

DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233

EXAM 2

Spring 2006

NAME: _____ Student ID# _____

Section Number: _____ Instructor's Name: _____

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations. Definite integrals must be solved symbolically, not by calculator.

You can leave answers in terms of fractions and square roots, but if approximate numerical answers are used, they should be rounded off to 4 significant figures.

1.	(20)	_____
2.	(18)	_____
3.	(17)	_____
4.	(15)	_____
5.	(15)	_____
6.	(15)	_____
Total		_____

Perfect Paper → 100 Points.

There are six pages, including this one, in this exam and six problems. Make sure you have them all before you begin!

- (1) Let $f(x, y, z) = x^2y - yz + z$.
- a) (6 points) Give a vector that points in the direction that f is *decreasing* fastest at the point $(0, 2, -1)$.
- b) (7 points) Give an equation for the tangent plane to the surface given by the equation $f(x, y, z) = 1$ at the point $(0, 2, -1)$.

c) (7 points) Find a point (x, y, z) on the surface from part b) where the tangent plane is parallel to the yz plane. Note that since the point is on the surface, it must satisfy the equation $f(x, y, z) = 1$.

(2) (18 points) Find all the critical points of the function

$$f(x, y) = y^2 - x^2y + y$$

and for each one, determine if it is a local maximum, local minimum, or a saddle point.

- (3) (17 points) Find the maximum and minimum values of the function

$$f(x, y) = x^2 + 2y$$

on the circle $x^2 + y^2 = 4$, and the points where they occur.

- (4) a) (7 points) Let $f(x, y) = x^2y$, and suppose that $x = x(t)$ and $y = y(t)$ are functions of a variable t , and that $x(0) = -1$, $y(0) = 5$ and $x'(0) = 3$, $y'(0) = 2$. Use this information to calculate the derivative $\left. \frac{df}{dt} \right|_{t=0} = \left. \frac{d}{dt} f(x(t), y(t)) \right|_{t=0}$.

b) (8 points) Suppose that a function $g(x, y)$ has continuous partial derivatives, and that $g(2, 1) = -3$, $g_x(2, 1) = 2$, $g_y(2, 1) = -1$. Use the local linear approximation or differentials to estimate $g(1.98, 1.05)$.

(5) (15 points) Find the volume of the solid in the first octant bounded by $y = 3x$, $y = x^2$, $z = 0$ and $z = 2xy$.

- (6) (15 points) D is the triangle with vertices at $(0,0)$, $(3,0)$ and $(3,9)$. Consider the double integral

$$I = \iint_D e^{x^2} dA.$$

- a) Write down the two iterated integrals for I .

- b) Find the value of I .