Statistical Properties of Nonequilibrium Dynamical Systems

July 27 - August 2, 2016

Speakers and Abstracts

(1) Wael Bahsoun, Loughborough University Linear response for non-uniformly expanding maps

We provide a general framework to study differentiability of SRB measures for one dimensional non-uniformly expanding maps. Our work covers systems that admit a finite SRB measure and it also covers systems that admit an infinite SRB measure. In particular, we obtain a linear response formula for both finite and infinite SRB measures. We apply our results to interval maps with a neutral fixed point (Pomeau-Manneville maps) to prove differentiability of the corresponding SRB measure. This is a joint work with Benoit Saussol.

(2) Peter Balint, Technical University of Budapest Mean field coupling of doubling maps

N points (or sites) on the circle are investigated, evolved by the composition of the doubling map acting on the individual points, and a mean field coupling. For finitely many sites, two distinct bifurcation values of the coupling strength have been identified in the literature, corresponding to the loss of contraction and, specifically for N=3, to the loss of ergodicity. On the one hand, these phenomena are reconsidered and an interpretation is provided in terms of the synchronization of the sites. On the other hand, by focusing on the evolution of distributions, a new viewpoint is initiated which can be regarded as a possible generelization to the case when N is infinite. This is joint work with Fanni Selley.

(3) Federico Bonetto, Georgia Institute of Technology Uniform Approximation of a Maxwellian Thermostat by Finite Reservoirs

We study the evolution of a system of M particles in contact with a large reservoir of $N \gg M$ particles. The reservoir is initially in equilibrium at temperature T. The evolution of the system and reservoir is described via a suitable Kac-style collision process. We show that for large N, this evolution can be effectively described by replacing the reservoir with a Maxwellian thermostat at temperature T. This description provides an approximation that is uniform in time both in a suitable L^2 norm and in the Gabetta-Toscani-Wennberg (GTW) distance.

(4) Leonid Bunimovich, Georgia Institute of Technology When and where orbits of extremely chaotic systems prefer to go

Some important features of dynamics of most chaotic systems like coin tossing, von Neumann-Ulam map, etc can be predicted. Such questions as which elements of a Markov partition will more likely be visited first, which out of two never visited elements will more likely be visited first, etc can be answered. This allows to understand some features of transport in phase space and, e.g. chose an optimal Young tower.

(5) Keith Burns, Northwestern University Unique equilibrium states for geodesic flows in nonpositive curvature The geodesic flow for a compact Riemannian manifold with negative curvature has a unique equilibrium state for every Holder continuous potential function. This is no longer true if the curvature is only nonpositive, but there is a large class of potentials with unique equilibrium states. For compact surfaces of nonpositive curvature (other than flat tori) this class includes multiples of the geometric potential by factors between 0 and 1 This is joint work with Vaughn Climenhaga, Todd Fisher and Dan Thompson, and is closely related to the ideas in Dan Thompson's talk.

(6) Jianyu Chen, University of Massachusetts, Amherst Statistical properties of one-dimensional expanding maps with poor singularities

We consider one-dimensional non-Markov uniformly expanding maps, whose derivatives may have unbounded variations. Under certain conditions, we are able to show the existence of absolutely continuous invariant measure, exponential decay or correlations, central limit theorem and large deviation principle. Our method applies to the tent map, the beta transformation, the Gauss map, and their C^1 perturbations.

(7) Jean-Pierre Conze, University of Rennes Limit theorems for multi-dimensional actions on some algebraic models

For multi-dimensional actions, algebraic models yield a convenient framework to compare some of the methods in the study of distribution limits (martingale, cumulant, characteristic function). We consider limits for normalized ergodic sums either on rectangles or for actions driven by a random walk (quenched limit theorems). One of the models will be a family of endomorphisms on shift-invariant subgroups of $\mathbb{F}_p^{\mathbb{Z}^d}$, for which mixing of all orders is not satisfied.

(8) Maria Fatima Correia, Universidade de Évora Ergodicity and periodic orbits of a two-parameter family of billiards

We construct a two-parameter family of moon-shaped billiard tables with boundary made of two circular arcs. These tables fail the defocusing mechanism and other known mechanisms that guarantee and hyperbolicity. We analytically study the stability of some periodic orbits and prove there is a class of billiards in this family with elliptic periodic orbits. We also numerically observe a subclass of moon-shaped billiards with a single ergodic component.

(9) Christopher Cox, Washington University in St. Louis On the Ergodicity of Umbrella and No-slip Billiards

parameters, even though they exhibit neither of the well-known mechanisms for constructing chaotic billiards, dispersing or defocusing. We consider 'umbrella' perturbations, in which one of the two boundary arcs is divided and the curvature of the newly created sub-arcs is varied. For parameters corresponding to non-ergodic lemon and moon billiards, small perturbations may transform elliptic periodic points into a cascade of higher order elliptic points, which either stabilize as the perturbation is increased or dissipate. In the latter case, phase portraits suggest new ergodic examples are created. No-slip billiards are based on an alternative to the specular model in which momentum may be exchanged at boundary collisions. In contrast to standard billiards, no known ergodic examples exist. Phase portraits of polygonal no-slip billiards are dominated by quasiperiodic neighborhoods of elliptic points, precluding ergodicity. Portraits of curvilinear polygons and Sinai dispersers show some evidence of chaotic behavior, but significant and possibly insurmountable obstructions to constructing ergodic examples remain.

(10) Mark Demers, Fairfield University

Exponential decay of correlations for Sinai billiard flows

While billiard maps for large classes of dispersing billiards are known to enjoy exponential decay of correlations, the corresponding flows have so far resisted such analysis. We describe recent results, based on the construction of function spaces on which the associated transfer operator has good spectral properties, which provide a description of the spectrum of the generator of the semi-group. This construction, together with a Dolgopyat-type cancellation argument to eliminate certain eigenvalues, proves that the generator has a spectral gap and that the Sinai billiard flow with finite horizon has exponential decay of correlations. This is joint work with V. Baladi and C. Liverani.

(11) Yanxia Deng, Queens University

Stability of periodic orbits by Conley-Zehnder index theory

Roughly speaking, the Conley-Zehnder index measures the number of half windings of a fundamental solution for a periodic linear Hamiltonian system. This index, and its closely related cousin the Morse index for the equivariant action functional, can be used to give non perturbative arguments for linearized stability and instability for families of periodic orbits in Hamiltonian systems. We will give several examples of this from the setting of parametric resonance in parametrized families, to minimum distance lines in kinetic plus potential systems. Using a necessary and sufficient condition for elliptic stability of periodic orbits in two degrees of freedom systems, we outline the global argument for families of hyperbolic orbits in the Henon-Heiles system.

(12) Dmitry Dolgopyat, University of Maryland Limit theorems for random walk in random environment on a strip

1 dimensional random walks in random environment exhibit a wide range of different behaviors depending on the strength of traps. In this talk I present recent results for walk on the strip, showing how dynamical system methods can be used to analyze this system. Based on a joint work with Ilya Goldsheid.

(13) Davor Dragicevic, University of New South Wales Hölder continuity of Oseledets splitting for semi-invertiblle operator cocycles

For Hölder continuous cocycles over an invertible, Lipschitz base, we establish the Hlder continuity of Oseledets subspaces on compact sets of arbitrarily large measure. This extends a result of Arajo, Bufetov, and Filip by considering possibly noninvertible cocycles, which in addition may take values in the space of compact operators on a Hilbert space. The talk is based on a joint work with G. Froyland.

(14) Renato Feres, Washington University in St. Louis Geometry and dynamics of no-slip billiards We extend earlier results by Gutkin and Wojtkowski on billiard systems known as noslip. These are natural, physically well-motivated alternatives to the standard kind. We first indicate how they are derived from a general classification of rigid bodies collisions, then give sufficient conditions for the invariance of the standard (canonical) billiard measure, and show some of the dynamical features that set them apart from standard billiards. Our main results provide evidence for the conjecture that non-slip billiards in dimension two are never ergodic. This is joint work with Christoffer Cox.

(15) Gary Froyland, University of New South Wales

Transfer operator co-cycle analysis of non-autonomous dynamical systems

Non-autonomous dynamical systems arise in situations where nonlinear dynamics is forced or driven by an external process. Naturally associated with a non-autonomous dynamical system is a transfer operator cocycle, where the cocycle is driven by the same external process. Transfer operator theory for single transformations can be extended to the non-autonomous situation: for example, to treat stability and approximation of (random) invariant measures, and to use isolated Lyapunov spectral values to quantify mass transport and coherence. We summarise recent work in this area and present applications to geophysical models and datasets.

(16) Michele Gianfelice, University of Calabria

Some results on the asymptotic behaviour of finite connection probabilities in percolation and related models

We review the results on the exact asymptoics of connection probabilities for the Bernoulli percolation and for the random-cluster model appearing in a series of papers authored by M. Campanino, D. Ioffe and Y. Velenik, when considering the subcritical regime in two or more lattice dimensions, and by M. Campanino and M. Gianfelice, when considering the highly supercritical regime in three or more lattice dimensions. We will show that the Ornstein-Zernike behaviour of connectivity functions for the independent percolation stems from a multidimensional renewal theorem, while, for the random-cluster model, from a local limit theorem result for a random process via thermodynamic formalism.

(17) Cecilia Gonzalez-Tokman, University of Queensland Non-autonomous dynamical systems and multiplicative eraodic theorems

Non-autonomous dynamical systems yield flexible models for the study of time-dependent systems, with driving mechanisms allowed to range from deterministic forcing to stationary noise. Multiplicative ergodic theorems encompass fundamental information for the study of transport phenomena in such systems, including Lyapunov exponents, invariant measures and coherent structures. In this talk, we will motivate and discuss recent developments on multiplicative ergodic theorems and address related stability questions, which arise naturally in the context of non-autonomous systems from the use of numerical approximation schemes, as well as from the presence of modelling errors and noise. (This talk is based on joint work with Gary Froyland and Anthony Quas.)

(18) Alex Grigo, University of Oklahoma Applications of billiard-like systems In this talk we will consider billiard-like models that arise in certain models of gas-like interacting particle systems. The aim of the talk will be to present how one can implement perturbation theory, particularly averaging theory, to compute statistical properties. In particular, we will be interested in studying transport properties.

- (19) Yves Guivarch, University of Rennes TBA
- (20) Nicolai Haydn, University of Southern California Entry and Return Times for Bowen Balls For mixing maps it has been shown that the first return times are in the limit exponentially distributed and the higher order returns converge in the limit to the Poisson distribution on a full measure set if the measure is α -mixing and if the limit is taken along cylinder sets. According to an example by Lacroix and Kupsa, one can obtain arbitrary limiting distributions if one allows for arbitrary sets. For C^2 on manifolds it was shown that for balls the limiting distribution is exponential or Poissonian. In joint work with Fan Yang we explore here the case of continuous maps that are α or ϕ -mixing and show that the limiting behaviour of entry and return times along sequences of dynamic balls (Bowen balls) are of the same kind.
- (21) Yunping Jiang, Queen's College, CUNY Ruelles Transfer Operators and Decay of Correlations for Uniformly and Non-Uniformly Expanding Dynamical Systems

Ruelle's transfer operators play an important role in the study of dynamical systems (as well as iterated function systems). For examples, the existence and uniqueness of an equilibrium state and studying the decay rate of correlation, etc.. In this talk, I will first give a general introduction to Ruelles transfer operators and then a review of our work jointly with Aihua Fan and Yuan-Ling Ye on Ruelles transfer operators associated with uniformly expanding and non-uniformly expanding dynamical systems (as well as uniformly contracting and non-uniformly contracting iterated function systems) and Dini continuous potentials. Some estimations of decay of correlations are given by using this study.

(22) Paul Jung, University of Alabama at Birmingham A Stable Limit Theorem for Billiards with Cusps

We will present recent joint work with D. Dolgopyat and H. Zhang in which we establish a Stable Limit Theorem for dispersing billiards with cusps.

(23) Konstantin Khanin, University of Toronto On global solutions to the parabolic Anderson model

We show that almost surely there exists a unique (up to a multiplicative constant) global solution to the parabolic Anderson model. We also discuss connection with a more general setting of the random Hamilton-Jacobi equation. The talk is based on a joint work with Tobias Hurth and Beatriz Navarro.

(24) Marco Lenci, Universita di Bologna Uniformly expanding Markov maps of the real line: exactness and infinite mixing We give a fairly complete characterization of the exact components of a large class of uniformly expanding Markov maps of R. Using this result, for a class of Z-invariant maps and finite modifications thereof, we prove certain properties of infinite mixing recently introduced by the author.

(25) Bing Li, South University of Science and Technology of China Zero-one law of Hausdorff dimensions of the recurrent sets

Let (Σ, σ) be the one-sided shift space with m symbols and $R_n(x)$ be the first return time of $x \in \Sigma$ to the *n*-th cylinder containing x. Denote

$$E_{\alpha,\beta}^{\varphi} = \left\{ x \in \Sigma : \liminf_{n \to \infty} \frac{\log R_n(x)}{\varphi(n)} = \alpha, \ \limsup_{n \to \infty} \frac{\log R_n(x)}{\varphi(n)} = \beta \right\},$$

where $\varphi : \mathbb{N} \to \mathbb{R}^+$ is a monotonically increasing function and $0 \leq \alpha \leq \beta \leq +\infty$. We show that the Hausdorff dimension of the set $E^{\varphi}_{\alpha,\beta}$ admits a dichotomy: it is either zero or one depending on φ, α and β .

(26) Yao Li, University of Massachusetts, Amherst Polynomial convergence to nonequilibrium steady-state

In this talk I will present my recent result about the ergodic properties of nonequilibrium steady-state (NESS) for a stochastic energy exchange model. The energy exchange model can be seen as a stochastic approximation of a certain billiards-like deterministic particle system that models the microscopic heat conduction in a 1D chain. By using a technique called the induced chain method, I proved the existence, uniqueness, and t^{-2} speed of convergence to the NESS for the stochastic energy exchange model. All of these are consistent with the numerical simulation results of the original deterministic particle model.

(27) Carlangelo Liverani, University of Rome Fast-slow partially hyperbolic systems

> I will give a summary of several results recently obtained in the study of fast-slow partially hyperbolic systems.

(28) Yiming Long, Nankai University Periodic solutions of the generalized anisotropic Lennard-Jones systems in \mathbb{R}^n

In this talk, I shall introduce the recent results obtained by Dr. Bowen Liu and myself jointly on periodic solutions of the following generalized anisotropic Lennard-Jones systems in \mathbb{R}^n . There are three families of special solutions of this system: the constant solutions, the circular solutions and the oscillating line solutions. A natural question is whether these special solutions have exhausted all the periodic solutions of the system. We prove that the answer is yes for every periodic solution of this system with zero topological degree, i.e., it must be either a constant solution or an oscillating line solution. And we prove that the answer is no for periodic solutions of this system with non-zero topological degree by showing that for every large enough T_i0 there exists a non-special T-periodic solution with non-zero topological degree. (29) Kening Lu, Bringham Young University

SRB measures, entropy, and horseshoes for infinite dimensional dynamical systems

This talk contains three parts: (1) the existence of SRB measures and their properties for infinite dimensional dynamical systems; (2) The existence of strange attractors with SRB measures for parabolic PDEs undergoing Hopf bifurcations driven by a periodic forcing with applications to the Brusselator; (3) Positive entropy implying the existence of horseshoes for infinite dimensional dynamical systems. The first part is based on joint works with Zeng Lian and Peidong Liu, the second part is a joint work with Qiudong Wang and Lai-Sang Young, and the third part is a joint work with Wen Huang.

(30) Ian Melbourne, University of Warwick The Lorenz attractor is exponentially mixing

> We show that the Lorenz attractor (for the Lorenz equations with the classical parameters) has exponential decay of correlations. Joint work with Vitor Araujo and Paulo Varandas.

(31) Peter Nandori, University of Maryland Toward the rare interaction limit of some hard ball systems

We prove that the time of the first collision between two particles on a Sinai billiard table converges weakly to an exponential distribution when time is rescaled by the inverse of the radius of the particles (joint work with Dmitry Dolgopyat). This result provides a first step in studying the energy evolution of hard ball systems in the rare interaction limit. I will also explain that in some spatially extended toy models, forced out of equilibrium, but with no or much simplified stochastic interaction, we can prove the emergence of local equilibrium.

(32) Mark Pollicott, University of Warwick Asymptotic escape rates for one-dimensional maps

Given a dynamical system with an invariant measure, the escape rate measures the speed at which measure escapes from a ball of radius r, say. The asymptotic escape rate (when it exists) describes asymptotic behaviour of the ratio of the escape rate to r, as r tends to zero. We will consider the asymptotic escape rates for a broad class of interval maps and rational maps, beyond the small family of uniformly hyperbolic maps for which the result was previously known. This is joint work with Mariusz Urbanski (UNT).

(33) Federico Rodriguez-Hertz, Penn State Equilibrium states for partially hyperbolic systems with isometric center direction

Joint with Pablo Carrasco we are exploring the equilibrium state theory for partially hyperbolic systems with isometric centre direction. This includes hyperbolic (Anosov) systems, some ergodic automorphisms of the torus, time 1 maps of some partially hyperbolic homogeneous flows, as well as time one maps of Anosov flows. One can also take some perturbations (we shall discuss this matter). We use Margulis type construction of entropy maximizing measure to build (explicitly) equilibrium states and we derived some geometric consequences, in particular uniqueness properties. Finally we apply this to solve the cohomological equation (Livisic equation). In the talk we plan to discuss this topic and give some related problems.

(34) Richard Sharp, University of Warwick Fluctuation theorems with shrinking intervals

Fluctuation theorems arise in statistical mechanics and describe systems away from equilibrium. They have been rigorously proved for hyperbolic dynamical systems and are related to large deviations of ergodic averages. We will discuss a version where averages lie in intervals that shrink (at an appropriately slow rate) as the systems evolves. (This is joint work with Mark Pollicott.)

(35) Dan Thompson, The Ohio State University Uniqueness of equilibrium states for geodesic flows

We discuss some recent progress on the thermodynamic formalism of geodesic flows. The talk will cover some aspects of the following three topics:

1) General machinery developed by Vaughn Climenhaga and myself, which applies for flows satisfying non-uniform versions of expansivity and the specification property, yielding uniqueness of equilibrium states for a wide class of potential functions.

2) Joint work with Keith Burns, Vaughn Climenhaga and Todd Fisher, where we apply this machinery to geodesic flow on rank one manifolds. This setting is a canonical case of non-uniform hyperbolicity for flows. Our methods are completely different from those used by Knieper in his seminal proof of uniqueness of the measure of maximal entropy in this setting.

3) Joint work with Jean-Francois Lafont and Dave Constantine, where we establish the weak specification property for geodesic flow on CAT(-1) spaces, and use this to prove results on uniqueness of equilibrium states and large deviations.

(36) Xueting Tian, Fudan University Specification in dynamical systems with certain hyperbolicity and applications

In this talk I will mainly study dynamical systems with specification property or various variants and discuss many related applications, including density of periodic measures, existence and variational principle of saturated sets and its relation to different recurrence, the relation of Poincare recurrence and Lyapunov exponents, dynamical topological complexity of points without Lyapunov exponents etc.

(37) Dmitry Todorov, University of Aix-Marseille Random averaging

> Averaging theory allows to approximate a motion of the slow variable in a fast-slow system by a single ODE. I will that the similar thing happens if one perturbs the fast system randomly. This corresponds to the addition of a third (infinitely fast) time-scale.

(38) Raul Ures, Universidad de la Republica - Uruguay Ergodicity and partial hyperbolicity on unit tangent bundles of surfaces

A diffeomorphism is partially hyperbolic if the tangent bundle of the ambient manifold splits into three invariant sub-bundles, $TM = E^s \oplus E^c \oplus E^u$, in such a way that df contracts the vectors of E^s and expands the vectors of E^u while the vectors of E^c have an intermediate behavior. We will present some new advances in the study of the ergodicity of volume preserving partially hyperbolic diffeomorphisms acting on the unit tangent bundle of hyperbolic surfaces.

- (39) Sandro Vaienti, University of Toulon and CPT Marseille TBA
- (40) Carlos Vasquez, Pontificia Universidad Catlica de Valparaiso On the non-robustness of intermingled basins

It is well-known that it is possible to construct a partially hyperbolic diffeomorphism on the 3-torus in a similar way than in Kan's example. It has two hyperbolic physical measures with intermingled basins supported on two embedded tori with Anosov dynamics. A natural question is how robust is the intermingled basins phenomenon for diffeomorphisms defined on boundaryless manifolds? In this work we study partially hyperbolic diffeomorphisms on the 3-torus and show that the intermingled basins phenomenon is not robust.

(41) Benjamin Webb, Bringham Young University Self-Limiting Motion of a Particle Moving Deterministically in Random Media

We study the motion of a particle moving on a two-dimensional hexagonal lattice, whose sites are randomly occupied by either right or left rotators. These rotators deterministically scatter the particle to the right or left, additionally changing orientation from left to right or from right to left after scattering the particle. In this way the particle effects the medium through which it travels, i.e. lattice's configuration of scatters. In the model we consider, the scatterers are each independently oriented to the right with probability $p \in [0,1]$ at time zero. For $p \in (0,1)$, we show that as the particle moves through the lattice, it creates a number of reflecting structures. These structures ultimately limit the particle's motion, causing it to have a periodic trajectory. As p approaches either 0 or 1, and the medium becomes increasingly homogenous, the particle's dynamics undergoes a discontinuous transition from this self-limiting, periodic motion to an unbounded self-avoiding motion, where the particle's trajectory, away from its initial position, is a self-avoiding walk. This is joint work with Eddie Cohen.

(42) Lan Wen, Peking University A review on the stability and density problems

Structural stability and density are two problems that are related to each other in a natural way. There have been quite some works about the two problems in the C^1 category. In this talk we review some of these works and ideas along this line.

- (43) Amie Wilkinson, University of Chicago TBA
- (44) Zhihong Xia, South University of Science and Technology of China TBA
- (45) Dawei Yang, Soochow University SRB measures for singular hyperbolic attractors

We prove that for C^2 vector field X on any dimensional manifold, there is a unique physical SRB measure supported on a singular hyperbolic attractor. This is a joint work with R. Leplaideur.

(46) Fan Yang, UFRJ/IMPA

Decay of correlation for DA maps with measure of maximum entropy

We consider a C^1 diffeomorphism f on three dimensional torus which is in the isotopy class of an Anosov diffeomorphism A. Let μ be the measure of maximal entropy. We prove that μ has exponential decay of correlations for Hölder continuous observables. This is a joint work with Jiagang Yang.

(47) Xiangdong Ye, South University of Science and Technology of China Around the pointwise convergence of multiple ergodic averages

In this talk, we first review the recent progress on the study of the convergence of multiple ergodic averages. Then we discuss some topological aspects of the problem. This talk is based on joint works with Huang-Shao and Gutman-Huang-Shao.

(48) Ke Zhang, University of Toronto Random Hamiltonian systems and Hamilton-Jacobi equations

In Hamiltonian systems, a special class of orbits are those that minimize the Lagrangian action. We show that in a class of random Hamiltonian systems, there is a unique minimizing orbit, which is also non-uniformly hyperbolic. We use this information to obtain results concerning the associated Hamilton-Jacobi PDE. With K. Khanin, we show that the stationary solutions are locally smooth. More recently, with K. Khanin and R. Iturriaga, we show that the convergence to stationary solutions are exponentially fast.

(49) Pengfei Zhang, University of Mississippi Homoclinic Intersections for Geodesic Flows on Convex Spheres

Transverse homoclinic intersection was discovered by Poincare in the study of stability properties of periodic orbits of n-body problem, whose existence not only destroys the stability of periodic orbits but also leads the existence of chaos in the phase space. In this talk, we will study the geodesic flows on convex spheres. We will show that, generically, every closed geodesic is either hyperbolic or irrationally elliptic. Moreover, every hyperbolic closed geodesic admits some transverse homoclinic intersections. Therefore, (everywhere) chaotic geodesic dynamics can happen generically on manifolds with simple/trivial topology.

(50) Weinian Zhang, Sichuan University Roughness of Tempered Exponential Dichotomies

This talk is concerning the roughness of tempered exponential dichotomies for linear random dynamical systems in Banach spaces. Such a dichotomy has a tempered bound and describes nonuniform hyperbolicity. The roughness is proved without assuming their invertibility and the integrability condition of the Multiplicative Ergodic Theorem. An explicit bound is given for the linear perturbation such that the dichotomy is persistent. Explicit forms are obtained for the exponent and the bound of tempered exponential dichotomy of the perturbed random system in terms of the original ones and the perturbations.