Math 233 **Practice Final 2** **Spring 2006**

1.Consider the function $f(x, y) = xe^{xy}$. Let P be the point $(1, 0)$.
(a) Find the rate of change of the function f at the point P in the direction of the point $(3, 2)$.
(b) Give a direction in terms of a unit vector (there are two possibilities) for which the rate of change of f at P in that direction is zero.

2. (a) Find the work done by the vector field $\mathbf{F}(x, y) = \langle x - y,\, x\rangle$ over the circle $\mathbf{r}(t) = \langle \cos t,\, \sin t\rangle$, $0 \le t \le 2\pi$.
(b) Use Green's Theorem to calculate the line integral $\int_C (-y^2)dx + xydy$, over the positively (counterclockwise) oriented closed curve defined by $x = 1\,, y = 1$ and the coordinate axes.

3. (a) Show that the vector field $\mathbf{F}(x, y) = \langle x^2y,\, \frac{1}{3}x^3\rangle$ is conservative and find a function f such that $\mathbf{F} = \nabla f$.
(b) Using the result in part (a) calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, along the curve C which is the arc of $y = x^4$ from $(0, 0)$ to $(2, 16)$.

4. Consider the surface $x^2 + y^2 - \frac{1}{4}z^2 = 0$ and the point $P(1,\, 2,\, -2\sqrt{5})$ which lies on the surface.
(a) Find the equation of the tangent plane to the surface at the point P.
(b) Find the equation of the normal line to the surface at the point P.

5. A flat circular plate has the shape of the region $x^2 + y^2 \le 1$. The plate (including the boundary $x^2 + y^2 = 1$) is heated so that the temperature at any point (x, y) on the plate is given by

$$T(x, y) = x^2 + 2y^2 - x$$

Find the temperatures at the hottest and the coldest points on the plate, including the boundary $x^2 + y^2 = 1$.

6. The acceleration of a particle at any time t is given by

$$\mathbf{a}(t) = \langle -3\cos t,\, -3\sin t,\, 2\rangle\,,$$

while its initial velocity is $v(0) = \langle 0,\, 3,\, 0\rangle$. At what times, if any, are the velocity and the acceleration of the particle orthogonal?

7. Find parametric equations for the line in which the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ intersect.