1. The line L through the points A and B is parallel to the vector $\overrightarrow{AB} = \langle 3, 2, -1\rangle$ and has parametric equations $x = 3t+2$, $y = 2t+1$, $z = -t-1$. Therefore, the intersection point of the line with the plane should satisfy:

$$2(3t+2) - 3(2t+1) + 4(-t-1) = 13.$$

Solving the last equation with respect to the parameter t, we obtain $t = -4$ and the coordinates of the intersection point are

$$(3(-4)+2, 2(-4)+1, -(-4)-1) = (-10, -7, 3).$$

2. (a) The curves meet at the point C if and only if components of the vectors $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ coincide, i.e.

$$\begin{aligned} t &= 3 - s, \\ 1 - t &= s - 2, \\ 3 + t^2 &= s^2. \end{aligned}$$

From the first and last equations, we have $3 + (3-s)^2 = s^2$, solving which we obtain $s = 2$. Now from the first equation $t = 1$ which also satisfy the second equation. The conclusion is that the point $(1, 0, 4)$ is the point of intersection of the curves.

(b) Let's find the tangent vectors of the curves: for C_1 the tangent vector is $\mathbf{r}'(t) = \langle 1, -1, 2t\rangle$ and for the curve C_2 the tangent vector is $\mathbf{r}'(s) = \langle -1, 1, 2s\rangle$. At the intersection of C_1 and C_2 one has $t = 1$ and $s = 2$, so L_1 is parallel to the vector $\langle 1, -1, 2\rangle$ and L_2 is parallel to $\mathbf{r}'(s) = \langle -1, 1, 4\rangle$. Since both lines contain the point $(1, 0, 4)$, the equations of the lines are

$$L_1: \quad x = t+1,\ y = -t,\ z = 2t+4,$$

and

$$L_2: \quad x = -s+1,\ y = s,\ z = 4s+4.$$

3. Suppose that C has coordinates (x, y, z). Then from the vector equality $\overrightarrow{AC} = \overrightarrow{BD}$, one has

$$\langle x-2, y-5, z-1\rangle = \langle 5-3, 2-1, -3-4\rangle,$$

and $x = 4$, $y = 6$, $z = -6$.

4. (a) Since $\overrightarrow{AB} = \langle -1, -1, 2\rangle$ and $\overrightarrow{AC} = \langle -2, 1, 1\rangle$, the scalar projection of $\overrightarrow{AC}$ onto $\overrightarrow{AB}$ is

$$\mathrm{comp}_{\overrightarrow{AB}}\overrightarrow{AC} = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|} = \frac{(-1)(-2)+(-1)(1)+2(1)}{\sqrt{(-1)^2+(-1)^2+2^2}} = \frac{3}{\sqrt{6}}$$

Therefore, the vector projection is this scalar projection times the unit vector in the direction of $\overrightarrow{AB}$:

$$\mathrm{proj}_{\overrightarrow{AB}}\overrightarrow{AC} = \frac{3}{\sqrt{6}}\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{1}{2}\overrightarrow{AB} = \langle -1/2, -1/2, 1\rangle$$

(b) The area S of the triangle ABC is the half of the area of the parallelogram with adjacent sides AB and AC which is the length of the cross product:

$$S = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}|\langle -3, -3, -3 \rangle| = \frac{3\sqrt{3}}{2}$$

(c) The line L containing $A(2,1,0)$ and parallel to the vector $\overrightarrow{AB} = \langle -1, -1, 2 \rangle$ has parametric equations

$$x = -t + 2, \quad y = -t + 1, \quad z = 2t$$

The square of the distance from the arbitrary point D on the line to the point C is a function depending on the parameter t:

$$F(t) = |\overrightarrow{CD}|^2 = (-t+2)^2 + (-t-1)^2 + (2t-1)^2 = 6t^2 - 6t + 6$$

To minimize the distance, set $\frac{d}{dt}F(t) = 0$, and solve for t to obtain $t = 1/2$. Hence, the required distance from the point C to the line L is $d = F(1/2) = 9/2$.

5. First, note that any direction vector of the line L is perpendicular both of the normal vectors $\mathbf{n}_1 = (1, -2, 1)$ and $\mathbf{n}_2 = (2, 1, 1)$. Hence, L is parallel to the vector

$$\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = (-3, 1, 5)$$

Now, it is sufficient to find any point on L, i.e. a point satisfying $x - 2y + z = 1$ and $2x + y + z = 1$. Let, for example, $x = 0$. Then, solving the system of equations $-2y + z = 1$ and $y + z = 1$, we obtain solutions $y = 0$ and $z = 1$, so the point $(0,0,1)$ is on the line and the parametric equations of L are

$$x = -3t, \quad y = t, \quad z = 5t + 1$$

6. The line L_1 is parallel to the vector

$$\mathbf{v}_1 = \langle -1-1, 4-0, 1-1 \rangle = 2 \cdot \langle -1, 2, 0 \rangle,$$

and L_2 is parallel to the vector

$$\mathbf{v}_2 = \langle 4-2, 4-3, -3-(-1) \rangle = \langle 2, 1, -2 \rangle$$

We see that the lines L_1 and L_2 are not parallel to each other, because the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are not proportional: there is no such a $k \neq 0$ so that $\mathbf{v}_1 = k \cdot \mathbf{v}_2$ (one can also check that $\mathbf{v}_1 \times \mathbf{v}_2 \neq \mathbf{0}$). The parametric equations of L_1 through $(1,0,1)$ are

$$x = -t + 1, \quad y = 2t, \quad z = 1$$

and the line L_2 containing $(2,3,-1)$ has parametric equations

$$x = 2s + 2, \quad y = s + 3, \quad z = -2s - 1$$

To find intersection point of L_1 and L_2, we should find values of t and s such that

$$\begin{aligned} -t+1 &= 2s+2, \\ 2t &= s+3, \\ 1 &= -2s-1 \end{aligned}$$

Solving the last two equations, we get $t = 1$ and $s = -1$ which satisfy the first equation. Therefore, the lines L_1 and L_2 intersect at the point $(0, 2, 1)$.

7. (a) First, compute the components of the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$:

$$\begin{aligned} \overrightarrow{AB} &= \langle 3-1, 1-4, -2-2\rangle = \langle 2, -3, -4\rangle \\ \overrightarrow{AC} &= \langle 4-1, 3-4, -3-2\rangle = \langle 3, -1, -5\rangle \\ \overrightarrow{AD} &= \langle 1-1, 0-4, -1-2\rangle = \langle 0, -4, -3\rangle. \end{aligned}$$

The volume V of the parallelepiped is the absolute value of the scalar triple product

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AB}) = \begin{vmatrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{vmatrix} = -13$$

and V is equal to 13.

(b) The equation of the plane through A, B, and D is given by the determinant

$$\begin{vmatrix} x-1 & y-4 & z-2 \\ 3-1 & 1-4 & -2-2 \\ 1-1 & 0-4 & -1-2 \end{vmatrix} = \begin{vmatrix} x-1 & y-4 & z-2 \\ 2 & -3 & -4 \\ 0 & -4 & -3 \end{vmatrix} = 0,$$

or $-7x + 6y - 8z = 1$.

(c) The plane through A, B, and C is

$$\begin{vmatrix} x-1 & y-4 & z-2 \\ 3-1 & 1-4 & -2-2 \\ 4-1 & 3-4 & -3-2 \end{vmatrix} = \begin{vmatrix} x-1 & y-4 & z-2 \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix} = 0,$$

or $11x-2y+7z-17 = 0$ with the normal vector $\mathbf{n} = \langle 11, -2, 7\rangle$. The normal vector to the plane xy is $\mathbf{k} = (0, 0, 1)$ and cosine of the angle between the planes is

$$\cos\varphi = \frac{\mathbf{n}\cdot\mathbf{k}}{|\mathbf{n}|\cdot|\mathbf{k}|} = \frac{7}{\sqrt{174}}$$

8. (a) The position vector of the particle whose velocity is $\mathbf{v}(t)$ and initial position is $\mathbf{r}_0 = \mathbf{r}(t_0)$ can be found as

$$\mathbf{r}(t) = \mathbf{r}_0 + \int_{t_0}^{t} \mathbf{v}(t)\,dt.$$

Since $t_0 = 0$, and $\mathbf{r}_0$ and $\mathbf{v}$ are given, one has

$$\mathbf{r}(4) = \langle -1, 5, 4\rangle + \int_0^4 \langle 2t, 2\sqrt{t}, 1\rangle\, dt = \langle -1, 5, 4\rangle + \langle 16, 32/3, 4\rangle = \langle 15, 47/3, 8\rangle.$$

(b) The tangent line to the curve at $t = 4$ is parallel to the vector $\mathbf{v}(4) = \langle 8, 4, 1\rangle$, and the equation of the line is

$$x = 8t + 15,\ y = 4t + 47/3,\ z = t + 8.$$

(c) The position vector of the particle is

$$\mathbf{r}(t) = \mathbf{r}_0 + \int_{t_0}^{t} \mathbf{v}(t)\, dt = \langle -1, 5, 4\rangle + \langle t^3/3, 4/3\, t^{3/2}, t\rangle,$$

so the particle pass through P: $\mathbf{r}(9) = \langle 80, 41, 13\rangle$.

(d) The length of the arc traveled from $t = 1$ to $t = 2$ is

$$L = \int_1^2 \sqrt{(2t)^2 + (2\sqrt{t})^2 + 1^2}\, dt = \int_1^2 (2t + 1)\, dt = 4.$$

9. The surface is a hyperboloid of one sheet

$$\frac{x^2}{1^2} + \frac{y^2}{(1/\sqrt{3})^2} - \frac{z^2}{(1/\sqrt{2})^2} = 1$$

10. The generic equation of the tangent plane to the graph of $z = y \ln x$ at the point where $(x, y) = (x_0, y_0)$ is

$$z = y_0 \ln x_0 + \frac{y_0}{x_0}(x - x_0) + \ln x_0 (y - y_0)$$

and at $(1, 4, 0)$ it is $z = 4(x - 4)$, or $4x - z - 4 = 0$.

11. To find the distance between the planes, fix any point on the first plane (i.e. any point $A(x, y, z)$ such that $z = 2x + y - 1$) and find a distance from this point to the second plane. Let, for example $A(1, 0, 1)$. Then the distance d from the point A to the plane is

$$d = \frac{|(1)(-4) + 0(-2) + (1)(2) - 3|}{\sqrt{(-4)^2 + (-2)^2 + 2^2}} = \frac{5}{\sqrt{24}}.$$

Another method to solve the problem is the following. Find the equations of the line through A perpendicular to the second plane (parallel to it's normal vector):

$$x = -4t + 1, \quad y = -2t, \quad z = 2t + 1$$

and the point of intersection with the plane B solving the equation

$$-4(-4t + 1) - 2(-2t) + 2(2t + 1) = 3$$

for t. It's easy to see that $t = 5/24$ and $B(1/6, -5/12, 17/12)$ and

$$d = \sqrt{(1/6 - 1)^2 + (-5/12 - 0)^2 + (17/12 - 1)^2} = \frac{5}{\sqrt{24}}.$$

12. The surface is a hyperboloid of one sheet. Indeed, by completing the squares, we have

$$4x^2 + 4(y^2 - 2y + 1) - 4 - z^2 = 0,$$

and after division by 4:

$$\frac{x^2}{1^2} + \frac{(y-1)^2}{1^2} - \frac{z^2}{2^2} = 1$$

14. The limit

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{2x^4 + y^4}$$

does not exist because if we consider two different directions along two different lines $x = 0$ and $y = x$, we obtain different answers:

$$\text{along } x = 0: \quad \frac{3 \cdot 0^2 x^2}{2 \cdot 0^4 + y^4} = 0 \to 0$$

$$\text{along } y = x: \quad \frac{3x^2x^2}{2x^4 + x^4} = \frac{3x^4}{3x^4} \to 1$$

15. It is easy to find two different points on the line by letting $t = 0$ and $t = 1$: $Q(2, 1, 2)$ and $R(5, 0, 4)$. Now the plane through the points P, Q and R is

$$\begin{vmatrix} x-1 & y-1 & z-0 \\ 2-1 & 1-1 & 2-0 \\ 5-1 & 0-1 & 4-0 \end{vmatrix} = 0,$$

or $2x + 4y - z = 6$.

16. (a) $f_x = 3x^2 - y^2$, $f_y = -2xy + 1$, $f_{xy} = -2y$.

(b) $f_x = \frac{1}{\sqrt{x^2+y^2}}$, $f_y = \frac{y}{\sqrt{x^2+y^2}(x+\sqrt{x^2+y^2})}$, and $f_{xy} = -\frac{y}{(x^2+y^2)^{3/2}}$.

(c) Let $z = f(x, y)$ be $f(x, y) = x^2 \cos x^2 y$. Then

$$f_x = 2x\cos x^2 y - 2x^3 y \sin x^2 y, \quad f_y = -x^4 \sin x^2 y,$$

and

$$f_{xy} = -4x^3 \sin x^2 y - 2x^5 y \cos x^2 y$$

17. The linear approximation of $f(x, y)$ at $(1, 1)$ is

$$L(x, y) = f(1, 1) + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) = e + 2e(x - 1) + e(y - 1)$$

since $f_x = ye^x(1 + x)$ and $f_y = xe^x$. Therefore $f(1.1, 0.9)$ is approximately $L(1.1, 0.9) = 2.99$.

18. Find the parametrization of the curve: let $x = t$, then $y = t^2$ and $z = 3t^2$, so $\mathbf{r}(t) = (t, t^2, 2t^2 + t^4)$.