7.2: Substitution: $u = g(x)$ (change of variable)

$$\int f(x)\,dx$$

Chain Rule: $\frac{d}{dx}\left(F(g(x))\right) = F'(g(x)) \cdot g'(x)$

$$\Rightarrow \boxed{\int F'(g(x))\,g'(x)\,dx} = \int \frac{d}{dx}\left(F(g(x))\right)dx = F(g(x)) + C$$

$$\int F'(g(x))\,\underbrace{\boxed{g'(x)\,dx}}^{du} = F(g(x))$$

Let: $u = g(x)$

$$\frac{du}{dx} = g'(x)$$

$$du = g'(x)\,dx$$

$$\Rightarrow \int F'(u)\,du = F(u) + C$$

$$1 \cdot \int f(x)\,dx = \frac{1}{5} \cdot 5 \int f(x)\,dx$$

$$= \frac{1}{5} \int 5 f(x)\,dx$$

Ex: $\int 2x(x^2+1)\,dx = \int \underbrace{2x^3+2x^1}\,dx$

multiplied

$= 2\frac{x^{3+1}}{3+1} + 2\frac{x^{1+1}}{1+1} + C$

$= \boxed{\frac{1}{2}x^4 + x^2 + C}$

Check! $\frac{d}{dx}\left(\frac{1}{2}x^4 + x^2 + c\right)$

$= \frac{1}{2}\cdot 4x^{4-1} + 2x^{2-1} + 0$

$= 2x^3 + 2x$

$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$

$r \neq -1$

$\int f+g = \int f + \int g$

$\int kf = k\int f$

Alt:

$$\int 2x\,\underbrace{(x^2+1)}_{u}\,dx = \int \underbrace{(x^2+1)}_{u}\,\underbrace{2x\,dx}_{du}$$

$$\boxed{u = x^2+1}$$

$$\frac{du}{dx} = \frac{d}{dx}(x^2+1) = 2x \implies \frac{du}{dx} = 2x \text{ or } du = 2x\,dx$$

or $\int u\,du = \dfrac{u^{1+1}}{1+1} + c = \dfrac{1}{2}u^2 + C$

answer in u!

$$= \frac{1}{2}(x^2+1)^2 + C = \frac{1}{2}(x^4+2x^2+1) + C$$

$$= \boxed{\frac{1}{2}x^4 + x^2 + \frac{1}{2} + C}$$

$$\boxed{\frac{1}{2}x^4 + x^2 + C}$$

Ex: $\int \underbrace{(x^2+1)^{100}}_{u^{100}} \underbrace{2x\,dx}_{du} = \int u^{100}\,du$

$u = x^2+1$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$= \frac{u^{100+1}}{100+1} + C$

$= \frac{1}{101} u^{101} + C$

$= \boxed{\frac{1}{101} (x^2+1)^{101} + C}$

Ex:

$$\int \underbrace{e^{2x}}_{e^u} \underbrace{1 \cdot dx}_{\frac{1}{2}du} = \boxed{\frac{1}{2}e^{2x} + C}$$

$$\boxed{\int e^{ax}dx = \frac{1}{a}e^{ax} + C}$$

$u = 2x$

$\frac{du}{dx} = 2$ or $du = 2dx$

or $\frac{1}{2}du = dx$

$$= \int e^u \cdot \frac{1}{2}du = \frac{1}{2}\int e^u du = \frac{1}{2}(e^u + c)$$

$$= \boxed{\frac{1}{2}e^{2x} + C}$$

Yes!

a) $\int x(x^2+1)^{10}\,dx$

b) $\int \frac{\ln t}{t}\,dt$

c) $\int \frac{x^3}{x^4+5}\,dx$

d) $\int \sqrt{\cos(3t)}\,\sin(3t)\,dt$

Ex a): $\int x\,(x^2+1)^{10}\,dx = \int \underbrace{(x^2+1)^{10}}_{u^{10}}\,\underbrace{x\,dx}_{1/2\,du}$

$\boxed{u = x^2+1}$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$

So $= \int u^{10}\frac{1}{2}du = \frac{1}{2}\int u^{10}du$

$= \frac{1}{2}\frac{u^{11}}{11} + C = \frac{1}{22}u^{11} + C$ answer in terms of u

$= \boxed{\frac{1}{22}(x^2+1)^{11} + C}$

c) $\int \frac{x^3}{x^4+5}\,dx = \int \underbrace{(x^4+5)^{-1}}_{u^{-1}} \underbrace{x^3 dx}_{1/4\,du}$

$\boxed{u = x^4 + 5}$

$\frac{du}{dx} = 4x^3$

or

$du = 4x^3 dx$

or

$\frac{1}{4} du = x^3 dx$

$= \int u^{-1} \frac{1}{4} du = \frac{1}{4} \int \frac{1}{u} du$

$= \frac{1}{4} \ln|u| + C$

$= \frac{1}{4} \ln|x^4+5| + C$

$= \boxed{\frac{1}{4} \ln(x^4+5) + C}$

$\boxed{\int \frac{1}{x} dx = \ln|x| + C}$

d) $\int \sqrt{\cos(3t)} \cdot \sin 3t \, dt$

$= \int \left((\cos(3t))^{1/2} \right) \left(\underbrace{\sin 3t \, dt} \right)$

$\boxed{u = \cos(3t)}$

$\frac{du}{dt} = -3 \sin(3t)$

or

$du = \sin(3t)\, dt$

$\frac{du}{-3}$ or

$-\frac{1}{3} du = \sin(3t) dt$

$= \int u^{1/2} \left(-\frac{1}{3} du \right)$

$= -\frac{1}{3} \boxed{\int u^{1/2} du}$

$= -\frac{1}{3} \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= -\frac{2}{9} u^{3/2} + C$

$= -\frac{2}{9} (\cos(3t))^{3/2} + C$

b) $\int \frac{\ln t}{t}\,dt$

$$\frac{d}{dt}(t) = 1$$

$$\frac{d}{dt}(\ln t) = \frac{1}{t}$$

$$= \int \boxed{\ln t}\,\boxed{\left(\frac{1}{t}\right)dt}$$

$$\boxed{u = \ln t}$$

$$\boxed{du = \frac{1}{t}\,dt}$$

$$= \int u\,du = \frac{u^2}{2} + C$$

$$= \boxed{\frac{1}{2}(\ln t)^2 + C}$$

"u universe"