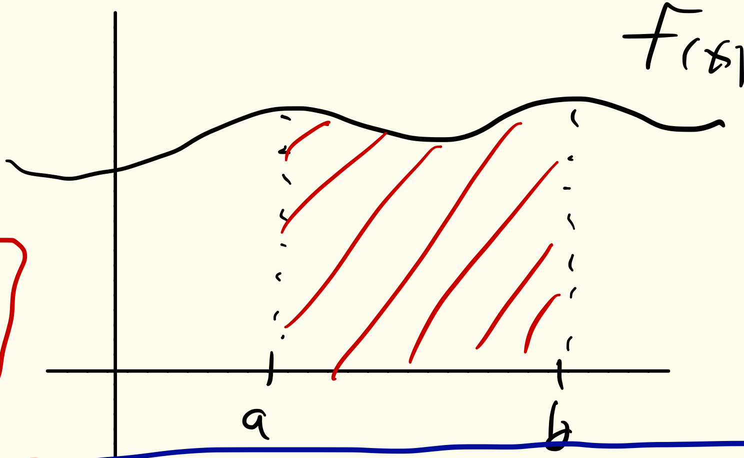
Chapter 5 - Integration ("Area" problem)

Given: • $f(x)$

• $[a,b]$



\# Definite Integral

Area $= \int_a^b f(x)\,dx \underset{FTC}{=} F(b) - F(a) = \#$

where $F'(x) = f(x)$ or $F(x) = \underbrace{\int f(x)\,dx}_{\text{Indefinite Integral}}$

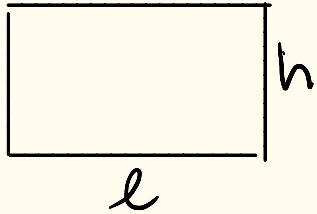
HARD

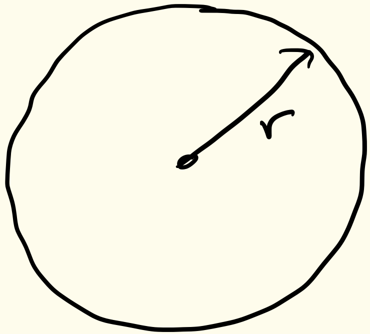
Indefinite Integral = Function

"Area"

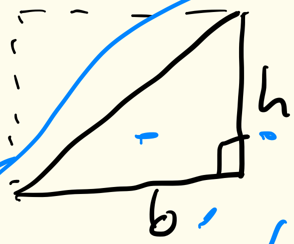
$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$A = \pi r^2$$

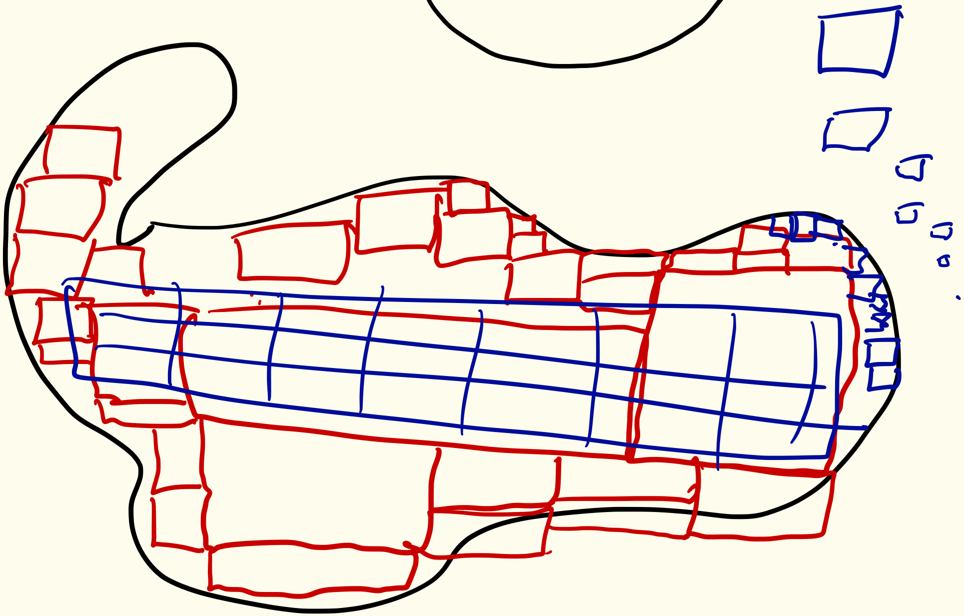




$$\text{Area} = l \cdot h$$



$$\text{Area} = \frac{1}{2} b \cdot h$$



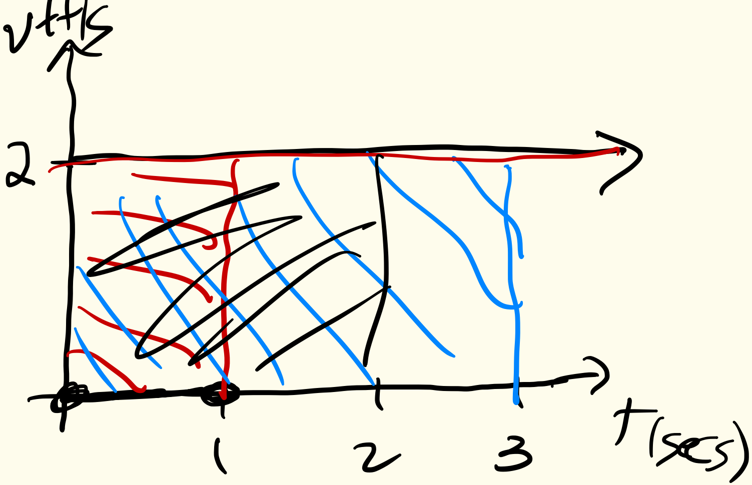
Ex: $s(t)$ - position
$v(t)$ - velocity $= s'(t)$
$a(t)$ - accel $= v'(t) = s''(t)$

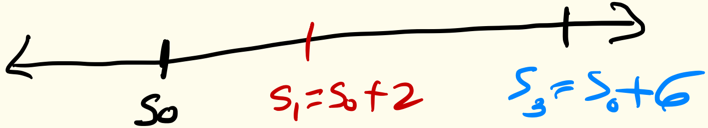
$\boxed{\begin{array}{l} F = ma = ms''(t) \\ \text{Find } s(t) \end{array}}$

"undo" derivatives

$v(t) = 2$, $s_0 =$ initial position $= s(0)$

1D





$t \geq 0$

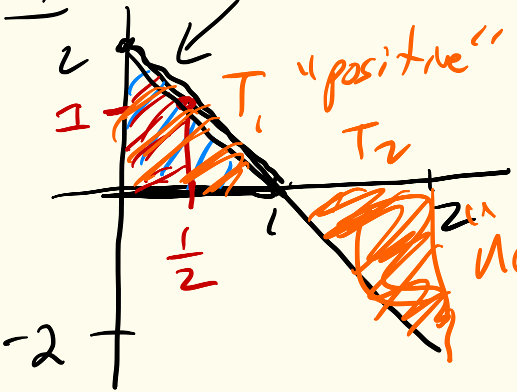
$s_1 = s(1) = s_0 + \text{Area}$
$= s_0 + \int_0^1 v(t)\,dt$
$= s_0 + \underline{2}$

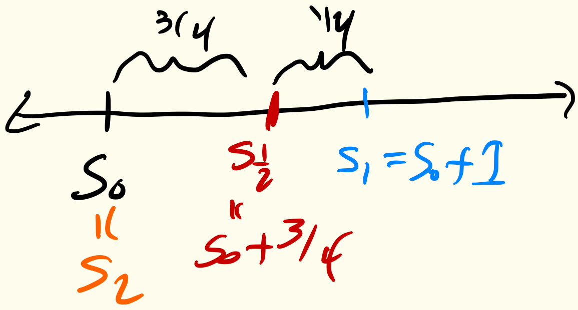
$s_3 = s(3) = s_0 + \int_0^3 v(t)\,dt$
$= s_0 + \underline{6}$

$s_2 = s_0 + 4$

Ex!

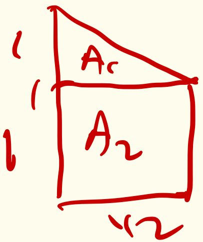
$v(t) = -2t + 2$





$$S(1) = S_1 = \underline{S_0} + \underbrace{\int_0^1 v(t)\,dt}_{\text{total displacement}} = S_0 + \frac{1}{2}\cdot 1 \cdot 2 = S_0 + 1$$

$$S\left(\tfrac{1}{2}\right) = S_0 + \underbrace{\frac{1}{2}\cdot\frac{1}{2}\cdot 1}_{\triangle} + \underbrace{\frac{1}{2}\cdot 1}_{\square} = S_0 + \frac{1}{4} + \frac{1}{2} = \underline{S_0 + 3/4}$$

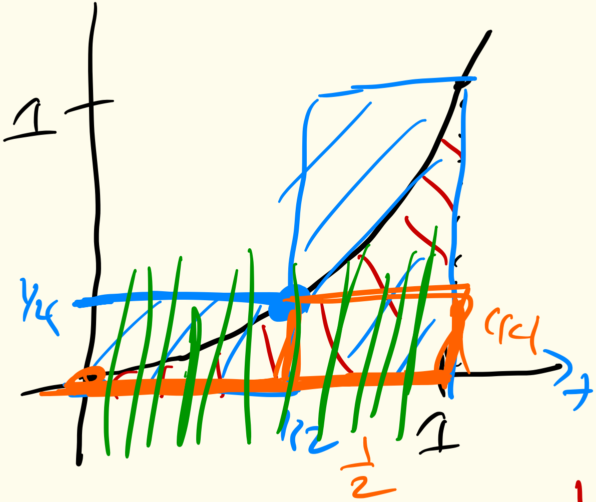


$$S(2) = S_2 = S_0 + \int_0^2 v(t)\,dt$$

$$= S_0 + 0 = S_0$$

Ex: $v(t) = t^2$

$$s(1) = s_1 = s_0 + \underbrace{\int_0^1 v(t)\,dt}_{\text{total displacement}}$$



$$\frac{1}{8} = 0 + \frac{1}{8} \leq \int_0^1 v(t)\,dt \leq \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{2}\cdot 1 = \frac{5}{8}$$

$$\frac{3}{24} \leq 1/3 = \frac{8}{24} \leq \frac{15}{24}$$

$$\int_0^1 t^2\,dt = \underline{V}(1) - \underline{V}(0)$$
$$= \left(\frac{1}{3}\cdot 1^3 + \cancel{C}\right) - \left(\frac{1}{3}0^3 + \cancel{C}\right)$$
$$= \boxed{\frac{1}{3}}$$

$v(t) = t^2$

$\underline{V}(t) = \underline{\frac{1}{3}} t^3 + C$

check:

$\underline{V}'(t) = \frac{1}{3}\cdot 3\cdot t^2 + 0$

$= t^2$ Yes!