Math 233 **<u>Practice Final 3</u>** **Spring 2006**

1. a) Find the equation of the plane containing the points $P(1, 3, 0)$, $Q(2, -1, 2)$ and $R(0, 0, 1)$.
b) Find all points of intersection of the parametric curve $\mathbf{r}(t) = \langle 2t^2 - 2, t, 1 - t - t^2 \rangle$ and the plane $x + y + z = 3$.

2. Find the absolute maximum and minimum of the function $f(x, y) = x^2 + 2y^2 - 2y$ on the closed disc $x^2 + y^2 \leq 5$ of radius $\sqrt{5}$.

3. Evaluate

$$\iint_R xy \; dA$$

where R is the region in the first quadrant bounded by the line $y = 2x$ and the parabola $y = x^2$.

4. Consider the vector field $\mathbf{F}(x, y) = \langle 2xy + \sin y, x^2 + x \cos y + 1 \rangle$.

a) Show that $\mathbf{F}(x, y)$ is conservative by finding a potential function $f(x, y)$ for $\mathbf{F}(x, y)$.
b) Use your answer to (a) to evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the arc of the parabola $y = x^2$ going from $(0, 0)$ to $(2, 4)$. (Even if you did not get (a), you can still get partial credit by explaining how to use a potential function to evaluate this integral.)

5. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F} = \langle y^2 + \sin x, xy \rangle$ and C is the unit circle oriented counterclockwise.

6. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F} = \langle y^2, 2xy + x \rangle$ and C is the curve starting at $(0, 0)$, travelling along a line segment to $(2, 1)$ and then travelling along a second line segment to $(0, 3)$.