

DEPARTMENT OF MATHEMATICS AND STATISTICS  
UNIVERSITY OF MASSACHUSETTS

**MATH 233**

**EXAM 1**

**Fall 2006**

NAME: \_\_\_\_\_

Section Number: \_\_\_\_\_ Instructor's Name: \_\_\_\_\_

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations.

You can leave answers in terms of fractions and square roots, but if approximate numerical answers are used, they should be rounded off to 4 significant figures.

$\langle x, y, z \rangle$ ,  $[x, y, z]$ ,  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ,  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ; are all permissible notations for the vector  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ .

1. (20) \_\_\_\_\_
2. (20) \_\_\_\_\_
3. (20) \_\_\_\_\_
4. (20) \_\_\_\_\_
5. (20) \_\_\_\_\_
- Total \_\_\_\_\_

**Perfect Paper  $\rightarrow$  100 Points.**

*There are seven pages, including this one, in this exam and five problems. Make sure you have them all before you begin!*

## FORMULAS

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

$$|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

where

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta = \text{area of parallelogram with sides } \mathbf{a}, \mathbf{b}.$

The vector projection of  $\mathbf{b}$  onto (in the direction of)  $\mathbf{a}$  is  $proj_{\mathbf{a}} \mathbf{b} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \right) \mathbf{a}$

The scalar projection (component) of  $\mathbf{b}$  onto  $\mathbf{a}$  is  $comp_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

The volume of the “box” determined by  $\mathbf{a}, \mathbf{b}$  and  $\mathbf{c}$  is  $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$

Arc length of parametrized curve:

$$L = \int_a^b |\mathbf{r}'(t)| dt.$$

1. a) (8 points) Find parametric equations for the line which contains  $A(2, 0, 1)$  and  $B(-1, 1, -1)$ .

- b) (12 points) Determine whether the lines  $l_1 : x = 1 + 2t, y = 3t, z = 2 - t$  and  $l_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$  are parallel, skew or intersecting.

2. a) (10 points) Find an equation of the plane which contains the points  $P(-1, 2, 1)$ ,  $Q(1, -2, 1)$  and  $R(1, 1, -1)$ .

- b) (10 points) Find the distance from the point  $(1, 2, -1)$  to the plane  $2x + y - 2z = 1$ .

3. a) (10 points) Let two space curves

$$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle,$$

be given where  $t$  and  $s$  are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point  $(1, 0, 1)$ .

b) (10 points) Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position  $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$ . Find the position vector function  $\mathbf{r}(t)$ .

4. a) (12 points) Let  $f(x, y) = e^{x^2-y} + x\sqrt{4-y^2}$ . Find partial derivatives  $f_x$ ,  $f_y$  and  $f_{xy}$ .

b) (8 points) Find an equation for the tangent plane of the graph of

$$f(x, y) = \sin(2x + y) + 1$$

at the point  $(0, 0, 1)$ .

5. a) (7 points) Let  $g(x, y) = ye^x$ . Estimate  $g(0.1, 1.9)$  using the linear approximation of  $g(x, y)$  at  $(x, y) = (0, 2)$ .

- b) (6 points) Find the center and radius of the sphere  $x^2 + y^2 + z^2 + 6z = 16$ .

- c) (7 points) Let  $f(x, y) = \sqrt{16 - x^2 - y^2}$ . Draw a contour map of level curves  $f(x, y) = k$  with  $k = 1, 2, 3$ . Label the level curves by the corresponding values of  $k$ .