Math 233 **Practice Exam 1** **Spring 2006**

1. (a) Consider the line L through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the intersection of the line L and the plane given by $2x - 3y + 4z = 13$.
(b) Find the distance of the point $(2, 1, -1)$ and the plane given by $2x - 3y + 4z = 13$.
(c) Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (3, 5, 1)$, $B = (5, 1, 4)$, $D = (-5, 2, -3)$, find the point C.

2. Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$.
(a) Find the orthogonal projection $proj_{\overrightarrow{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.
(b) Find the point P such that $\overrightarrow{AP} = proj_{\overrightarrow{AB}}(\overrightarrow{AC})$.
(c) Find the distance d from the point C to the line L that contains points A and B.

3. (a) Find paramteric equations for the line of intersection of the planes $3x + 2y - z = 4$ and $2x + z = 1$.
(b) Let L_1 denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let L_2 denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines L_1 and L_2 intersect? If not, are they skew or parallel?

4. (a) Find the volume of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.
(b) Find an equation of the plane through A, B, D.
(c) Find the angle between the plane through A, B, C and the xy plane.

5. The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. (a) At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.
(b) Find an equation of the tangent line to the curve at the time $t = 4$.
(c) Does the particle ever pass through the point $P = (80, 41, 13)$?
(d) Find the length of the arc traveled from time $t = 1$ to time $t = 2$.

6. Consider the vector valued function $f(x, y) = 6x^3y/(2x^4 + y^4)$.
(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Why or why not?
(b) Compute the second partial derivatives of $f(x, y)$ and verify that $f_{xy} = f_{yx}$.