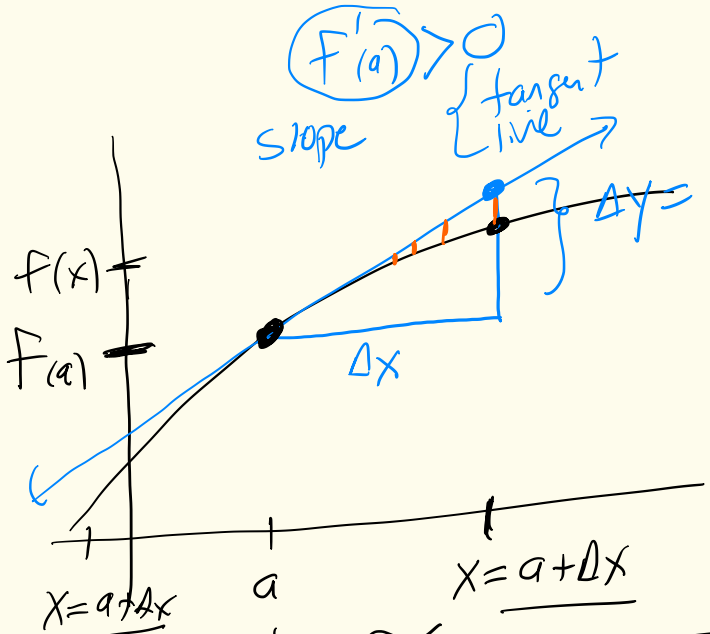
Local Linear Approximation

$y = f(x)$

MAIN IDEA

Follow the TANGENT LINE

"Near" x the tangent is a "good" approx to $f(x)$



$\Delta x = x - a$

$f(x) \approx f(a) + \Delta y$ (tangent line)

or

$f(x) \approx f(a) + f'(a) \cdot \Delta x$

Local Linear Approx

Ex:

$y = 2x + 1$

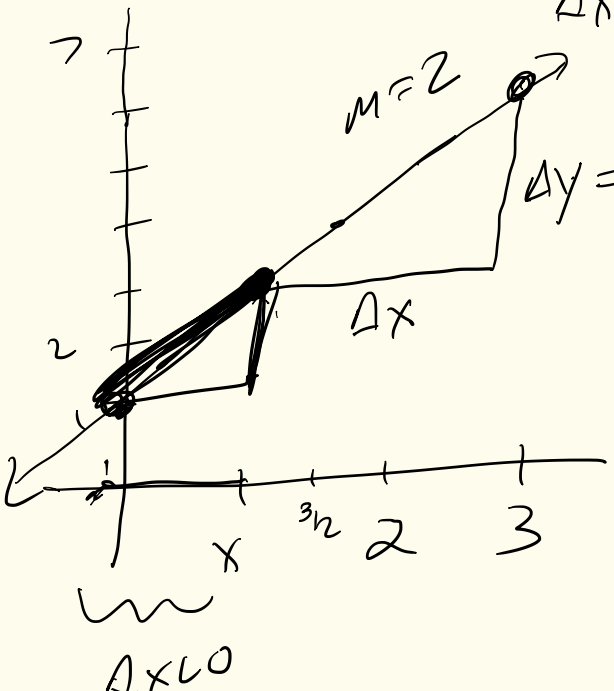
x	y
0	1
1	3
3/2	$3 + \frac{1}{2} \cdot 2 = 4$

$3 - 1 = 2$

$\Delta x = 1 = 1 - 0$

$m = 2$

$\Delta y = 2 \cdot \Delta x$



Chapter 2, Web Quiz, Question 25

If $f(1) = -1.24$ and $f'(1) = -0.002$ estimate $f(2)$.

$a = 1$, $x = 2$ $\Rightarrow \Delta x = x - a$
$= 2 - 1 = 1$

- -1.244
- 1.236
- 1.238
- -1.242

Local Linear Approx!

$f(2) \approx f(1) + f'(1)(2-1)$

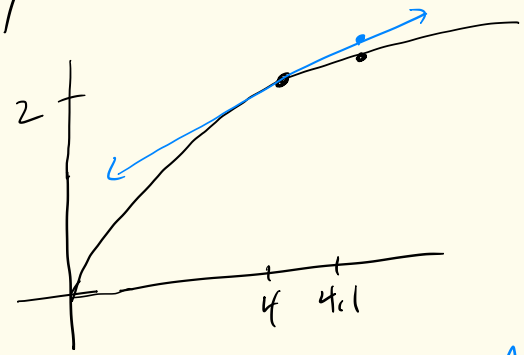
$f(x)$ $f(a)$ $f'(a)$ $\Delta x = x - a$

$= -1.24 + (-0.002) \cdot (1)$

$= -1.242$

Ex: Estimate $\sqrt{4.1}$

$y = f(x) = \sqrt{x} = x^{1/2}$



Fact: $f(x) = x^r$

$f'(x) = r\,x^{r-1}$ Power Rule

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

$a = 4 \quad x = 4.1 \Rightarrow \Delta x = 0.1$

$f(a) = f(4) = 2$, $f'(a) = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\sqrt{4.1} = f(4.1) \approx f(4) + f'(4)\cdot\Delta x$

$= 2 + \frac{1}{4}\cdot 0.1 = 2.025$

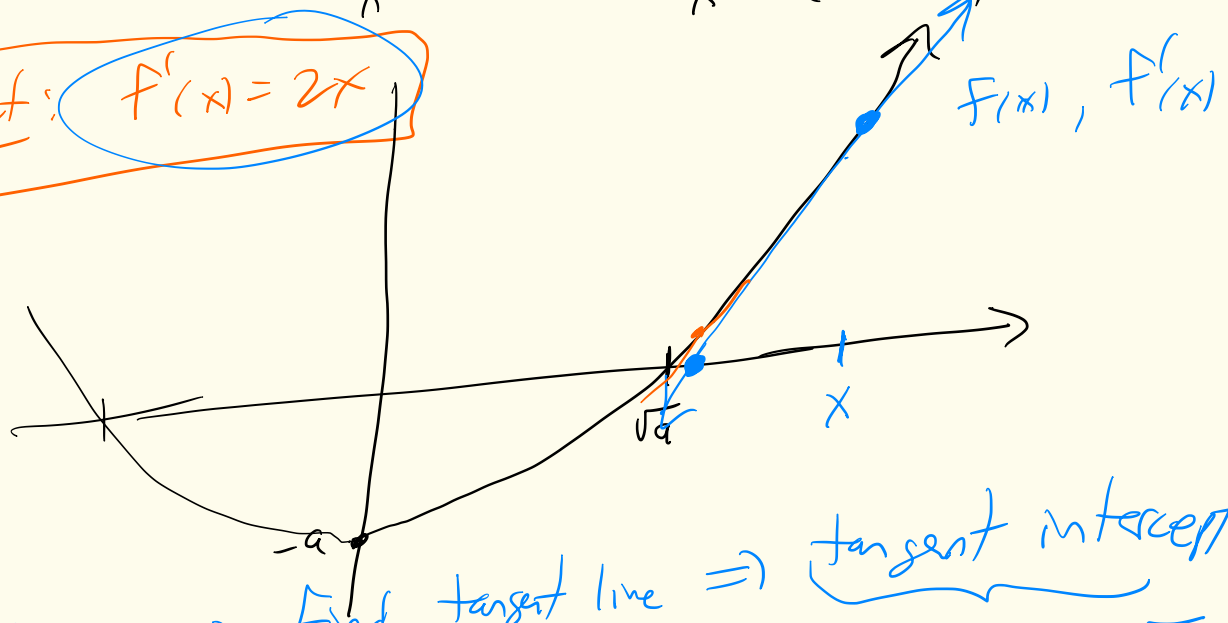
Exact: $\sqrt{4.1} = 2.024845\ldots$

Aside: What does your $\sqrt{\ }$ button do?

$\sqrt{a}$: let $f(x) = x^2 - a$ $\Rightarrow$ $f(\pm\sqrt{a}) = 0$

$x^2 - a = 0 \Rightarrow x^2 = a$ or $x = \pm\sqrt{a}$

Fact: $f'(x) = 2x$



given x $\Rightarrow$ find target line $\Rightarrow$ tangent intercept

an approx to $\sqrt{a}$

Newton's Method

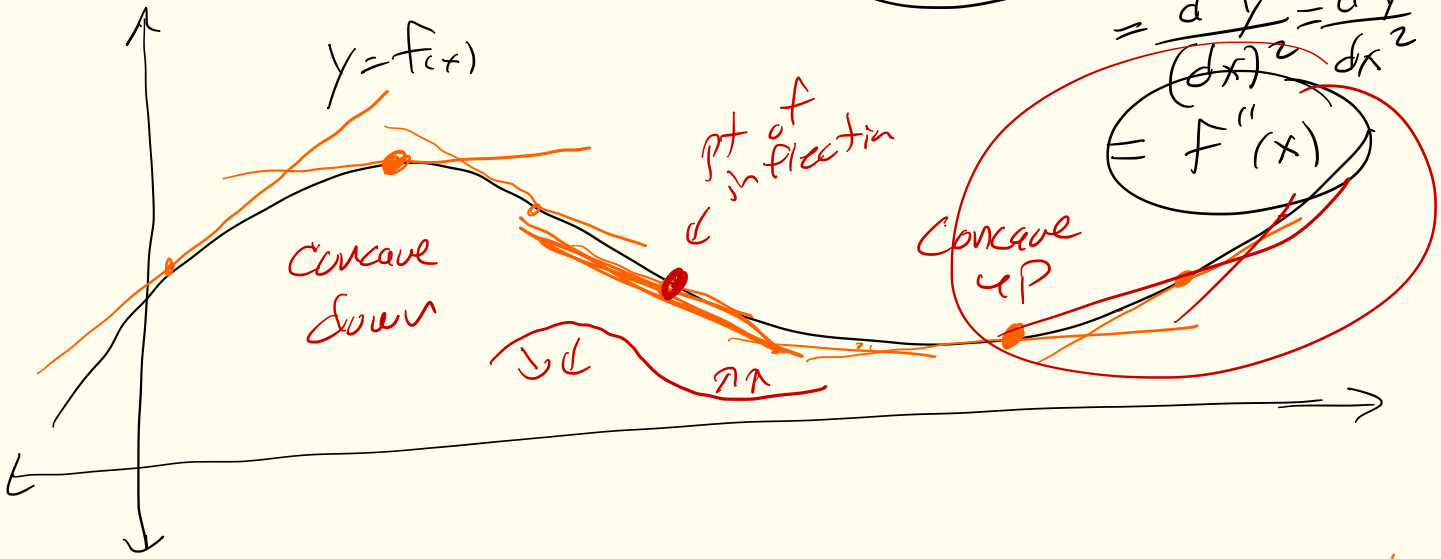
2.4: 2nd Derivative

CONCAVITY

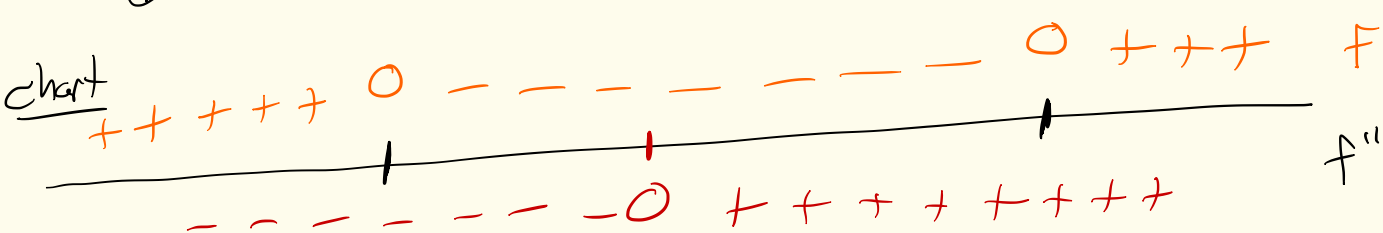
$y = f(x)$

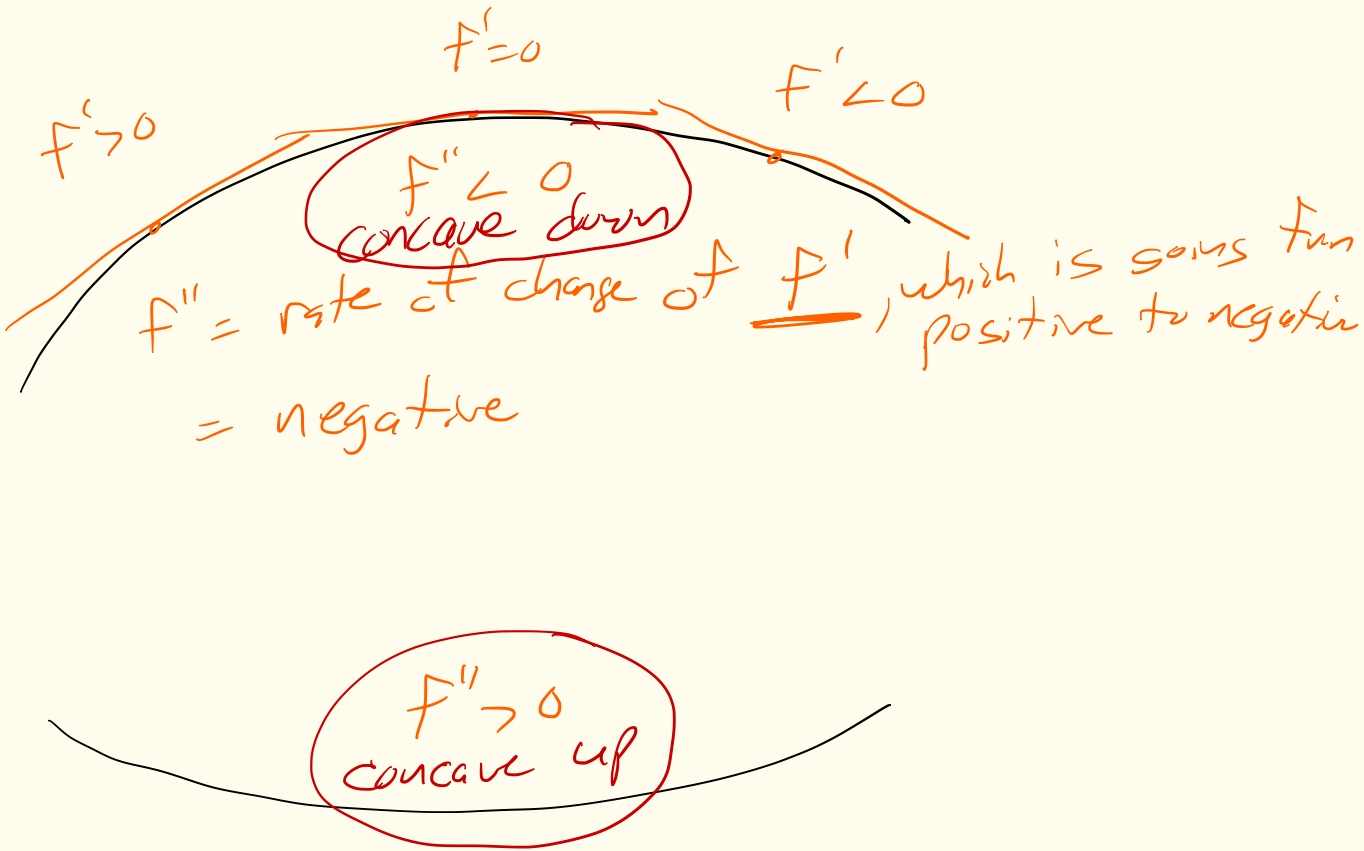
1st derivative: $\frac{dy}{dx} = f'(x)$

2nd derivative: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{(dx)^2} = \frac{d^2y}{dx^2} = f''(x)$



chart




f'=0
f'>0
f'<0
f''<0
concave down
f'' = rate of change of f', which is going from positive to negative
= negative
f''>0
concave up

Chapter 2, Section 2.4, Question 11

Use the values given for the function.

t	0	1	2	3	4	5
$s(t)$	9	14	24	36	49	64

+5 +10 +12 +13 +15

(a) Does the derivative of the function appear to be positive or negative over the given interval?

It appears to be [ ] over the given interval.

positive

(b) Does the second derivative of the function appear to be positive or negative over the given interval?

It appears to be [ ] over the given interval.

positive

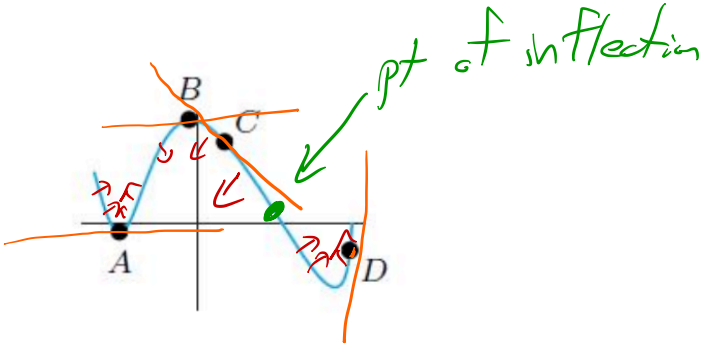
t	0	1	2	3
$s(t)$	9	16	19	20

7 3 1

$f'' < 0$

Chapter 2, Section 2.4, Question 19

At exactly two of the labeled points in the figure below, the derivative f' is 0; the second derivative f'' is not zero at any of the labeled points. In the table, give the signs of f, f', f'' at each marked point.



Point	f	f'	f''
A	−	0	+
B	+	0	−
C	+	−	−
D	−	+	+