

1. (15 points) Let $f(x, y)$ be a differentiable function with the following values of the partial derivatives $f_x(x, y)$ and $f_y(x, y)$ at certain points (x, y) :

x	y	$f_x(x, y)$	$f_y(x, y)$
0	0	3	2
2	-1	-2	5
1	2	-1	4

(You are given more values than you will need for this problem.) Suppose that x and y are functions of variables r and s :

$$x = r^2s; \quad y = r - s,$$

so that we may regard f as a function of r and s . Compute the partial derivative f_r of f with respect to r when $r = 1$ and $s = 2$.

2. (10 points) Consider the function $f(x, y) = x^2y - xy$. Find a unit vector \mathbf{u} such that the directional derivative of $f(x, y)$ at the point $(1, 4)$ in the direction of \mathbf{u} equals 2.

3. (20 points) Let

$$f(x, y) = \frac{1}{3}x^3 + x^2 - xy + \frac{1}{4}y^2 - 4x.$$

Find and classify (as maxima, minima or saddle points) all critical points of $f(x, y)$.

4. (20 points) Find the minimum of the function $f(x, y) = x^2y$ subject to the constraint $x^2 + 2y^2 = 6$.

5. (15 points) Find the volume above the rectangle $-3 \leq x \leq 3$, $2 \leq y \leq 4$ and below the surface $z = 1 + x^2 + y$. (You must show your work.)

6. (15 points) Evaluate the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3) dy dx.$$

(Hint: Change the order of integration.) You must show your work.