Representations of Semisimple Lie Algebras in the BGG Category $\mathcal{O}$ (Revisions)

On the dedication page, the list of names should be: Rowan Leland Gerlis, Zoë Humphreys, Asher Leland Gerlis, Emily Hunter, Miranda Hunter

xvi In last paragraph of Preface, read "Stroppel" in place of "Stropple".

8 In line -11, delete the minus sign.

10 In line -9, replace $L(3)$ by $L(2)$.

14 In line 7, replace 0.7 by 0.8.

25 In line 3, replace μ by $w\mu$.

27–28 The proofs of (b) and (c) in 1.10 are out of focus at several points and should be revised as follows:

27 In line -6, replace $W \cdot \lambda$ by $W(\lambda + \rho)$ and $W \cdot \mu$ by $W(\mu + \rho)$.

27 In line -4, replace "dot orbits" by "W-orbits".

27 In lines $-4, -3$, replace "Using the assumption about ψ, take any pre-image ..." by "Using part (a), take the pre-image ...".

27 In line -2, the expression should be $\chi_\lambda(z) = (\lambda+\rho)(\psi(z))$ and similarly for μ.

28 In lines 11–12, replace the three occurrences of λ by $\lambda + \rho$, and similarly for the first occurrence of λ in line 13.

29 In line -19, replace "for $M \in \mathcal{O}$" by "for isomorphism class representatives $M \in \mathcal{O}$".

35 In line -17, replace "$\lambda \geq 0$" by "$\lambda \in \mathbb{Z}^+$".

35 In line -6, the last symbol in the Exercise should be $[M(\lambda)]$.

55 In line -6, replace $w' \cdot (\lambda - s_\alpha \cdot \lambda)$ by $(w' \cdot \lambda - w \cdot \lambda)$. Then in line -4, replace $w' \cdot (\lambda - s_\alpha \cdot \lambda) = w(s_\alpha \cdot \lambda - \lambda)$ by $w' \cdot \lambda - w \cdot \lambda = ws_\alpha \cdot \lambda - w \cdot \lambda$.

60 In line -18, replace "the right exact functor ... is also left exact." by "the contravariant functor ... is exact."

60 In lines $-11, -10$, read "is dominant if $-\lambda$ is antidominant."

64 In the proof of Corollary 3.10, delete the sentence "Use induction on the length ..." at the end of the first paragraph. Then replace the last line of the proof by "After discarding these summands, induction on $\sum c_\mu$ takes over for the remaining summand of P."

65 Reword Exercise 3.11: "If $\lambda \in \Lambda$, prove that $M(\lambda)$ is projective only when λ is dominant. [Using results of Chapter 4, this can be proved for all $\lambda \in \mathfrak{h}^*$.]"

80 In line -16, replace "at least" by "an".

91 In the paragraph of the Notes starting with "Following ...", replace the second sentence by: "Lutsyuk [**204**] independently develops a recursive formula for the elements of $U(\mathfrak{n}^-)$ inducing such embeddings."

95 In line -18, replace the occurrences of the symbol $<$ by $\uparrow$ and $\leq$ respectively.

97 In line -6, replace "cases $w = s_\beta s_\alpha$ and $w = w_\circ$" by "case $w = s_\beta s_\alpha$, while the case $w = w_\circ$ follows by comparing the alternating sum formula for $\operatorname{ch} L(w_\circ \cdot \lambda)$ in 2.4"

98 In lines 9–10, replace the long sentence "By induction, ..." by the shorter "By induction, $\mu \uparrow s_\alpha \cdot \lambda$."

101 In lines $-9, -8$, reword the sentence: "Since $M(\lambda)$ has finite length, it follows that ..."

101 In line -5,. replace "3.14" by "Exercise 3.14".

118 In the last two lines, replace "case $k = m$" by "case $k = m - 1$" and "let $k < m$" by "let $k < m - 1$".

121 In line 15, the second $M(w \cdot 0)$ should be $M(w' \cdot 0)$.

121 The ideas sketched in the last paragraph for the proof of the theorem stated at the top of page 122 are probably not adequate for the purpose. It is safer to rely on the arguments given by Mazorchuk [214], though it would be interesting to minimize the prerequisites for the theorem.

122 In line -3, delete the extra right parenthesis.

124 In line 8 of (1) in the proof, the first Ext term in the exact sequence should be $\mathrm{Ext}_{\mathcal{O}}$.

130-131 Revise the last paragraph of 7.1: At the end of the existing text in line -9, insert the first sentence at the top of page 131 "The rationale ..." In line -8, replace "a couple of" by "some". Then reword the proposition as follows: "Let $\lambda, \mu \in \mathfrak{h}^*$.

(a) The exact functor T_λ^μ commutes with the duality functor.

(b) T_λ^μ takes projective modules to projective modules.

(c) $T_\lambda^\mu M(\lambda)$ has a standard filtration involving $M(\mu)$ as a subquotient."

For the proof, follow the current wording for the first two parts and the current wording on page 131 for (c) with obvious modifications (omitting parentheses around the last sentence):

(a) "To see that ... in that section."

(b) "This follows from ... to a direct summand."

(c) "When $M = M(\lambda)$, a standard filtration ... This module also has a standard filtration, thanks to Proposition 3.7(b)."

134 In line -15, begin (c) with: "Suppose $\Phi_{[\lambda]} = \Phi_{[\mu]} = \Phi_{[\lambda+\mu]}$." Then in line -6, begin (c) with: "By assumption, $\Phi_{[\lambda+\mu]} = \Phi_{[\lambda]}$.

136 In line 14, replace C' by $\overline{C'}$.

136 In line -14, delete "and (7)".

136 In line -1, read: $\xi = \lambda^\natural + \nu = \mu^\natural$.

137 Delete the bottom text on the page, starting at line -10: "But it ..."

141 Replace line 5 by "$\alpha \in w\Phi_{F'}^0$, with F' the facet of μ." Then at the end of line 6, replace F by F', and in line 8 replace F by F'.

143 In line 3, replace W_μ by W_μ°.

144 In line 7, replace "Corollary 7.6" by "Proposition 7.1(c)".

144 In line -3, replace $\mathcal{O}$ by $\mathcal{O}_{\chi_\mu}$.

146 In line 1, replace $P(w_\lambda \cdot \lambda)$ by $P(w_\lambda w_{\mu^\circ} \cdot \lambda)$.

147 In line 7, replace "Corollary 7.6" by "Proposition 7.1(c)".

147 In line -2 expand "> 0" to "> 0 if $T_\lambda^\mu L(w' \cdot \lambda) \neq 0$".

150 In line -1, replace "bonce" by "once".

151 In line 2, replace "just one" by "no". Then in lines $-16, -15$, remove the sentence "Using the projective property ..."

151-152 In lines 151 : -2 and 152:1, replace $L(\lambda_\circ)$ by $M(\lambda_\circ)$.

163 In line 14, replace "which means" by "which implies". Then in the lemma replace "$\ell(w) - \ell(x) = 1$" by "$w = rx$ for some reflection r".

176 In the second paragraph of (2), lines 11–15 are seriously out of focus in the discussion of C_3. The reader should work out the details as an exercise, keeping in mind from 8.4(2) how to relate the composition factor multiplicities of Verma modules with the values at 1 of certain KL polynomials. Here $x < w$ in W is equivalent to $w_\circ w < w_\circ x$, while $\ell(w_\circ w) = \ell(w_\circ) - \ell(w) = 9 - \ell(w)$, etc.

177 In line 3, replace $\ell(w_\circ) + 1$ by $\ell(w_\circ)$.

178 In line 9, replace $\mathfrak{h}$ by $\mathfrak{h}^*$.

191 In line 6, read "Conjecture".

200 In the third line of 9.15, read "more refined partition".

233 My reading of Soergel's notation was too hasty, so several lines must be modified:

In (1), replace the term on the right side by $[M(w \cdot \lambda) : L(x \cdot \lambda)]$, and in line 15, replace "Theorem 3.9" by "Theorem 3.7".

In the statement of Soergel's Theorem, replace $P_{x,w}(1)$ by $P_{w_\circ w, w_\circ x}(1)$. Then in the Corollary, delete all but the first sentence.

243 Revise the third paragraph of 12.7, starting with line 4:

"and $N_w = \mathbb{C}[y_\alpha]$. The algebra N_w is $\mathbb{Z}$-graded: the standard grading of U by Λ_r induces a $\mathbb{Z}$-grading on U if all simple root vectors are placed in U_1. The graded dual N_w^*, with nth graded piece the dual space $(N_w)^*_{-n}$, then becomes a $\mathbb{Z}$-graded N_w-bimodule. Define $S_w := U \otimes_{N_w} N_w^*$. Somewhat miraculously, ..."

265 In line -4, replace "on W" by "on V".

269 In line -15, replace W_I by W_{I}.

(May 28, 2019) Many of these revisions were suggested by Brian Boe, Andreas Glang, Chun-ju Lai, Visnambhara Makam, David Nies, David Savitt.