5.1 and 5.2

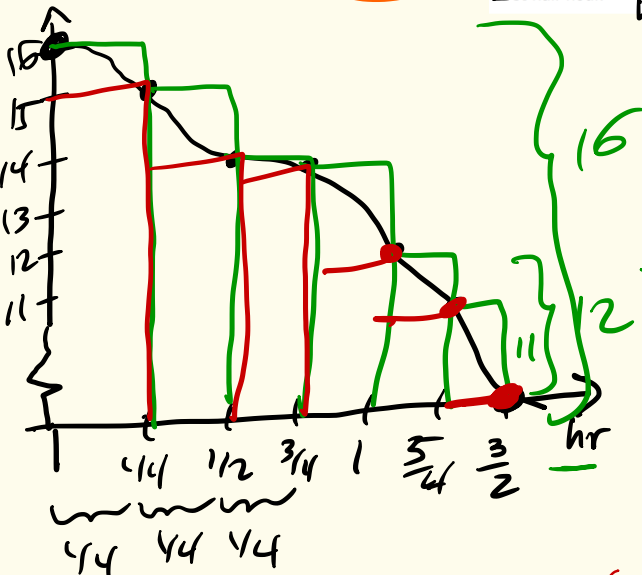
Better estimate $= \frac{20\frac{1}{2} + 16\frac{1}{2}}{2} = \frac{37}{2} = 18\frac{1}{2}$

Roger runs a marathon. His friend Jeff rides behind him on a bicycle and clocks his speed every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. Jeff's data follow:

(hr) 0 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ 1 $\frac{5}{4}$ $\frac{3}{2}$

Time since start (min)	0	15	30	45	60	75	90
Speed (mph)	16	15	14	14	12	11	0

(a) Assuming that Roger's speed is never increasing, give upper and lower estimates for the distance Roger ran during the ~~first half hour.~~ hour and a half of his run



16 (LHS)

12

11

total displacement $= \int_{t_{beg}}^{t_{end}} v(t)\,dt$

upper estimate

$\frac{1}{4}(16+15+14+14+12+11)$

$= \frac{1}{4}(82) = \frac{82}{4} = \frac{41}{2} = 20\frac{1}{2}$ miles

(hr)(miles/hr)

lower estimate (RHS in this case)

$= \frac{1}{4}(15+14+14+12+11+0) = \frac{66}{4} = \frac{33}{2}$

$= 16\frac{1}{2}$

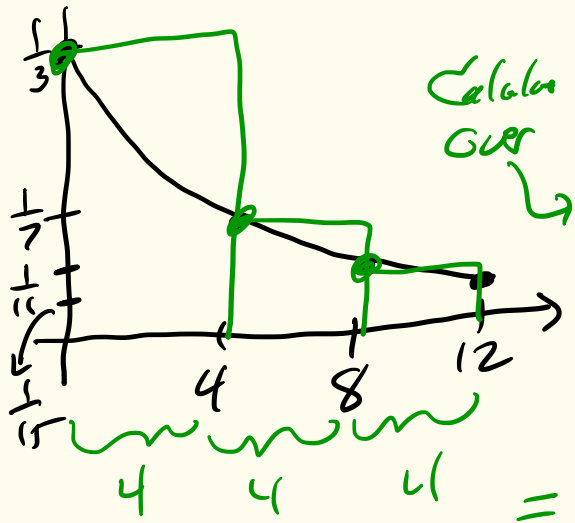
Chapter 5, Section 5.2, Question 2

Estimate $\int_0^{12} \frac{1}{x+3}\,dx$ using a left-hand sum with $n = 3$.

Enter your answer to three decimal places.

$f(x) = \frac{1}{x+3}$

$\frac{3}{9} + \frac{34}{5 \cdot 4} = \frac{3.5}{20} + \frac{342}{20}$



LHS:

$4(f(0) + f(4) + f(8)) \quad \frac{277}{231}$

Calculator over

$= 4\left(\frac{1}{3} + \frac{1}{7} + \frac{1}{11}\right)$

$= 4\left(\frac{7 \cdot 11}{3 \cdot 7 \cdot 11} + \frac{3 \cdot 11}{3 \cdot 7 \cdot 11} + \frac{3 \cdot 7}{3 \cdot 7 \cdot 11}\right)$

$= 4\left(\frac{77 + 33 + 21}{231}\right)$

$= 4\left(\frac{131}{231}\right) \approx 2.2683\ldots$

Calculator

$$\int_0^{12} \frac{1}{x+3}\,dx \approx 1.6094\ldots$$

$f(x)$

$$\overset{\text{exact}}{\underset{\text{FTC}}{=}} F(12) - F(0)$$

$$= \ln(12+3) - \ln(0+3)$$

$$= \ln(15) - \ln(3)$$

$$= \ln\left(\frac{15}{3}\right) = \ln(5)$$

$$\approx 1.6094\ldots$$

Looking Forward

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}\underbrace{\left(\ln(x+3)\right)}_{F(x)} = \frac{1}{x+3}\cdot 1 = \frac{1}{x+3} = f(x)$$