Math 233 **<u>Practice Final 1</u>** **Spring 2006**

1) Express the double integral

$$\iint_R x^2y - x \;\; dA$$

as an interated integral and evaluate it, where R is the first quadrant region enclosed by the curves $y = 0$, $y = x^2$ and $y = 2 - x$. **b)** Find an equivalent iterated integral expression for the double integral in **1a)**, where the order of intergration is reversed from the order used in part **1a)**. (Do **not** evaluate this integral.)

2) Calculate the line integral

$$\int_C \mathbf{F} \cdot \mathbf{dr},$$

where $\mathbf{F}(x, y) = y^2x\mathbf{i} + xy\mathbf{j}$, and C is the path starting at $(1, 2)$, moving along a line segment to $(3, 0)$ and then moving along a second line segment to $(0, 1)$.

3) Evaluate the integral

$$\iint_R y\sqrt{x^2 + y^2}dA$$

with R the region $\{(x, y) : 1 < x^2 + y^2 < 2, \quad 0 < y < x.\}$

4a) Show that the vector field

$$\mathbf{F}(x, y) = \left\langle \frac{1}{y} + 2x, -\frac{x}{y^2} + 1 \right\rangle$$

is conservative by finding a potential function $f(x, y)$.
4b) Let C be the path described by the parametric curve $\mathbf{r}(t) =< 1 + 2t, 1 + t^2 >$ for $0 \leq t \leq 1$. Use your answer from **4a)** to determine the value of the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

5a) Find the equation of the tangent plane at the point $P = (1, 1, -1)$ in the level surface $f(x, y, z) = 3x^2 + xyz + z^3 = 1$.
b) Find the directional derivative of the function $f(x, y, z)$ at $P = (1, 1, -1)$ in the direction of the tangent vector to the space curve $\mathbf{r}(t) = \langle 2t^2 - t, t^{-2}, t^2 - 2t^3 \rangle$ at $t = 1$.

6) Find the absolute maxima and minima of the function

$$f(x, y) = x^2 - 2xy + 2y^2 - 2y$$

in the region bounded by the lines $x = 0$, $y = 0$ and $x + y = 7$.