Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

Ex: $\int x \sin x \, dx$

$u = x \qquad v = -\cos x$

$\qquad\qquad dv = \sin x\, dx$

$du = 1 \cdot dx$

$$= (x)(-\cos x) - \int (-\cos x)\, dx$$

$$= -x\cos x + \int \cos x\, dx$$

$$= \boxed{-x\cos x + \sin x + C}$$

Ex: $\boxed{\int e^x \sin x \, dx} = -e^x \cos x - \int (-\cos x) e^x \, dx$

$u = e^x \qquad v = -\cos x$

$du = e^x dx \qquad dv = \sin x \, dx$

$= -e^x \cos x + \boxed{\int e^x \cos x \, dx}$

$= \boxed{-e^x \cos x + e^x \sin x} - \boxed{\int e^x \sin x \, dx}$

$$\int e^x \sin x \, dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

$\int e^x \cos x \, dx = \boxed{e^x \sin x - \boxed{\int e^x \sin x \, dx}}$

$u = e^x \qquad v = \sin x$

$du = e^x dx \qquad dv = \cos x \, dx$

$\square = \square - \square$

$$\underline{\int e^x \sin x\,dx} = -e^x\cos x + e^x \sin x - \underline{\int e^x \sin x\,dx}$$

$$\Rightarrow 2\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x + C$$

$$\boxed{\int e^x \sin x\,dx = \frac{1}{2}\left(-e^x\cos x + e^x\sin x\right) + C}$$

Ex: Find the eqn of the tangent line to $y = x^3 - 3x^2 - 5x + 2$ at $x = 2$

ans: $\frac{dy}{dx} = 3x^2 - 6x - 5 \Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = 3\cdot 2^2 - 6\cdot 2 - 5 = \boxed{-5}$

slope

pt: $(2, 2^3 - 3\cdot 2^2 - 10 + 2) = \boxed{(2, -12)}$

eqn: $y - (-12) = -5(x - 2)$

$y + 12 = -5x + 10$

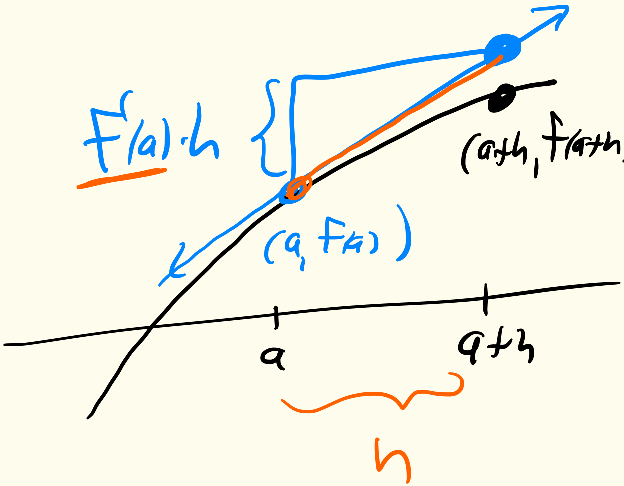
$\boxed{y = -5x - 2}$

Ex: Suppose $f(3) = 3$ and $f'(3) = -1$

Use a local linear approx. to estimate $f(\underline{3.5})$

$$f(a+h) \approx f(a) + f'(a) \cdot h$$

$a = 3$, $h = \frac{1}{2}$



$$f\left(3+\tfrac{1}{2}\right) \approx f(3) + f'(3) \cdot \frac{1}{2}$$

or

$$f(3.5) \approx 3 + (-1) \cdot \frac{1}{2}$$

$$= \boxed{2.5}$$

Ex: Find $F(x)$ such that

$F'(x) = 10\cos(2x) - e^{-x}$, $\boxed{F(0) = 10}$

ans:

$$\boxed{\begin{aligned} F(x) &= \int 10\cos(2x) - e^{-x}\,dx \\ &= 10\,\tfrac{1}{2}\sin(2x) - (-e^{-x}) + C \\ &= 5\sin(2x) + e^{-x} + C \end{aligned}}$$

$= 9$

$10 = F(0) = 5 \cdot \sin(2 \cdot 0) + e^{-0} + C$

$= 5 \cdot 0 + 1 + C \quad = 1 + C$

$C = 9$