Math 233 **Practice Exam 2** **Fall 2005**

1. Find the critical points of $f(x,y) = x^3 + y^2 + 2xy - 4x - 3y + 5$ and classify each as a relative maximum, relative minimum or saddle point.

2. An open (no top) rectangular box must have a volume of 6 cubic feet. Find the dimensions of the box that will minimize the amount of material used to construct the box.

3. a) Find the directional derivative of $F(x,y,z) = x^2 + 2y^2 + 3z^2 - 12$ at the point $(2,2,0)$ in the direction toward the point $(3,2,1)$.

   b) Find the equation of the tangent plane to the surface $F(x,y,z) = x^2 + 2y^2 + 3z^2 - 12 = 0$ at the point $(2,2,0)$.

   c) Find a point (a,b,c) on the above surface (there are actually two such points) at which the normal line to the surface is parallel to the line $x = 1 + t,\ y = -1 + 4t,\ z = 2 - 3t$. (NOTE: Since (a,b,c) is on the above surface, its coordinates must satisfy the equation of the surface.)

4. a) If $G(x,y,z) = x^2z + yz^3 - x^3y^2 - 5z = 0$ defines z as an implicit function of x and y, find $\dfrac{\partial z}{\partial y}$.

   b) Suppose $z = f(x,y)$, where $x = s^2t$ and $y = t^2$. If $f_x(2,-1) = 3$, $f_x(-4,1) = 1,\ f_y(2,-1) = 3$ and $f_y(-4,1) = 2$, find the value of $\dfrac{\partial z}{\partial t}$ when $s = 2$ and $t = -1$. (Note that there is more information given than is needed.)

   c) Let $h(x,y)$ have continuous partials and suppose that the maximum value of the directional derivative of h at $P(1,2)$ has magnitude $= 50$ and is attained in the direction from $P(1,2)$ to $Q(3,-4)$, Find $\bigtriangledown h(1,2)$

5. a) Let $I = \displaystyle\int_0^2 \int_0^{x^2} 6\pi \cos\left(\frac{\pi x^3}{16}\right)\, dy\, dx$

      i) Sketch the region of integration.

      ii) Write I with the order of integration reversed.

      iii) Evaluate <u>either</u> the integral I as given or with the order reversed.

   b) Set up, but DO NOT evaluate, the <u>iterated</u> integral to find the volume of the solid in the first octant that is bounded by $z = 4 - y^2,\ z = 0,\ x = 0$ and $y = 2x$.