Math 127 : Final 2013

1. $f(x) = 2x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 2x^2}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$$

$$= \frac{\cancel{2x^2} + 4x\cancel{h} + 2h^{\cancel{2}} - \cancel{2x^2}}{\cancel{h}}$$

$$= 4x + 2h$$ = answer does not appear

2. $f(x) = \ln(5x+1)$

$g(x) = 2x - 3$

$$f(g(x)) = f(2x-3)$$

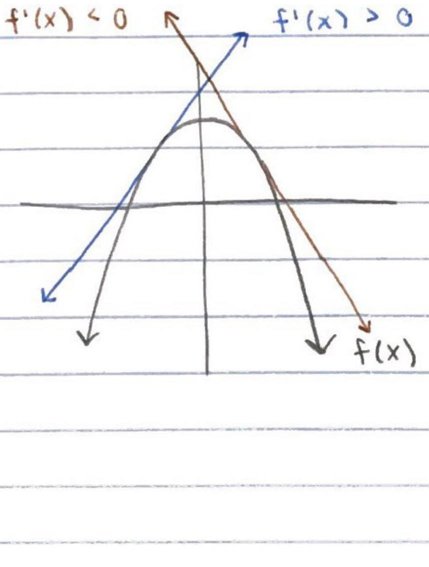
$$= \ln[5(2x-3)+1]$$

$$= \ln(10x - 15 + 1)$$

$$= \ln(10x - 14)$$ = $\boxed{A}$

3. $h(g(-2)) = h(0) = 2$ = $\boxed{D}$

4.



$f'(x) > 0$ for $x < 0$

$f'(x) < 0$ for $x > 0$ } $\boxed{A}$

5. $C = f(q)$ cost to produce q baskets

$C' = f'(q)$ rate of cost

$f'(100) = 2.3$ ⟶ \$2.30 per basket $= \boxed{C}$

↓

100 baskets

6. $y = -2\sin(4\theta) + 1$ ⟶ period $= \frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$

↓

amplitude $= |-2| = 2$

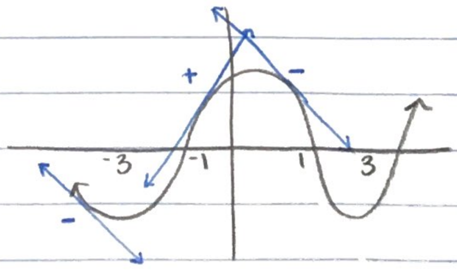
$= \boxed{D}$

7. $$\lim_{x\to5}\frac{x^2-25}{x-5} = \lim_{x\to5}\frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}$$

$$= \lim_{x\to5} x+5$$

$$= 5+5$$

$$= \textcircled{10} \quad = \boxed{D}$$

8. 

~~A.~~ f'(x) NEG [-1, 1]

B. f'(x) NEG [0, 3]

~~C.~~ f'(x) POS $(-\infty, 2)$ + (1, 3)

~~D.~~ f'(x) POS $(0, \infty)$

$= \boxed{B}$

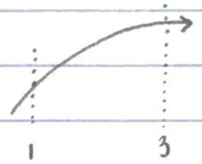
9. $f(8) = 10$

$f'(8) = -2$

$$
\begin{aligned}
f(10) \approx L(10) &= f(a) + f'(a)\,\Delta x \\
&= f(8) + f'(8)(10-8) \\
&= 10 + (-2)(2) \\
&= 10 - 4 \\
&= 6 \quad = \boxed{D}
\end{aligned}
$$

10. pos $f'(x)$ = temp is rising

neg $f''(x)$ = concave down



$= \boxed{A}$

11. $f(x) = x^3 - 4x - 4$ ⓐ $x = 2$

① slope

$$
\begin{aligned}
f'(x) &= 3x^2 - 4 \\
f'(2) &= 3(2)^2 - 4 \\
&= 3(4) - 4 \\
&= 12 - 4 \\
&= 8
\end{aligned}
$$

② point

$$
\begin{aligned}
f(2) &= 2^3 - 4(2) - 4 \\
&= 8 - 8 - 4 \\
&= -4
\end{aligned}
$$

$\downarrow$

$(2, -4)$

③ tangent line

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y + 4 &= 8(x - 2) \\
&= 8x - 16 \\
y &= 8x - 20 \quad = \boxed{C}
\end{aligned}
$$

12.

x :	0	2	4	6	8	10	12	14
f'(x) :	8	6	-1	3	-5	-2	-1	-2

local max ↓ local max

local min

local max between $x = 2$, $x = 4$ $= \boxed{A}$

$x = 6$, $x = 8$

local min between $x = 4$, $x = 6$

13.

$y = \cos(x^2)$

$y' = -\sin(x^2)(2x) = -2x\sin(x^2)$

$y'' = -2\sin(x^2) + (-2x)\cos(x^2)(2x)$

$= -2\sin(x^2) - 4x^2\cos(x^2)$ $= \boxed{B}$

in exam, it is marked as "$2\sin x^2$" but should be negative (what I got!)

14.

$f(x) = \sin x$

$g(x) = e^x$

$h(x) = g(f(x))$

$= g(\sin x)$

$= e^{\sin x}$

$h'(x) = e^{\sin x}(\cos x)$ $= \boxed{A}$

15. $f(x) = -2xe^{4x}$

$f'(x) = -2e^{4x} + (-2x)e^{4x}(4)$

$= -2e^{4x} - 8xe^{4x}$

$f''(x) = -2e^{4x}(4) - 8e^{4x} + (-8x)e^{4x}(4)$

$= -8e^{4x} - 8e^{4x} - 32xe^{4x}$

$= -8e^{4x}[1 + 1 + 4x]$

$0 = \underbrace{-8e^{4x}}_{\neq 0}[2 + 4x]$

$0 = 2 + 4x$

$-4x = 2$

$x = -\frac{1}{2}$

f'' $+$ | $-$

$\underbrace{-\frac{1}{2}}_{\text{inflection point}}$

$= \boxed{B}$

10. $f(x) = x^3 - 3x^2 - 8x + 1$

$f'(x) = 3x^2 - 6x - 8$

$0 = (3x \quad)(x \quad) \Rightarrow$ no factors

? this question does not seem correct

17. $R = 750q$ $\quad\quad$ $C = 8500 + 5q^2$

$R' = 750$ $\quad\quad$ $C' = 10q$

$P = R - C$

$P' = R' - C'$

$0 = 750 - 10q$

$10q = 750$

$q = 75$ $\quad = \boxed{D}$

18. $P(0) = 30$ $\quad\quad$ $C = ?$

$r = 1.04$

$L = 1200$

$$P = \frac{L}{1 + ce^{-rt}} = \frac{1200}{1 + ce^{-1.04t}}$$

$$P(0) = \frac{1200}{1 + ce^{-1.04(0)}}$$

$$30 = \frac{1200}{1 + c}$$

$30(1 + c) = 1200$

$1 + c = 40$

$c = 39$

Logistic Equation: $P = \dfrac{1200}{1 + 39e^{-1.04t}}$ $\quad = \boxed{A}$

19. $C(t) = 5te^{-0.1t}$

$C'(t) = 5e^{-0.1t} + 5te^{-0.1t}(-0.1)$

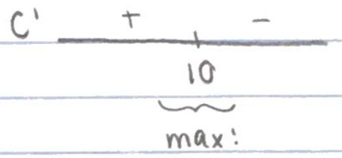
$= 5e^{-0.1t} - 0.5te^{-0.1t}$

$0 = \underbrace{e^{-0.1t}}_{\neq 0}(5 - 0.5t)$

$0 = 5 - 0.5t$

$0.5t = 5$

$t = 10$



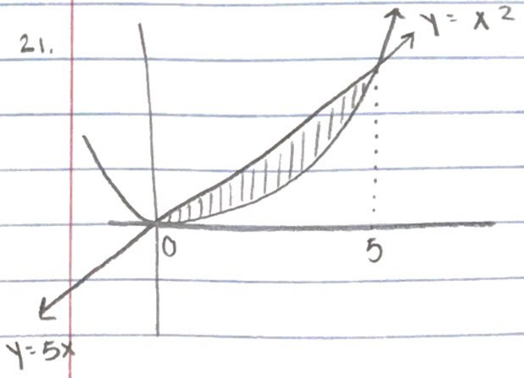
$C(10) = 5(10)e^{-0.1(10)} = 18.39 \text{ mg} = \boxed{C}$

20. $\int_0^4 \sin x \, dx$

x:	0	1	2	3	4
sinx:	0	sin1	sin2	sin3	sin4

$RHS = 1[\sin 1 + \sin 2 + \sin 3 + \sin 4] = \boxed{C}$

21.



$\int_0^5 5x - x^2 \, dx = \boxed{B}$

22. $f(x) = \sin 3x + 100$

$\int f(x)\,dx = \int \sin 3x + 100 \ dx$

$= -\cos 3x \left(\frac{1}{3}\right) + 100x + C$ $= \boxed{D}$

23. $\int x^3 + e^{5x} + \frac{1}{x} \ dx = \frac{x^4}{4} + \frac{1}{5}e^{5x} + \ln|x| + C$ $= \boxed{A}$

24. $\int 10x(5x^2 + 6)^{10} \ dx$

$u = 5x^2 + 6$

$du = 10x \ dx$ $= \boxed{C}$

25. $\int x^2 \sin(5x) \ dx$

$u = x^2$ $\quad dv = \sin 5x \ dx$

$du = 2x$ $\quad v = -\frac{1}{5}\cos 5x$ $= \boxed{C}$

26. $\int x^3(x^4 - 5)^4 \ dx = \int (x^4 - 5)^4 x^3 \ dx$

$u = x^4 - 5$

$du = 4x^3 \ dx$

$\frac{1}{4} dx = x^3 \ dx$

$= \int u^4 \cdot \frac{1}{4} \ du$

$= \frac{1}{4} \int u^4 \ du$

$= \frac{1}{4}\left[\frac{1}{5}u^5\right] + C$

$= \frac{1}{20}\left[u^5\right] + C$

$= \frac{1}{20}(x^4 - 5)^5 + C$ $= \boxed{C}$

27. $\int x^3 \sqrt{x^4 + 2}\ dx \qquad = \int \sqrt{x^4 + 2}\ \ x^3\ dx$

$u = x^4 + 2 \qquad = \int \sqrt{u} \cdot \frac{1}{4}\ du$

$du = 4x^3\ dx \qquad = \frac{1}{4} \int \sqrt{u}\ du$

$\frac{1}{4}\ du = x^3\ dx \qquad = \frac{1}{4} \int u^{1/2}\ du$

$= \frac{1}{4}\left[\frac{2}{3} u^{3/2}\right] + C$

$= \frac{1}{6}\left[u^{3/2}\right] + C$

$= \frac{1}{6}(x^4 + 2)^{3/2} + C \qquad = \boxed{B}$

28. $\int 25x \sin x\ dx$

$u = 25x \searrow \qquad dv = \sin x\ dx$

$du = 25\ dx \qquad v = \ominus \cos x$

note: a $(-)(-) = +$

$= -25x\cos x + \int 25\cos x\ dx$

$= -25x\cos x + 25\int \cos x\ dx$

$= -25x\cos x + 25[\sin x] + C$

$= -25x\cos x + 25\sin x + C \qquad = \boxed{A}$

29. $\int -4x\cos 9x\ dx$

$u = -4x \searrow \qquad dv = \cos 9x\ dx$

$du = \ominus 4dx \qquad v = \frac{1}{9}\sin 9x\ dx$

note: a $(-)(-) = +$

$= -\frac{4}{9}x\sin 9x + \int \frac{4}{9}\sin 9x\ dx$

$= -\frac{4}{9}x\sin 9x + \frac{4}{9}\int \sin 9x\ dx$

$= -\frac{4}{9}x\sin 9x + \frac{4}{9}\left[-\frac{1}{9}\cos 9x\right] + C$

$= -\frac{4}{9}x\sin 9x - \frac{4}{81}\cos 9x + C \qquad = \boxed{C}$

30. $\int \ln(10x)\,dx$

$u = \ln(10x)$ $\qquad$ $dv = dx$

$du = \frac{1}{10x} \cdot 10\,dx$ $\qquad$ $v = x$

$= \frac{1}{x}\,dx$

$= x\ln(10x) - \int x \cdot \frac{1}{x}\,dx$

$= x\ln(10x) - \int 1\,dx$

$= x\ln(10x) - [x] + C$

$= \boxed{x\ln(10x) - x + C}$ $\quad = \boxed{D}$