7.4: Integration By Parts

$$\underbrace{\int u\,dv}_{\text{original integral}} \boxed{=} \underbrace{uv}_{\text{Known}} - \underbrace{\int v\,du}_{\text{"easier" integral}}$$

$du = g'(x)dx$

$\frac{du}{dx} = g'(x)$

where does it come from?

product Rule: $\frac{d}{dx}(f(x)g(x)) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

mighty equal sign

$$\int f(x)g'(x)dx = \int \frac{d}{dx}(f(x)g(x))dx - \int g(x)f'(x)dx$$

$$\int \underbrace{f(x)}_{u}\underbrace{g'(x)dx}_{dv} = \underbrace{f(x)}_{u}\cdot\underbrace{g(x)}_{v} - \int \underbrace{g(x)}_{v}\underbrace{f'(x)dx}_{du}$$

Ex: $\boxed{\int \underbrace{x}_{u}\, \underbrace{e^x\, dx}_{dv}}$

$\int u\,dv = uv - \int v\,du$

$u = x$

$\frac{du}{dx} = 1$ or $du = 1\,dx$

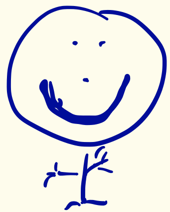
$v = e^x$

$dv = e^x\,dx$ or $\frac{dv}{dx} = e^x$

$= xe^x - \int e^x\, 1 \cdot dx = \boxed{xe^x - e^x + C}$

check: $\frac{d}{dx}\left(xe^x - e^x + c\right) = \frac{d}{dx}\left(xe^x\right) - \frac{d}{dx}\left(e^x\right) + 0$

$= xe^x + e^x \cdot 1 - e^x = \boxed{xe^x}$ ✓



Again:

$$\int x^1 e^x \, dx \qquad \int u \, dv = uv - \int v \, du$$

$$u = e^x \qquad v = \frac{x^2}{2}$$

$$du = e^x \, dx \qquad dv = x \, dx$$

$$= e^x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} e^x \, dx$$

$$= \frac{1}{2} x^2 e^x - \frac{1}{2} \int x^2 e^x \, dx$$

No!

Ex: $\int 2y \ln y \, dy$

$\int u\,dv = uv - \int v\,du$

$= \underline{2} \int \underline{y} \ln y \, dy$

$u = y \quad v = ?$ No!

$du = 1\,dy \quad dv = \ln y\,dy$

let's try ↓

$u = \ln y \quad v = \frac{y^2}{2}$

$du = \frac{1}{y}\,dy \quad dv = y\,dy$

$= 2\left[(\ln y)\frac{y^2}{2} - \int \frac{y^2}{2} \cdot \frac{1}{y}\,dy\right]$

$= 2\left[\frac{1}{2} y^2 \ln y - \frac{1}{2}\int y\,dy\right] = y^2 \ln y - \int y\,dy$

$$= \boxed{y^2 \ln y - \frac{y^2}{2} + C}$$

$\int u\,dv = uv - \int v\,du$

Ex: $\int \underbrace{\ln x}_{u}\, \underbrace{1\,dx}_{dv}$

$u = \ln x \qquad v = x$

$du = \frac{1}{x}dx \qquad dv = 1\,dx$

$$= (\ln x)(x) - \int x \cdot \frac{1}{x}dx = x\ln x - \int 1\,dx$$

$$= x\ln x - x + C$$

check: $\frac{d}{dx}\left(x\ln x - x\right)$

$$= x \cdot \frac{1}{x} + (\ln x)\cdot(1) - 1 = 1 + \ln x - 1$$

$$= \boxed{\ln x}$$