1. Basic Probability (p.1).
   Interpretations, experiment and sample space, mutually exclusive events, Borel field, probability measure, basic properties including law of total probability, Boole’s and Bonferonni’s inequalities, how to assign probabilities, counting rules (with a focus on with and without replacement sampling), the hypergeometric distribution, multinomial counting, conditional probability, generalized multiplication rule, Bayes rule, independent events, the multinomial distribution with the binomial as a special case.

2. Random Variables (17)
   Definition of a R.V., the C.D.F., discrete random variables and p.m.f., continuous random variables and p.d.f, the exponential distribution, percentiles.

3. Expected values and moments (21)
   Definition of expected value, moments (including mean and variance with illustration using common distributions), Chebychev’s Inequality, the relationship between moments of a random variable and descriptive measures for a set of numerical values. Properties of expectations, moment generating functions.

4. Parametric families (27)
   Uniform, Poisson, Negative Binomial, Geometric, Normal, Gamma, Chi-square.

5. Random vectors, joint and conditional distributions (34)
   - Random vector, Stochastic Process, Joint CDF, joint pmf, joint pdf, the bivariate normal
   - Marginal distributions, an example getting marginals, marginals of the bivariate normal
   - Conditional distributions (for discrete or continuous random vectors), conditional mean and variance, defining a joint distribution via marginal and conditional (can handle mixed random vectors with some discrete and some continuous), independence, conditioning and hierarchical models, mixtures of normals, double expectations with application to hierarchical sampling.
   - Expected values of functions of random vectors, Covariance and correlation, Joint MGF

6. FUNCTIONS OF RANDOM VARIABLES: (52)
   (a) Mean and variance of linear combinations and sums (with the mean as a special case)
   (b) Distributions of functions. Sums of independent random variables via MGF technique, direct method via CDF, probability integral transform, change of variables (univariate and multivariate) derivation of the t-distribution, linear transformations, linear functions of a multivariate normal.
   (c) Approximate mean and variance of nonlinear functions via Taylor Series

7. Large sample concepts. (61)
   - Convergence in probability, convergence in distribution, continuous mapping theorem, Slutzky’s lemma.

8. AN OVERVIEW OF STATISTICAL INFERENCE (62)
   Populations and parameters, samples and statistics, sampling distribution. Point estimation (estimates and estimators, bias, variance, standard error, mean squared error, consistency). Basic definitions for confidence intervals and sets and hypothesis testing. Basic ideas in Bayesian inference.
9. ONE SAMPLE PROBLEMS (No distributional assumptions). (67)
- Exchangeable random variables and one sample problems.
- Simple Random Samples From Finite Populations and “Random Samples”.
- Sample moments, mean and variance of $\bar{X}$, consistency of $\bar{X}$ for $\mu$ (the weak law of large numbers), consistency of sample moments for population counterpart, expected value and consistency of $S^2$.
- Properties of proportions as a special case of the mean, estimating cumulative probabilities, definition of empirical CDF.
- Large Sample properties of $\bar{X}$. The Central Limit Theorem, sample size determination via Chebychev bound and based on normal approximation.
- Approximate large sample normal based tests and confidence intervals for $\mu$. Approximate power function and accuracy of confidence intervals for large $n$.

10. Random Sample from the Normal distribution. (75)
Sampling properties of $\bar{X}$ and $S^2$, exact confidence intervals and tests for $\mu$ and $\sigma^2$.

11. Further topics in testing. (80)
- Duality of Confidence intervals and tests, P-values.

12. Sampling from two independent normals. (82)
Major sampling distribution results given in notes. Proofs and development of related tests and confidence intervals for difference in means and ratio of variances in homework.

13. Method of moments in one sample problems. (83)
The method of moments approach. Statement of general multivariate CLT and WLLN with application to the large sample behavior of collection of sample moments.
- The Delta Method for finding the asymptotic distribution of functions of things which are asymptotically normal (relates back to approximation to variances of nonlinear functions). Application to the large sample properties of $S^2$ and development of large sample inferences for population variance.

14. Maximum Likelihood Estimation. (86)
The likelihood function, MLE’s (in general), score equations, one sample examples and one example with exponential mean changing proportional to a constant. The invariance principle for maximum likelihood estimators.

Some important concepts introduced in Homework.