Math 233 **Practice Problems for Exam 1** **Fall 2006**

1. Consider the line L through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the intersection of the line L and the plane given by $2x - 3y + 4z = 13$.

2. Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) =< t, 1-t, 3+t^2 >, \mathbf{r}_2(s) =< 3-s, s-2, s^2 >$$

where t and s are two independent real parameters.
(a) Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.
(b) Find parametric equations for the tangent line to each of the two space curves at the intersection point.

3. Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point C.

4. Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$.
(a) Find the orthogonal projection $proj_{\overrightarrow{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.
(b) Find the area of triangle ABC.
(c) Find the distance d from the point C to the line L that contains points A and B.

5. Find paramteric equations for the line of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

6. Let L_1 denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let L_2 denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines L_1 and L_2 intersect? If not, are they skew or parallel?

7. (a) Find the volume of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.
(b) Find an equation of the plane through A, B, D.
(c) Find the angle between the plane through A, B, C and the xy plane.

8. The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
(a) At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.
(b) Find an equation of the tangent line to the curve at the time $t = 4$.
(c) Does the particle ever pass through the point $P = (80, 41, 13)$?
(d) Find the length of the arc traveled from time $t = 1$ to time $t = 2$.

9. Consider the surface $x^2 + 3y^2 - 2z^2 = 1$.
(a) What are the traces in $x = k, y = k, z = k$? Sketch a few.
(b) Sketch the surface in the space.

10. Find an equation for the tangent plane to the graph of $f(x, y) = y \ln x$ at $(1, 4, 0)$.

11. Find the distance between the given parallel planes

$$z = 2x + y - 1, -4x - 2y + 2z = 3.$$

12. Identify the surface given by the equation $4x^2 + 4y^2 - 8y - z^2 = 0$. Draw the traces and sketch the curve.

13. A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
a) Write an equation for the acceleration vector.
b) Write a vector for initial velocity.
c) Write a vector for initial position.
d) At what time does the projectile hit the ground?
e) How far did it travel, horizontally, before it hit the ground?

14. Explain why the limit of $f(x, y) = (3x^2y^2)/(2x^4 + y^4)$ does not exist as (x, y) approaches $(0, 0)$.

15. Find an equation of the plane that passes through the point $P(1, 1, 0)$ and contains the line given by parametric equations $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

16. Find all of the first order and second order partial derivatives of the function.

(a) $f(x, y) = x^3 - xy^2 + y$

(b) $f(x, y) = \ln(x + \sqrt{x^2 + y^2})$

17. Find the linear approximation of the function $f(x, y) = xye^x$ at $(x, y) = (1, 1)$, and use it to estimate $f(1.1, 0.9)$.

18. Find a vector function which represents the curve of intersection of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.