f(x) = 1^x = 1$

ex: $a = \frac{1}{2}, \quad f(x) = \left(\frac{1}{2}\right)^x$

$\left(\frac{1}{2}\right)^{-1} = \frac{1}{1/2} = 2, \quad \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Logarithms! Inverse of exponentials

$$y = F(x) = a^x$$

$$y = F^{-1}(x) = \log_a x$$

Note! $f^{-1}(x) \neq \frac{1}{f(x)}$

Note! $a^{\log_a x} = x$

$$\log_a a^x = x$$

Ex! $y = f(x) = 2^x \to F^{-1}(x) = \log_2 x$

$$4 \xrightarrow{2^x} 16 \xrightarrow{\log_2 x} 4$$

$f(x)$ $\qquad$ $f^{-1}(x)$

$\pi + 4 = \underline{3}^x$ solve for x

$$x = \log_3 (\pi + 4)$$

Special Base: $a = e \approx 2.7182\ldots$

$y = f(x) = e^x$ exponential natural base

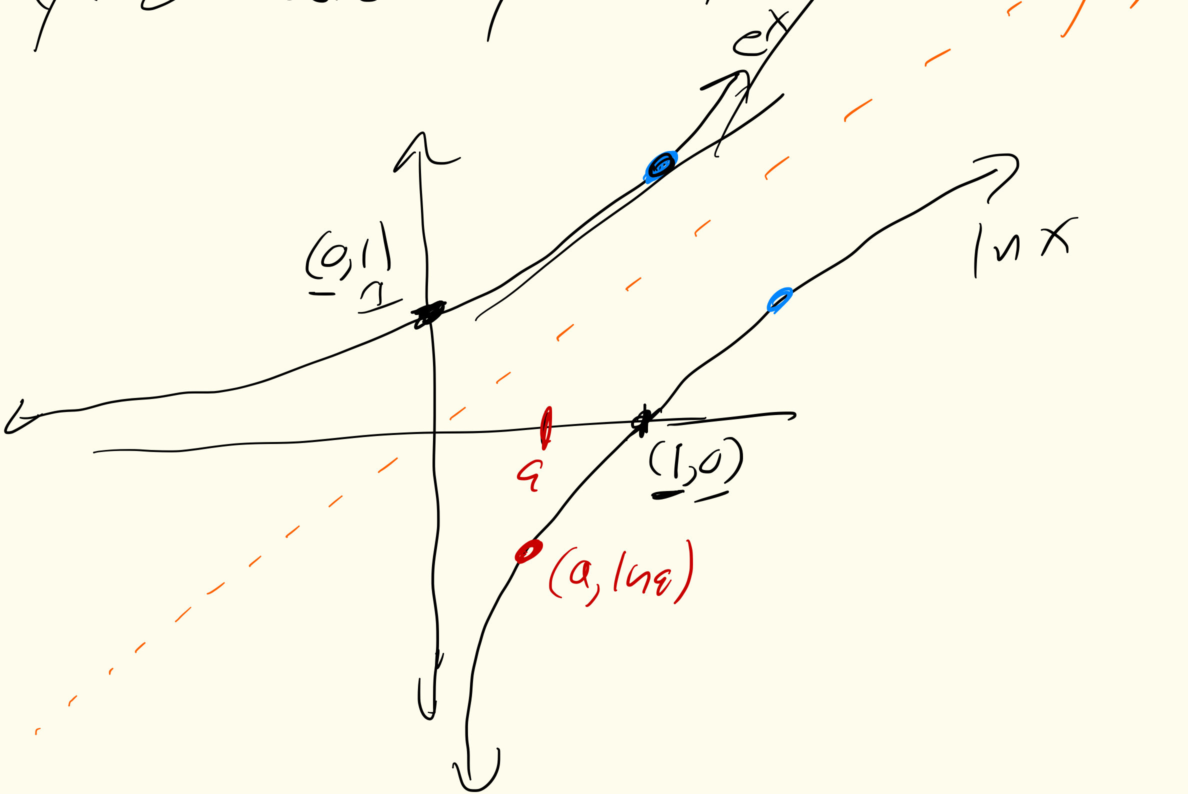
$y = f^{-1}(x) = \log_e x$

$= \ln x$

Fact: $\log_a x = \dfrac{\ln x}{\ln a}$

So you can compute $\log_a x$ for any base a using only the $\ln x$ function.

$y = e^x$ and $y = \ln x$



$$\frac{d}{dx}(a^x) = (\ln a)\, a^x$$

constant

$$e^? = e \Rightarrow ? = 1$$

$$\frac{d}{dx}(e^x) = (\ln e)\cdot e^x = \log_e e \cdot e^x = 1 \cdot e^x$$

$$\boxed{\frac{d}{dx}(e^x) = e^x}$$

Big Deal!