Integration:

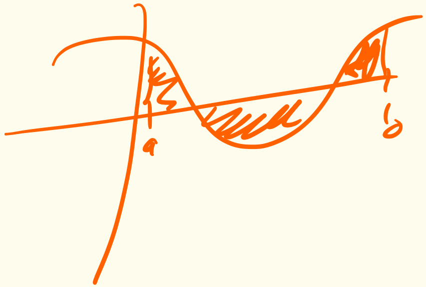
Given $f(x)$

Find $\underbrace{F(x)}_{\text{anti-derivative}} = \underbrace{\int f(x)\,dx}_{\text{indefinite integral}}$ so $F'(x) = f(x)$

What is it good for?

"Area" problems

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$$\frac{d}{dx}(3x) = 3$$

$$\boxed{\frac{d}{dx}(kx) = k} \Rightarrow \boxed{\int k\,dx = kx + C}$$

$$\frac{d}{dx}(x^2) = 2x \Rightarrow$$

$$\frac{d}{dx}(x^r) = r x^{r-1}$$

$$\frac{d}{dx}(x^{r+1}) = (r+1)\,x^r$$

$$\boxed{\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \qquad r \neq -1}$$

$$r = -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\sin(3x)) = 3\cos 3x$$

$$\frac{d}{dx}(\sin(ax)) = a \cdot \cos(ax)$$

$$\Rightarrow \boxed{\int \cos ax \, dx = \frac{1}{a} \sin(ax) + C}$$

$$\frac{d}{dx}\left(\frac{1}{a}\sin(ax)\right) = \frac{1}{a}\frac{d}{dx}(\sin(ax)) = \frac{1}{\cancel{a}} \cdot \cancel{a} \cos(ax) + c$$

$$= \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\Rightarrow \boxed{\int \sin(ax) \cdot dx = -\frac{1}{a}\cos(ax) + C}$$

$$\frac{d}{dx}\left(e^x\right) = e^x \Rightarrow \int e^x\,dx = e^x + C$$

$$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax} \Rightarrow \boxed{\int e^{ax}\,dx = \frac{1}{a} e^{ax} + C}$$

$a=0$: $\int e^{0 \cdot x}\,dx = \int 1\,dx = x + C$

General properties

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

$$\Rightarrow \int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\frac{d}{dx}\left(k\,f(x)\right) = k \cdot \frac{d}{dx}\left(f(x)\right)$$

$$\Downarrow$$

$$\int k\,f(x)\,dx = k \int f(x)\,dx$$

sums, differences, and constant multiples

Ex:

$$\int \overbrace{3x^2 - \sqrt{x} + \cos(3x) + \pi}^{f(x)}\, dx$$

$\cancel{\int e^{\sin x}\, dx}$

$$= \boxed{\int 3x^2\, dx} - \int \sqrt{x}\, dx + \int \cos(3x)\, dx + \int \pi\, dx$$

$r=2$

$$= 3\int x^2\, dx - \int x^{1/2}\, dx + \int \cos(3x)\, dx + \int \pi\, dx$$

$$= 3\left(\frac{x^{2+1}}{2+1}\right) - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{3}\sin(3x) + \pi x + C$$

$$= x^3 - \frac{x^{3/2}}{3/2} + \frac{1}{3}\sin(3x) + \pi x + C$$

$$= \boxed{x^3 - \frac{2}{3}x^{3/2} + \frac{1}{3}\sin(3x) + \pi x + C} = F(x)$$

$$\frac{9}{6} = \frac{1}{6} \cdot 9$$

CHECK!

$$F'(x) = f(x)$$

$\int e^{\sin x}\,dx$?

$\frac{d}{dx}\left(\underline{e^{\sin x}}\right) = \left(e^{\sin x}\right)\cdot(\sin x)'$ Chain Rule

$= \cos x\, e^{\sin x}$

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