Examples

a) $\int \frac{\cos x}{1+\sin x}\,dx$

b) $\int \ln(2x)\,dx$ ✓

c) $\int \frac{x}{e^x}\,dx$

d) $\int x \sin x\,dx$

e) $x^3\sqrt{x^4+2}\,dx$ ✓

f) $\int e^x \sin x\,dx$

a) $\int (1+\sin x)^{-1}\cos x\,dx = \int u^{-1}\,du = \int \frac{1}{u}\,du$

$u = 1+\sin x$, $du = \cos x\,dx$

$\frac{dy}{dx} = \cos x$, $du = \cos x\,dx$

$= \ln|u| + C$

$= \ln|1+\sin x| + C$

$1+\sin x \geq 0$

e) $\int x^3\sqrt{x^4+2}\,dx = \int x^3(x^4+2)^{1/2}\,dx$

$u = x^4+2$

$\frac{du}{dx} = 4x^3 \Rightarrow du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$= \int u^{1/2}\frac{1}{4}\,du$

$= \frac{1}{4}\int u^{1/2}\,du = \frac{1}{4}\,\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{6}u^{3/2} + C = \boxed{\frac{1}{6}(x^4+2)^{3/2} + C}$

6) $\int \underbrace{\ln 2x}_{\ln u} \underbrace{dx}_{\frac{1}{2}du}$

$u = 2x$

$du = 2dx$ or $\frac{1}{2}du = dx$

$= \int \ln u \cdot \frac{1}{2} du = \frac{1}{2}\int \ln u \, du$

$= \frac{1}{2}(u \ln u - u + c)$

$= \frac{1}{2}(2x \ln 2x - 2x + c)$

$= \boxed{x \ln 2x - x + c}$

$\boxed{\int \ln x \, dx = x \ln x - x + c}$

$\frac{d}{dx}(\ln g(x)) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$

$\frac{d}{dx}(x \ln 2x - x) = x \cdot \frac{1}{2x} \cdot 2 + (\ln 2x) \cdot 1 - 1 = \boxed{\ln 2x}$

c) $\int \frac{x}{e^x}\,dx = \int \underbrace{x}_{u}\underbrace{e^{-x}\,dx}_{dv}$

$u = x \qquad v = -e^{-x}$

$du = 1\,dx \qquad dv = e^{-x}\,dx$

IBP $= x(-e^{-x}) - \int (-e^{-x})\,dx$

$= -xe^{-x} + \int e^{-x}\,dx$

$= \boxed{-xe^{-x} - e^{-x} + C = -e^{-x}(x+1) + C}$ $\boxed{\int e^{-x}\,dx = -e^{-x} + C}$

$\int u\,dv = uv - \int v\,du$

Check

$\frac{d}{dx}(-e^{-x}) = -\frac{d}{dx}(e^{-x})$

$= -(e^{-x}\cdot(-1)) = e^{-x}$