

## Homework 5

5.22, 5.25, 5.45, 5.46, 5.52, 5.81, 5.88 +3 problems

Extra credit – Case Study page 218

5.22

In 2006, the average combined SAT score for students in the United States was 1518 (out of 2400). Suppose that approximately 45% of all high school graduates took this test, and that 100 high school graduates are randomly selected from throughout the United States. Which of the following random variables has an approximate binomial distribution? If possible, give the values for  $n$  and  $p$ .

- The number of students who took the SAT
- The scores of the 100 students on the SAT
- The number of students who scored above average on the SAT
- The amount of time it took each student to complete the SAT

5.25

Car color preferences change over the years and according to the particular model that the customer selects. In a recent year, 10% of all luxury cars sold were black. If 25 cars of that year and type are randomly selected, Find the following probabilities:

- At least 5 cars are black
- At most 6 cars are black
- More than 4 cars are black
- Exactly 4 cars are black
- Between 3 and 5 cars (inclusive) are black.
- More than 20 cars are not black.

**5.45 Accident Prone** Parents who are concerned that their children are “accident prone” can be reassured, according to a study conducted by the Department of Pediatrics at the University of California, San Francisco. Children who are injured two or more times tend to sustain these injuries during a relatively limited time, usually 1 year or less. If the average number of injuries per year for school-age children is two, what are the probabilities of these events?

- A child will sustain two injuries during the year.
- A child will sustain two or more injuries during the year.
- A child will sustain at most one injury during the year.

**5.46 Accident Prone, continued** Refer to Exercise 5.45.

- Calculate the mean and standard deviation for  $x$ , the number of injuries per year sustained by a school-age child.
- Within what limits would you expect the number of injuries per year to fall?

5.52

A candy dish contains five blue and three red candies. A child reaches up and selects three candies without looking.

- What is the probability that there are two blue and one red candies in the selection?
- What is the probability that the candies are all red?
- What is the probability that the candies are all blue?

5.81

Tay-Sachs disease is a genetic disorder that is usually fatal in young children. If both parents are carriers of the disease, the probability that their offspring will develop the disease is approximately 0.25. Suppose a husband and wife are both carriers of the disease and the wife is pregnant on three different occasions. If the occurrence of Tay-Sachs in any one offspring is independent of the occurrence in any other, what are the probabilities of these events

- All three children will develop Tay-Sachs disease.
- Only one child will develop Tay-Sachs disease
- The third child will develop Tay-Sachs disease, given that the first two did not.

5.88

Evidence shows that the probability that a driver will be involved in a serious automobile accident during a given year is .01. A particular corporation employs 100 full-time traveling sales reps. Based on this evidence, use the Poisson Approximation to the binomial distribution to find the probability that exactly two of the sales reps will be involved in a serious automobile accident during the coming year.

### **Problem 1**

York Steel Corporation produces a special bearing that must meet rigid specifications. When the production process is running properly, 10% of the bearings fail to meet the required specifications. Sometimes problems develop with the production process that cause the rejection rate to exceed 10%. To guard against this higher rejection rate, samples of 15 bearings are taken periodically and carefully inspected. If more than 2 bearings in a sample of 15 fail to meet the required specifications, production is suspended for necessary adjustments.

- a. If the true rate of rejection is 10% (that is, the production process is working properly), what is the probability that the production will be suspended based on a sample of 15 bearings?
- b. What assumptions did you make in part a?

### **Problem 2**

The number of calls that come into a small mail-order company follows a Poisson distribution. Currently, these calls are serviced by a single operator. The manager knows from past experience that an additional operator will be needed if the rate of calls exceeds 20 per hour. The manager observes that 9 calls came into the mail-order company during a randomly selected 15-minute period.

- a. If the rate of calls is actually 20 per hour, what is the probability that 9 or more calls will come in during a given 15-minute period?
- b. If the rate of calls is really 30 per hour, what is the probability that 9 or more calls will come in during a given 15-minute period?
- c. Based on the calculations in parts a and b, do you think that the rate of incoming calls is more likely to be 20 or 30 per hour?
- d. Would you advise the manager to hire a second operator? Explain.

### **Problem 3**

A history teacher has given her class a list of seven essay questions to study before the next test. The teacher announced that she will choose four of the seven questions to give on the test, and each student will have to answer three of those four questions.

- a. In how many ways can the teacher choose four questions from the set of seven?
- b. Suppose that a student has enough time to study only five questions. In how many ways can the teacher choose four questions from the set of seven so that the four selected questions include both questions that the student did not study?
- c. What is the probability that the student in part b will have to answer a question that he or she did not study? That is, what is the probability that the four questions on the test will include both questions that the student did not study?