Hi students!

I am putting this version of my review for this Tuesday and Wednesday nights here on the website. **DO NOT PRINT!!**; it is very long!! **Enjoy!!**
Your course chair, **Bill**

PS. There are probably errors in some of the solutions presented here and for a few problems you need to complete them or simplify the answers; some questions are left to you the student. Also you might need to add more detailed explanations or justifications on the actual similar problems on your exam.
After our exam, I have added the solutions right after this slide.

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4}$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49}$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7.$
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2\rangle, \qquad \mathbf{b} = \langle 1, 2, 3\rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a}$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{ unit vector parallel to } \mathbf{a}.$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of **b** along **a**. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{ unit vector parallel to } \mathbf{a}.$$

- So,

$$\mathbf{proj_a b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{ unit vector parallel to } \mathbf{a}.$$

- So,

$$\mathbf{proj_a b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{ unit vector parallel to } \mathbf{a}.$$

- So,

$$\mathbf{proj_a b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} =$$

$$\frac{1}{49}\langle 1, 2, 3 \rangle \cdot \langle 3, 6, -2 \rangle \langle 3, 6, -2 \rangle$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2\rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3\rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{ unit vector parallel to } \mathbf{a}.$$

- So,

$$\mathbf{proj_a b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} =$$

$$\frac{1}{49}\langle 1, 2, 3\rangle \cdot \langle 3, 6, -2\rangle\langle 3, 6, -2\rangle = \frac{9}{49}\langle 3, 6, -2\rangle.$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3 \rangle$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49}\langle 3, 6, -2 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2\rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3\rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3\rangle = \langle 1, 2, 3\rangle - \frac{9}{49}\langle 3, 6, -2\rangle + \frac{9}{49}\langle 3, 6, -2\rangle$$

$$= \frac{1}{49}\langle 22, 44, 165\rangle + \frac{9}{49}\langle 3, 6, -2\rangle.$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49}\langle 3, 6, -2 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle$$

$$= \frac{1}{49}\langle 22, 44, 165 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle.$$

- Here

$$\frac{9}{49}\langle 3, 6, -2 \rangle \ \textbf{parallel to} \ \ \mathbf{a} = \langle 3, 6, -2 \rangle$$

and

$$\frac{1}{49}\langle 22, 44, 165 \rangle \ \textbf{orthogonal to} \ \mathbf{a} = \langle 3, 6, -2 \rangle.$$

$\square$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write

$$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write
$$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$
where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.
- Of course this is the same as
$$\mathbf{b} = (\mathbf{b} - \mathbf{proj}_{\mathbf{a}}\mathbf{b}) + \mathbf{proj}_{\mathbf{a}}\mathbf{b}.$$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write

$$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

  where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.

- Of course this is the same as

$$\mathbf{b} = (\mathbf{b} - \mathbf{proj}_{\mathbf{a}}\mathbf{b}) + \mathbf{proj}_{\mathbf{a}}\mathbf{b}.$$

  That is, we write $\mathbf{b}$ as $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$ plus “the rest”.

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write

$$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

  where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.

- Of course this is the same as

$$\mathbf{b} = (\mathbf{b} - \mathbf{proj}_{\mathbf{a}}\mathbf{b}) + \mathbf{proj}_{\mathbf{a}}\mathbf{b}.$$

  That is, we write $\mathbf{b}$ as $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$ plus "the rest". But "the rest" is orthogonal to $\mathbf{n}$ (and to $\mathbf{a}$), since

  $(\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n}) \cdot \mathbf{n}$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2\rangle, \qquad \mathbf{b} = \langle 1, 2, 3\rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write

$$\mathbf{b} = \mathbf{b} - (\mathbf{b}\cdot\mathbf{n})\mathbf{n} + (\mathbf{b}\cdot\mathbf{n})\mathbf{n}$$

where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.

- Of course this is the same as

$$\mathbf{b} = (\mathbf{b} - \mathbf{proj_a b}) + \mathbf{proj_a b}.$$

That is, we write $\mathbf{b}$ as $\mathbf{proj_a b}$ plus "the rest". But "the rest" is orthogonal to $\mathbf{n}$ (and to $\mathbf{a}$), since

$$(\mathbf{b} - (\mathbf{b}\cdot\mathbf{n})\mathbf{n})\cdot\mathbf{n} = \mathbf{b}\cdot\mathbf{n} - (\mathbf{b}\cdot\mathbf{n})(\mathbf{n}\cdot\mathbf{n})$$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write

$$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

  where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.

- Of course this is the same as

$$\mathbf{b} = (\mathbf{b} - \mathbf{proj_a b}) + \mathbf{proj_a b}.$$

  That is, we write $\mathbf{b}$ as $\mathbf{proj_a b}$ plus "the rest". But "the rest" is orthogonal to $\mathbf{n}$ (and to $\mathbf{a}$), since

$$(\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n}) \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} - (\mathbf{b} \cdot \mathbf{n})(\mathbf{n} \cdot \mathbf{n}) = 0, \text{ as } \mathbf{n} \cdot \mathbf{n} = 1.$$

$\square$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Solution:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Solution:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{|\langle 3, 6, -2 \rangle||\langle 1, 2, 3 \rangle|}$$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Solution:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{|\langle 3, 6, -2 \rangle||\langle 1, 2, 3 \rangle|} = \frac{9}{\sqrt{49}\sqrt{14}}$$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Solution:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{|\langle 3, 6, -2 \rangle||\langle 1, 2, 3 \rangle|} = \frac{9}{\sqrt{49}\sqrt{14}} = \frac{9}{7\sqrt{14}}.$$

$\square$

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line.

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB}$$

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5 \rangle + t \langle 4, -5, -3 \rangle$$

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5\rangle + t\,\langle 4, -5, -3\rangle = \langle -1 + 4t, 7 - 5t, 5 - 3t\rangle\,,$$

where O is the origin.

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5\rangle + t\,\langle 4, -5, -3\rangle = \langle -1 + 4t, 7 - 5t, 5 - 3t\rangle\,,$$

where O is the origin.

- The **parametric equations** are:

$$\left|\begin{array}{l} x = -1 + 4t \\ y = 7 - 5t, \\ z = 5 - 3t \end{array}\right. \qquad t \in \mathbb{R}.$$

$\square$

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D.

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB}$$

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$. A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle$$

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5\rangle + \langle 2, 0, -1\rangle = \langle 1, 7, 4\rangle,$$

where O is the origin.

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle = \langle 1, 7, 4 \rangle ,$$

where O is the origin. That is,

$$D = (1, 7, 4).$$

□

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Problem 3(a) - Spring 2009

Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector **n** and a point on it, say the point P, it suffices to find **n**.

Problem 3(a) - Spring 2009

Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point P, it suffices to find $\mathbf{n}$. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point P, it suffices to find $\mathbf{n}$. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix}$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector **n** and a point on it, say the point P, it suffices to find **n**. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point P, it suffices to find $\mathbf{n}$. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle = 2\langle 1, -6, 3 \rangle.$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector **n** and a point on it, say the point P, it suffices to find **n**. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle = 2\langle 1, -6, 3 \rangle.$$

So the **equation of the plane** is:

$$(x-1) - 6(y-3) + 3(z-5) = 0. \qquad \square$$

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Solution:

The area of the triangle $\boldsymbol{\Delta}$ with vertices P, Q, R can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2.

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Solution:

The area of the triangle $\mathbf{\Delta}$ with vertices P, Q, R can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2. Thus, using **a)**, we have:

$$\textbf{Area}(\mathbf{\Delta}) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2}$$

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Solution:

The area of the triangle $\boldsymbol{\Delta}$ with vertices P, Q, R can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2. Thus, using **a)**, we have:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2} = \frac{1}{2}\,|2\langle 1, -6, 3\rangle|$$

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Solution:

The area of the triangle $\boldsymbol{\Delta}$ with vertices P, Q, R can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2. Thus, using **a)**, we have:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2} = \frac{1}{2}\,|2\langle 1, -6, 3\rangle|$$

$$= \sqrt{1+36+9}$$

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Solution:

The area of the triangle $\boldsymbol{\Delta}$ with vertices P, Q, R can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2. Thus, using **a)**, we have:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2} = \frac{1}{2}\,|2\langle 1, -6, 3\rangle|$$

$$= \sqrt{1+36+9} = \sqrt{46}.$$

$\square$

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

  $$\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle$$

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, 1, 3\rangle \times \langle 1, -1, 2\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix}$$

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector **v** parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix} = \langle 5, 1, -2 \rangle.$$

- A point on **L** is any (x_0, y_0, z_0) that satisfies the equations of **both** planes.

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix} = \langle 5, 1, -2 \rangle.$$

- A point on $\mathbf{L}$ is any (x_0, y_0, z_0) that satisfies the equations of **both** planes.
- Setting $z = 0$, we obtain the equations $x + y = 1$ and $x - y = 1$ and find such a point $(\frac{1}{2}, \frac{1}{2}, 0)$.

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix} = \langle 5, 1, -2 \rangle.$$

- A point on $\mathbf{L}$ is any (x_0, y_0, z_0) that satisfies the equations of **both** planes.
- Setting $z = 0$, we obtain the equations $x + y = 1$ and $x - y = 1$ and find such a point $(\frac{1}{2}, \frac{1}{2}, 0)$. Therefore **parametric equations** for $\mathbf{L}$ are:

$$\left| \begin{array}{l} x = \frac{1}{2} + 5t \\ y = \frac{1}{2} + t \\ z = -2t. \end{array} \right.$$

$\square$

Problem 5(a) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \; t \in \mathbb{R}.$$

Set up an integral for the length of the arc between $t = 0$ and $t = 1$. Do **not** attempt to evaluate the integral.

Problem 5(a) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \ t \in \mathbb{R}.$$

Set up an integral for the length of the arc between $t = 0$ and $t = 1$. Do **not** attempt to evaluate the integral.

Solution:

- The **velocity field** is:

$$\mathbf{v}(t) = \mathbf{r}'(t)$$

Problem 5(a) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \ t \in \mathbb{R}.$$

Set up an integral for the length of the arc between $t = 0$ and $t = 1$. Do **not** attempt to evaluate the integral.

Solution:

- The **velocity field** is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \left\langle 1, 2t, 3t^2 \right\rangle.$$

Problem 5(a) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \; t \in \mathbb{R}.$$

Set up an integral for the length of the arc between $t = 0$ and $t = 1$. Do **not** attempt to evaluate the integral.

Solution:

- The **velocity field** is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \left\langle 1, 2t, 3t^2 \right\rangle.$$

- Then the **speed** is

$$|\mathbf{r}'(t)| = \sqrt{1 + 4t^2 + 9t^4}.$$

Problem 5(a) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \;\; t \in \mathbb{R}.$$

Set up an integral for the length of the arc between $t = 0$ and $t = 1$. Do **not** attempt to evaluate the integral.

Solution:

- The **velocity field** is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \left\langle 1, 2t, 3t^2 \right\rangle.$$

- Then the **speed** is

$$|\mathbf{r}'(t)| = \sqrt{1 + 4t^2 + 9t^4}.$$

- Therefore, the **length** of the arc is:

$$\mathbf{L} = \int_0^1 \sqrt{1 + 4t^2 + 9t^4}\, dt.$$

□

Problem 5(b) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \ t \in \mathbb{R}.$$

Write down the parametric equations of tangent line to $\mathbf{r}(t)$ at $(2, 4, 8)$.

Problem 5(b) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \; t \in \mathbb{R}.$$

Write down the parametric equations of tangent line to $\mathbf{r}(t)$ at $(2, 4, 8)$.

Solution:

- The parametrized curve passes through the point $(2, 4, 8)$ if and only if $$t = 2, \;\; t^2 = 4, \;\; t^3 = 8 \Longleftrightarrow t = 2.$$

Problem 5(b) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \; t \in \mathbb{R}.$$

Write down the parametric equations of tangent line to $\mathbf{r}(t)$ at $(2, 4, 8)$.

Solution:

- The parametrized curve passes through the point $(2, 4, 8)$ if and only if
$$t = 2, \quad t^2 = 4, \quad t^3 = 8 \iff t = 2.$$
- The **velocity vector field** to the curve is given by
$$\mathbf{r}'(t) = \left\langle \, 1, 2t, 3t^2 \right\rangle \text{ hence } \mathbf{r}'(2) = \langle 1, 4, 12 \rangle \, .$$

Problem 5(b) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \ t \in \mathbb{R}.$$

Write down the parametric equations of tangent line to $\mathbf{r}(t)$ at $(2, 4, 8)$.

Solution:

- The parametrized curve passes through the point $(2, 4, 8)$ if and only if
$$t = 2, \quad t^2 = 4, \quad t^3 = 8 \Longleftrightarrow t = 2.$$
- The **velocity vector field** to the curve is given by
$$\mathbf{r}'(t) = \left\langle \ 1, 2t, 3t^2 \right\rangle \ \text{hence} \ \ \mathbf{r}'(2) = \langle 1, 4, 12 \rangle .$$
- The **equation of the tangent line** in question is:
$$\left| \begin{array}{l} x = 2 + \tau \\ y = 4 + 4\tau, \quad \tau \in \mathbb{R} \\ z = 8 + 12\tau \end{array} \right.$$

Problem 5(b) - Spring 2009

Consider the parametrised curve

$$\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle, \ t \in \mathbb{R}.$$

Write down the parametric equations of tangent line to $\mathbf{r}(t)$ at $(2, 4, 8)$.

Solution:

- The parametrized curve passes through the point $(2, 4, 8)$ if and only if
$$t = 2, \quad t^2 = 4, \quad t^3 = 8 \iff t = 2.$$
- The **velocity vector field** to the curve is given by
$$\mathbf{r}'(t) = \left\langle\ 1, 2t, 3t^2 \right\rangle \ \text{hence} \ \ \mathbf{r}'(2) = \langle 1, 4, 12 \rangle\,.$$
- The **equation of the tangent line** in question is:
$$\left| \begin{array}{l} x = 2 + \tau \\ y = 4 + 4\tau, \quad \tau \in \mathbb{R} \\ z = 8 + 12\tau \end{array} \right.$$

Caution: The parameter along the line, τ, has nothing to do with the parameter along the curve, t. $\square$

Problem 6(a) - Spring 2009

Consider the sphere **S** in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

Find its center **C** and its radius **R**.

Problem 6(a) - Spring 2009

Consider the sphere **S** in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

Find its center **C** and its radius **R**.

Solution:

- Completing the square we get

$$(x-2)^2 - 4 + y^2 + (z-3)^2 - 9 - 3 = 0$$

$$\iff$$

$$(x-2)^2 + y^2 + (z-3)^2 = 16.$$

Problem 6(a) - Spring 2009

Consider the sphere **S** in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

Find its center **C** and its radius **R**.

Solution:

- Completing the square we get

$$(x-2)^2 - 4 + y^2 + (z-3)^2 - 9 - 3 = 0$$

$$\iff$$

$$(x-2)^2 + y^2 + (z-3)^2 = 16.$$

- This gives:

$$\mathbf{C} = (2, 0, 3) \qquad \mathbf{R} = 4$$

□

Problem 6(b) - Spring 2009

What does the equation $x^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.

Problem 6(b) - Spring 2009

What does the equation $x^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.

Solution:

- This a (straight, circular) **cylinder** determined by the circle in the xz-plane of radius 2 and center $(0, 0)$ and parallel to the y-axis. $\square$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the *xy*-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g \rangle$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g \rangle = \langle 0, -10 \rangle$.

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g \rangle = \langle 0, -10 \rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \rangle$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0,0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g \rangle = \langle 0, -10 \rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle$.

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0,0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g\rangle = \langle 0, -10\rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle 6\sqrt{2}, 6\sqrt{2}\rangle$.
  So the velocity function is

  $$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\, d\tau$$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g\rangle = \langle 0, -10\rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle 6\sqrt{2}, 6\sqrt{2}\rangle$. So the velocity function is

$$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\,d\tau = \mathbf{v}(0) + \mathbf{a}t$$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g\rangle = \langle 0, -10\rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle 6\sqrt{2}, 6\sqrt{2}\rangle$.
  So the velocity function is

$$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\,d\tau = \mathbf{v}(0) + \mathbf{a}t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t\rangle.$$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g \rangle = \langle 0, -10 \rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle$.
  So the velocity function is
  $$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\, d\tau = \mathbf{v}(0) + \mathbf{a}t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle.$$
  One can recover the position by integrating the velocity:
  $$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau)\, d\tau + \mathbf{r}(0).$$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0,0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g\rangle = \langle 0, -10\rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle 6\sqrt{2}, 6\sqrt{2}\rangle$.
  So the velocity function is
  $$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\,d\tau = \mathbf{v}(0) + \mathbf{a}t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t\rangle.$$
  One can recover the position by integrating the velocity:
  $$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau)d\tau + \mathbf{r}(0).$$
  Notice the initial position is $\mathbf{r}(0) = \langle 0, 2\rangle$.

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 $\mathrm{m/s^2}$ and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0,0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g\rangle = \langle 0, -10\rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle 6\sqrt{2}, 6\sqrt{2}\rangle$.
  So the velocity function is
  $$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\,d\tau = \mathbf{v}(0) + \mathbf{a}t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t\rangle.$$
  One can recover the position by integrating the velocity:
  $$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau)d\tau + \mathbf{r}(0).$$
  Notice the initial position is $\mathbf{r}(0) = \langle 0, 2\rangle$. This integral yields:
  $$\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}(0)t + \mathbf{a}\frac{t^2}{2}$$

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(a) Find the velocity function $\mathbf{v}(t)$ and the position function $\mathbf{r}(t)$ of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point $(0,0)$ and y represents the height.

Solution:

- Acceleration due to gravity is $\mathbf{a} = \langle 0, -g\rangle = \langle 0, -10\rangle$. Initial velocity is $\mathbf{v}(0) = 12\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle 6\sqrt{2}, 6\sqrt{2}\rangle$.
  So the velocity function is
  $$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\, d\tau = \mathbf{v}(0) + \mathbf{a}t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t\rangle.$$
  One can recover the position by integrating the velocity:
  $$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau)\, d\tau + \mathbf{r}(0).$$
  Notice the initial position is $\mathbf{r}(0) = \langle 0, 2\rangle$. This integral yields:
  $$\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}(0)t + \mathbf{a}\frac{t^2}{2} = \left\langle 6\sqrt{2}t, 2 + 6\sqrt{2}t - 5t^2\right\rangle.$$

Problem 7(b) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(b) Find the speed of the ball at its highest point.

Problem 7(b) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(b) Find the speed of the ball at its highest point.

Solution:

At the highest point, the vertical component of the velocity is zero, so we only need to calculate the horizontal component which is $6\sqrt{2}$.

Problem 7(b) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(b) Find the speed of the ball at its highest point.

Solution:

At the highest point, the vertical component of the velocity is zero, so we only need to calculate the horizontal component which is $6\sqrt{2}$. Thus the speed at the highest point is $6\sqrt{2}$. $\square$

Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(c) At what time **T** does the ball reach its highest point.

Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(c) At what time **T** does the ball reach its highest point.

Solution:

When the ball reaches its highest point, the vertical component of its velocity is zero.

Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(c) At what time **T** does the ball reach its highest point.

Solution:

When the ball reaches its highest point, the vertical component of its velocity is zero.That is,

$$6\sqrt{2} - 10t = 0,$$

Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45^o to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(c) At what time **T** does the ball reach its highest point.

Solution:

When the ball reaches its highest point, the vertical component of its velocity is zero. That is,

$$6\sqrt{2} - 10t = 0,$$

so $\mathbf{T} = \frac{3\sqrt{2}}{5}$. $\square$

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1,2,3)$ and $B(4,6,5)$.

Solution:

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

Solution:

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.
- Take the point to be A and the vector to be the $\overrightarrow{AB}$.

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1,2,3)$ and $B(4,6,5)$.

Solution:

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.
- Take the point to be A and the vector to be the $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB}$$

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

Solution:

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.
- Take the point to be A and the vector to be the $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle 1, 2, 3 \rangle + t \langle 3, 4, 2 \rangle$$

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1,2,3)$ and $B(4,6,5)$.

Solution:

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.
- Take the point to be A and the vector to be the $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle 1,2,3\rangle + t\,\langle 3,4,2\rangle = \langle 1+3t, 2+4t, 3+2t\rangle,$$

where O is the origin.

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

Solution:

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.
- Take the point to be A and the vector to be the $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle 1, 2, 3\rangle + t\,\langle 3, 4, 2\rangle = \langle 1 + 3t, 2 + 4t, 3 + 2t\rangle\,,$$

  where O is the origin.
- The **parametric equations** are:

$$\begin{aligned} x &= 1 + 3t \\ y &= 2 + 4t, \qquad t \in \mathbb{R}. \\ z &= 3 + 2t \end{aligned}$$

$\square$

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$.

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part **v** of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, -1\rangle$. Hence **v** can be taken to be:

  $\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle$

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, -1\rangle$. Hence $\mathbf{v}$ can be taken to be:
$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line $\mathbf{L}$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, -1\rangle$. Hence $\mathbf{v}$ can be taken to be:
$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$
- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero.

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, -1\rangle$. Hence $\mathbf{v}$ can be taken to be:
$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$
- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero. Setting $z = 0$, we obtain:
$$x - 2y = 10$$
$$2x + y = 0.$$

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $\mathbf{v}$ can be taken to be:
$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$
- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero. Setting $z = 0$, we obtain:
$$x - 2y = 10$$
$$2x + y = 0.$$
Solving, we find that $x = 2$ and $y = -4$.

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, -1\rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$

- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero. Setting $z = 0$, we obtain:

$$x - 2y = 10$$

$$2x + y = 0.$$

  Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0\rangle$ lies on the line **L**.

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $\mathbf{v}$ can be taken to be:
$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$
- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero. Setting $z = 0$, we obtain:
$$x - 2y = 10$$
$$2x + y = 0.$$
Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0 \rangle$ lies on the line **L**.
- The **parametric equations** are:

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part **v** of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, -1\rangle$. Hence **v** can be taken to be:
$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.$$
- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero. Setting $z = 0$, we obtain:
$$x - 2y = 10$$
$$2x + y = 0.$$
Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0\rangle$ lies on the line **L**.
- The **parametric equations** are:
$$x = 2 + t$$
$$y = -4 + 3t$$
$$z = 0 + 5t = 5t.$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

Method 1

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -2, 0 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -2 \rangle$ which lie parallel to the plane.

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 1

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-2,0\rangle$ and $\overrightarrow{PR} = \langle 3,0,-2\rangle$ which lie parallel to the plane.
- Then consider the normal vector:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 1

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-2,0\rangle$ and $\overrightarrow{PR} = \langle 3,0,-2\rangle$ which lie parallel to the plane.
- Then consider the normal vector:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix}$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 1

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-2,0\rangle$ and $\overrightarrow{PR} = \langle 3,0,-2\rangle$ which lie parallel to the plane.
- Then consider the normal vector:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 1

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-2,0\rangle$ and $\overrightarrow{PR} = \langle 3,0,-2\rangle$ which lie parallel to the plane.
- Then consider the normal vector:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

- So the **equation of the plane** is given by:

$$\langle 4,4,6\rangle \cdot \langle x+1, y, z-1\rangle = 4(x+1) + 4y + 6(z-1) = 0.$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

Method 2

- The plane consists of all the points $S(x, y, z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

Method 2

- The plane consists of all the points $S(x, y, z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).
- But this happens if and only if their box product is zero.

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 2

- The plane consists of all the points $S(x,y,z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).
- But this happens if and only if their box product is zero.
- So the **equation of the plane** is:

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 2

- The plane consists of all the points $S(x,y,z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).
- But this happens if and only if their box product is zero.
- So the **equation of the plane** is:

$$\begin{vmatrix} x+1 & y & z-1 \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix}$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

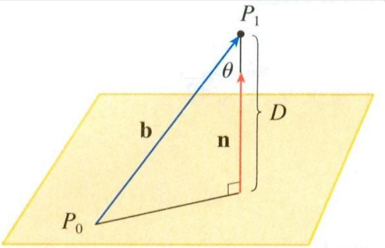
Solution:

Method 2

- The plane consists of all the points $S(x,y,z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).
- But this happens if and only if their box product is zero.
- So the **equation of the plane** is:

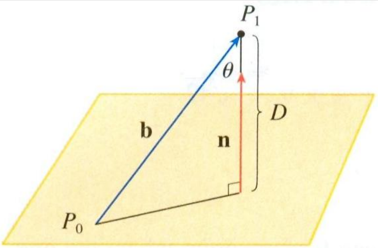
$$\begin{vmatrix} x+1 & y & z-1 \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix} = 4(x+1) + 4y + 6(z-1) = 0.$$

$\square$



Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

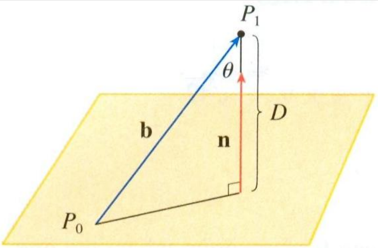


Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.

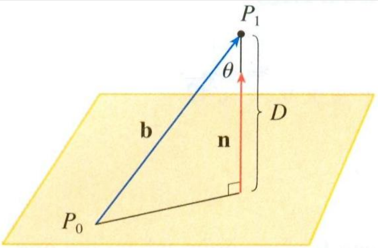


Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.

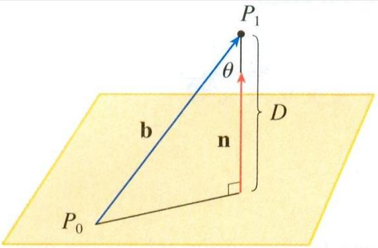


Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.
- So, the distance from $(1, 2, -1)$ to the plane is:



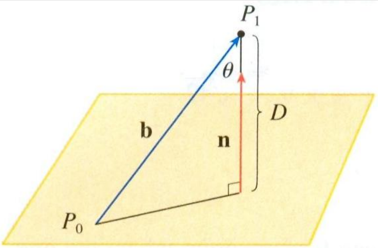
Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.
- So, the distance from $(1, 2, -1)$ to the plane is:

$$\mathbf{D} = \frac{|(2 \cdot 1) + (1 \cdot 6) + (-2 \cdot -1) - 19|}{\sqrt{2^2 + 1^2 + (-2)^2}}$$



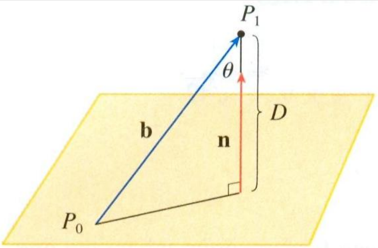
Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.
- So, the distance from $(1, 2, -1)$ to the plane is:

$$\mathbf{D} = \frac{|(2 \cdot 1) + (1 \cdot 6) + (-2 \cdot -1) - 19|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{|-9|}{\sqrt{9}}$$



Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.
- So, the distance from $(1, 2, -1)$ to the plane is:

$$\mathbf{D} = \frac{|(2 \cdot 1) + (1 \cdot 6) + (-2 \cdot -1) - 19|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{|-9|}{\sqrt{9}} = 3.$$

Problem 2(c) - Fall 2008

Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

Problem 2(c) - Fall 2008

Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

Solution:

- The line **L** in the Hint passes through $(1, 6, -1)$ and is parallel to $\mathbf{n} = \langle 2, 1, -2 \rangle$.

Problem 2(c) - Fall 2008

Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

Solution:

- The line **L** in the Hint passes through $(1, 6, -1)$ and is parallel to $\mathbf{n} = \langle 2, 1, -2 \rangle$.
- So, **L** has **parametric equations**:

$$\begin{aligned} x &= 1 + 2t \\ y &= 6 + t, \qquad\qquad t \in \mathbb{R}. \\ z &= -1 - 2t \end{aligned}$$

Problem 2(c) - Fall 2008

Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

Solution:

- The line **L** in the Hint passes through $(1, 6, -1)$ and is parallel to $\mathbf{n} = \langle 2, 1, -2 \rangle$.
- So, **L** has **parametric equations**:

$$\begin{aligned} x &= 1 + 2t \\ y &= 6 + t, \qquad\qquad t \in \mathbb{R}. \\ z &= -1 - 2t \end{aligned}$$

- **L** intersects the plane $2x + y - 2z = 19$ if and only if

$$2(1 + 2t) + (6 + t) - 2(-1 - 2t) = 19 \iff 9t = 9 \iff t = 1.$$

Problem 2(c) - Fall 2008

Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

Solution:

- The line **L** in the Hint passes through $(1, 6, -1)$ and is parallel to $\mathbf{n} = \langle 2, 1, -2 \rangle$.
- So, **L** has **parametric equations**:

$$\begin{aligned} x &= 1 + 2t \\ y &= 6 + t, \qquad\qquad t \in \mathbb{R}. \\ z &= -1 - 2t \end{aligned}$$

- **L** intersects the plane $2x + y - 2z = 19$ if and only if

$$2(1 + 2t) + (6 + t) - 2(-1 - 2t) = 19 \iff 9t = 9 \iff t = 1.$$

- Substituting $t = 1$ in the **parametric equations** of **L** gives the point $Q = (3, 7, -3)$. $\square$

Problem 3(a) - Fall 2008

Find the volume **V** of the **parallelepiped** such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Problem 3(a) - Fall 2008

Find the volume **V** of the **parallelepiped** such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The **parallelepiped** is determined by its edges

$$\overrightarrow{AB} = \langle 0, -3, -2 \rangle, \quad \overrightarrow{AC} = \langle 1, 1, -3 \rangle, \quad \overrightarrow{AD} = \langle -2, -4, -1 \rangle.$$

Problem 3(a) - Fall 2008

Find the volume **V** of the **parallelepiped** such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The **parallelepiped** is determined by its edges

$$\overrightarrow{AB} = \langle 0, -3, -2 \rangle\,, \quad \overrightarrow{AC} = \langle 1, 1, -3 \rangle\,, \quad \overrightarrow{AD} = \langle -2, -4, -1 \rangle\,.$$

Its volume can be computed as the absolute value of the box product $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})$,

Problem 3(a) - Fall 2008

Find the volume **V** of the **parallelepiped** such that the following four points $A = (3,4,0)$, $B = (3,1,-2)$, $C = (4,5,-3)$, $D = (1,0,-1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The **parallelepiped** is determined by its edges

$$\overrightarrow{AB} = \langle 0,-3,-2\rangle\,, \quad \overrightarrow{AC} = \langle 1,1,-3\rangle\,, \quad \overrightarrow{AD} = \langle -2,-4,-1\rangle\,.$$

Its volume can be computed as the absolute value of the box product $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})$, i.e.,

$$\mathbf{V} = \left\| \begin{matrix} 0 & -3 & -2 \\ 1 & 1 & -3 \\ -2 & -4 & -1 \end{matrix} \right\|$$

Problem 3(a) - Fall 2008

Find the volume $\mathbf{V}$ of the **parallelepiped** such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The **parallelepiped** is determined by its edges

$$\overrightarrow{AB} = \langle 0, -3, -2 \rangle \,, \quad \overrightarrow{AC} = \langle 1, 1, -3 \rangle \,, \quad \overrightarrow{AD} = \langle -2, -4, -1 \rangle \,.$$

Its volume can be computed as the absolute value of the box product $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})$, i.e.,

$$\mathbf{V} = \left\| \begin{matrix} 0 & -3 & -2 \\ 1 & 1 & -3 \\ -2 & -4 & -1 \end{matrix} \right\| = |3(-1-6) - 2(-4+2)|$$

Problem 3(a) - Fall 2008

Find the volume **V** of the **parallelepiped** such that the following four points $A = (3,4,0)$, $B = (3,1,-2)$, $C = (4,5,-3)$, $D = (1,0,-1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The **parallelepiped** is determined by its edges

$$\overrightarrow{AB} = \langle 0,-3,-2\rangle\,, \quad \overrightarrow{AC} = \langle 1,1,-3\rangle\,, \quad \overrightarrow{AD} = \langle -2,-4,-1\rangle\,.$$

Its volume can be computed as the absolute value of the box product $\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD})$, i.e.,

$$\mathbf{V} = \left|\begin{vmatrix} 0 & -3 & -2 \\ 1 & 1 & -3 \\ -2 & -4 & -1 \end{vmatrix}\right| = |3(-1-6)-2(-4+2)| = |-17| = 17.$$

□

Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere
$x^2 - 4x + y^2 + 4y + z^2 = 8.$

Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere
$x^2 - 4x + y^2 + 4y + z^2 = 8.$

Solution:

- Completing the square we get

$$x^2 - 4x + y^2 + 4y + z^2 = (x^2 - 4x + 4) - 4 + (y^2 + 4y + 4) - 4 + (z^2)$$

Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere
$x^2 - 4x + y^2 + 4y + z^2 = 8$.

Solution:

- Completing the square we get

$$x^2-4x+y^2+4y+z^2 = (x^2-4x+4)-4+(y^2+4y+4)-4+(z^2)$$

$$= (x-2)^2 - 4 + (y+2)^2 - 4 + z^2 = 8$$

$$\Longleftrightarrow$$

$$(x-2)^2 + (y+2)^2 + z^2 = 16.$$

Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere $x^2 - 4x + y^2 + 4y + z^2 = 8$.

Solution:

- Completing the square we get

$$x^2-4x+y^2+4y+z^2 = (x^2-4x+4)-4+(y^2+4y+4)-4+(z^2)$$

$$= (x-2)^2 - 4 + (y+2)^2 - 4 + z^2 = 8$$

$$\iff$$

$$(x-2)^2 + (y+2)^2 + z^2 = 16.$$

- This gives:

$$\textbf{Center} = (2, -2, 0) \qquad \textbf{Radius} = 4$$

$\square$

Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals
$\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
a) Find an **equation of the tangent line** to the curve at the point $(4, -4, 2)$.

Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.

a) Find an **equation of the tangent line** to the curve at the point $(4, -4, 2)$.

Solution:

- The parametrized curve passes through the point $(4, -4, 2)$ if and only if

$$t^2 = 4, \;\; -t^2 = -4, \;\; \frac{1}{2}t^2 = 2 \Longleftrightarrow t^2 = 4 \Longleftrightarrow t = \pm 2.$$

Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
a) Find an **equation of the tangent line** to the curve at the point $(4, -4, 2)$.

Solution:

- The parametrized curve passes through the point $(4, -4, 2)$ if and only if
$$t^2 = 4, \;\; -t^2 = -4, \;\; \frac{1}{2}t^2 = 2 \iff t^2 = 4 \iff t = \pm 2.$$
- Since we have that $t \geq 0$, we are left with the choice $t_0 = 2$.

Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
a) Find an **equation of the tangent line** to the curve at the point $(4, -4, 2)$.

Solution:

- The parametrized curve passes through the point $(4, -4, 2)$ if and only if
$$t^2 = 4, \;\; -t^2 = -4, \;\; \frac{1}{2}t^2 = 2 \iff t^2 = 4 \iff t = \pm 2.$$
- Since we have that $t \geq 0$, we are left with the choice $t_0 = 2$.
- The **velocity vector field** to the curve is given by
$$\mathbf{r}'(t) = \langle\, 2t, -2t, t\rangle \;\text{ hence }\; \mathbf{r}'(2) = \langle 4, -4, 2\rangle\,.$$

Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
a) Find an **equation of the tangent line** to the curve at the point $(4, -4, 2)$.

Solution:

- The parametrized curve passes through the point $(4, -4, 2)$ if and only if

$$t^2 = 4, \;\; -t^2 = -4, \;\; \frac{1}{2}t^2 = 2 \Longleftrightarrow t^2 = 4 \Longleftrightarrow t = \pm 2.$$

- Since we have that $t \geq 0$, we are left with the choice $t_0 = 2$.
- The **velocity vector field** to the curve is given by

$$\mathbf{r}'(t) = \langle\, 2t, -2t, t\rangle \;\text{ hence }\; \mathbf{r}'(2) = \langle 4, -4, 2\rangle\,.$$

- The **equation of the tangent line** in question is:

$$\begin{aligned} x &= 4 + 4t \\ y &= -4 - 4t, \quad t \geq 0. \\ z &= 2 + 2t \end{aligned}$$

$\square$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.

(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

- Since $t \geq 0$, the speed is:

$$\left|\mathbf{r}'(t)\right| = \sqrt{9t^2} \Longrightarrow \left|\mathbf{r}'(t)\right| = 3t.$$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length **L** of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

- Since $t \geq 0$, the speed is:

$$|\mathbf{r}'(t)| = \sqrt{9t^2} \Longrightarrow |\mathbf{r}'(t)| = 3t.$$

- Therefore, the length is:

$$\mathbf{L} = \int_1^4 3t\, dt$$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

- Since $t \geq 0$, the speed is:

$$\left|\mathbf{r}'(t)\right| = \sqrt{9t^2} \Longrightarrow \left|\mathbf{r}'(t)\right| = 3t.$$

- Therefore, the length is:

$$\mathbf{L} = \int_1^4 3t\,dt = \frac{3}{2}t^2\Big|_1^4$$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

- Since $t \geq 0$, the speed is:

$$\left|\mathbf{r}'(t)\right| = \sqrt{9t^2} \Longrightarrow \left|\mathbf{r}'(t)\right| = 3t.$$

- Therefore, the length is:

$$\mathbf{L} = \int_1^4 3t\,dt = \frac{3}{2}t^2\Big|_1^4 = \frac{3}{2}\cdot 16 - \frac{3}{2}\cdot 1$$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals
$\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

- Since $t \geq 0$, the speed is:

$$\left|\mathbf{r}'(t)\right| = \sqrt{9t^2} \Longrightarrow \left|\mathbf{r}'(t)\right| = 3t.$$

- Therefore, the length is:

$$\mathbf{L} = \int_1^4 3t\,dt = \frac{3}{2}t^2\Big|_1^4 = \frac{3}{2}\cdot 16 - \frac{3}{2}\cdot 1 = \frac{3}{2}\cdot 15$$

Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals $\mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ at any time $t \geq 0$.
(b) Find the length $\mathbf{L}$ of the arc traveled from time $t = 1$ to time $t = 4$.

Solution:

- The velocity field is:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle .$$

- Since $t \geq 0$, the speed is:

$$\left|\mathbf{r}'(t)\right| = \sqrt{9t^2} \Longrightarrow \left|\mathbf{r}'(t)\right| = 3t.$$

- Therefore, the length is:

$$\mathbf{L} = \int_1^4 3t\, dt = \frac{3}{2}t^2\Big|_1^4 = \frac{3}{2}\cdot 16 - \frac{3}{2}\cdot 1 = \frac{3}{2}\cdot 15 = \frac{45}{2}.$$

□

Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, 2\cos 2t, 3e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, 2\cos 2t, 3e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- One can recover the position, by integrating the velocity:

Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, 2\cos 2t, 3e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- One can recover the position, by integrating the velocity:

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) d\tau + \mathbf{r}(0)$$

Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, 2\cos 2t, 3e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- One can recover the position, by integrating the velocity:

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) d\tau + \mathbf{r}(0)$$

- Carrying out this integral yields:

$$\mathbf{r}(t) = \left\langle \, -\cos\tau\Big|_0^t, \;\; \sin 2\tau\Big|_0^t, \;\; 3e^t\Big|_0^t \right\rangle + \langle 1, 2, 0 \rangle$$

Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, 2\cos 2t, 3e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- One can recover the position, by integrating the velocity:

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) d\tau + \mathbf{r}(0)$$

- Carrying out this integral yields:

$$\mathbf{r}(t) = \left\langle -\cos\tau\Big|_0^t, \ \sin 2\tau\Big|_0^t, \ 3e^t\Big|_0^t \right\rangle + \langle 1, 2, 0 \rangle$$

$$= \left\langle 2 - \cos t, \ 2 + \sin 2t, \ 3e^t - 3 \right\rangle. \qquad \square$$

Problem 5(a) - Fall 2008

Consider the points $A(2, 1, 0)$, $B(3, 0, 2)$ and $C(0, 2, 1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Problem 5(a) - Fall 2008

Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\textbf{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|.$$

Problem 5(a) - Fall 2008

Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\textbf{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|.$$

- Since $\overrightarrow{AB} = \langle 1, -1, 2\rangle$ and $\overrightarrow{AC} = \langle -2, 1, 1\rangle$,

Problem 5(a) - Fall 2008

Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\textbf{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|.$$

- Since $\overrightarrow{AB} = \langle 1, -1, 2\rangle$ and $\overrightarrow{AC} = \langle -2, 1, 1\rangle$,

$$\textbf{Area} = \frac{1}{2}\left\|\begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ -2 & 1 & 1 \end{array}\right\| = \frac{1}{2}\left|\langle -3, -5, -1\rangle\right|$$

Problem 5(a) - Fall 2008

Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\textbf{Area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|.$$

- Since $\overrightarrow{AB} = \langle 1,-1,2\rangle$ and $\overrightarrow{AC} = \langle -2,1,1\rangle$,

$$\textbf{Area} = \frac{1}{2}\left\|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}\right\| = \frac{1}{2}\left|\langle -3,-5,-1\rangle\right| = \frac{1}{2}\sqrt{35}.$$

$\square$

Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(2, 1, 1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(2, 1, 1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $\mathbf{S}$.

Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(2,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by **S**. Then

$$\overrightarrow{O\mathbf{S}} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0,1,0 \rangle + \langle 2,2,0 \rangle = \langle 2,3,0 \rangle \, ,$$

where O is the origin.

Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(2,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by **S**. Then

$$\overrightarrow{O\mathbf{S}} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0,1,0\rangle + \langle 2,2,0\rangle = \langle 2,3,0\rangle\,,$$

where O is the origin. That is,

$$\mathbf{S} = (2,3,0).$$

□

Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point A, it suffices to find $\mathbf{n}$.

Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point A, it suffices to find $\mathbf{n}$. Note that:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \langle -1, 1, 2 \rangle.$$

Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point A, it suffices to find $\mathbf{n}$. Note that:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \langle -1, 1, 2 \rangle.$$

So the **equation of the plane** is:

$$-(x-1) + (y-2) + 2(z-3) = 0.$$

$\square$

Problem 6(b) - Spring 2008

Find the area of the triangle Δ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Problem 6(b) - Spring 2008

Find the area of the triangle Δ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$.

Problem 6(b) - Spring 2008

Find the area of the triangle $\boldsymbol{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2.

Problem 6(b) - Spring 2008

Find the area of the triangle $\boldsymbol{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2}$$

Problem 6(b) - Spring 2008

Find the area of the triangle $\boldsymbol{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $\;B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2}\left\| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{array} \right\|$$

Problem 6(b) - Spring 2008

Find the area of the triangle $\boldsymbol{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2}\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2}|\langle -1, 1, 2 \rangle|$$

Problem 6(b) - Spring 2008

Find the area of the triangle $\boldsymbol{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2}\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2}|\langle -1, 1, 2\rangle| = \frac{1}{2}\sqrt{1+1+4}$$

Problem 6(b) - Spring 2008

Find the area of the triangle $\mathbf{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
Hint: the area of this triangle is related to the area of a certain parallelogram

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\mathbf{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\mathbf{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2}\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2}|\langle -1, 1, 2\rangle| = \frac{1}{2}\sqrt{1+1+4} = \frac{1}{2}\sqrt{6}.$$

□

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Solution:

- The vector part of the line **L** is the normal vector $\mathbf{n} = \langle 3, -1, 5 \rangle$ to the plane.

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Solution:

- The vector part of the line **L** is the normal vector $\mathbf{n} = \langle 3, -1, 5\rangle$ to the plane.
- The **vector equation** of **L** is:

$$\mathbf{r}(t) = \langle 2, 4, 1\rangle + t\mathbf{n}$$

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Solution:

- The vector part of the line **L** is the normal vector $\mathbf{n} = \langle 3, -1, 5 \rangle$ to the plane.
- The **vector equation** of **L** is:

$$\mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}$$

$$= \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle$$

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Solution:

- The vector part of the line $\mathbf{L}$ is the normal vector $\mathbf{n} = \langle 3, -1, 5 \rangle$ to the plane.
- The **vector equation** of $\mathbf{L}$ is:

$$\mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}$$

$$= \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle = \langle 2 + 3t, 4 - t, 1 + 5t \rangle.$$

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Solution:

- The vector part of the line **L** is the normal vector $\mathbf{n} = \langle 3, -1, 5 \rangle$ to the plane.
- The **vector equation** of **L** is:

$$\mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}$$

$$= \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle = \langle 2 + 3t, 4 - t, 1 + 5t \rangle.$$

- The **parametric equations** are:

$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$

□

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have **L** has **parametric equations**:

$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have **L** has **parametric equations**:

$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$

- Plug these t-values into equation of plane and solve for t:

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t\rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have $\mathbf{L}$ has **parametric equations**:
$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$
- Plug these t-values into equation of plane and solve for t:

$$3(2+3t) - (4-t) + 5(1+5t) = 77,$$

$$6 + 9t - 4 + t + 5 + 25t = 77,$$

$$35t = 70;$$

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t\rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have $\mathbf{L}$ has **parametric equations**:

$$\begin{aligned} x &= 2+3t \\ y &= 4-t \\ z &= 1+5t. \end{aligned}$$

- Plug these t-values into equation of plane and solve for t:

$$3(2+3t) - (4-t) + 5(1+5t) = 77,$$

$$6 + 9t - 4 + t + 5 + 25t = 77,$$

$$35t = 70; \quad \implies t = 2.$$

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have $\mathbf{L}$ has **parametric equations**:

$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$

- Plug these t-values into equation of plane and solve for t:

$$3(2+3t) - (4-t) + 5(1+5t) = 77,$$

$$6 + 9t - 4 + t + 5 + 25t = 77,$$

$$35t = 70; \quad \implies t = 2.$$

So $\mathbf{L}$ intersects the plane at time $t = 2$.

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t\rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have **L** has **parametric equations**:

$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$

- Plug these t-values into equation of plane and solve for t:

$$3(2+3t) - (4-t) + 5(1+5t) = 77,$$

$$6 + 9t - 4 + t + 5 + 25t = 77,$$

$$35t = 70; \quad \Longrightarrow t = 2.$$

  So **L** intersects the plane at time $t = 2$.

- At $t = 2$, the **parametric equations** give the point:

$$\langle 2 + 3\cdot 2, 4 - 2, 1 + 5\cdot 2\rangle = \langle 8, 2, 11\rangle.$$

$\square$

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \ \ 0 \leq t \leq 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Solution:

- Rewriting $\mathbf{u}$, we get:

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle$$

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \;\; 0 \leq t \leq 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Solution:

- Rewriting $\mathbf{u}$, we get:

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle.$$

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \;\; 0 \le t \le 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Solution:

- Rewriting $\mathbf{u}$, we get:

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle.$$

- Since $\langle \cos t, \sin t \rangle$ is the **parametrization** of the circle of radius 1 centered at the origin,

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \;\; 0 \leq t \leq 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Solution:

- Rewriting $\mathbf{u}$, we get:

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle.$$

- Since $\langle \cos t, \sin t \rangle$ is the **parametrization** of the circle of radius 1 centered at the origin, then $\mathbf{u}$ is a circle of radius $\mathbf{r} = 1$ centered at $(1, 0)$.

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \;\; 0 \le t \le 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Solution:

- Rewriting **u**, we get:

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle.$$

- Since $\langle \cos t, \sin t \rangle$ is the **parametrization** of the circle of radius 1 centered at the origin, then **u** is a circle of radius $\mathbf{r} = 1$ centered at $(1, 0)$.
- So the answer is:

$$(x - 1)^2 + (y - 0)^2 = 1^2$$

Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi.$$

Find a single equation for the curve in terms of x and y by eliminating t.

Solution:

- Rewriting **u**, we get:

$$\mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle.$$

- Since $\langle \cos t, \sin t \rangle$ is the **parametrization** of the circle of radius 1 centered at the origin, then **u** is a circle of radius $\mathbf{r} = 1$ centered at $(1, 0)$.
- So the answer is:

$$(x - 1)^2 + (y - 0)^2 = 1^2$$

or

$$(x - 1)^2 + y^2 = 1.$$

□

Problem 8(b) - Spring 2008

Consider the *space* curve given by the graph of the vector function

$$\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle, \quad 0 \leq t \leq 2\pi.$$

Sketch the curve and indicate the direction of increasing t in your graph.

Problem 8(b) - Spring 2008

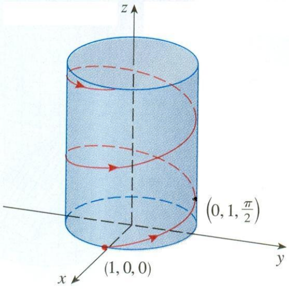
Consider the *space* curve given by the graph of the vector function

$$\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle, \quad 0 \leq t \leq 2\pi.$$

Sketch the curve and indicate the direction of increasing t in your graph.

Solution:

The sketch would be the following one translated 1 unit along the x-axis.



Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle$$

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $\mathbf{T}$ tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $\mathbf{T}$ in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$
- At $t = \frac{\pi}{3}$,
$$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3} \rangle$$

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t\rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1+\cos t)', (\sin t)', 1\rangle = \langle -\sin t, \cos t, 1\rangle.$$
- At $t = \frac{\pi}{3}$,
$$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3}\rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3}\rangle,$$

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
  $$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$
- At $t = \frac{\pi}{3}$,
  $$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,$$
  $$\mathbf{r}'(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, \cos\frac{\pi}{3}, 1 \rangle$$

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$
- At $t = \frac{\pi}{3}$,
$$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,$$
$$\mathbf{r}'(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, \cos\frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle.$$

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line **T** tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch **T** in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$
- At $t = \frac{\pi}{3}$,
$$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,$$
$$\mathbf{r}'(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, \cos\frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle.$$
- The vector part of tangent line **T** is $\mathbf{r}'(\frac{\pi}{3})$ and a point on line is $\mathbf{r}(\frac{\pi}{3})$.

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $\mathbf{T}$ tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t\rangle$ at $t = \pi/3$, and sketch $\mathbf{T}$ in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1\rangle = \langle -\sin t, \cos t, 1\rangle.$$
- At $t = \frac{\pi}{3}$,
$$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3}\rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3}\rangle,$$
$$\mathbf{r}'(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, \cos\frac{\pi}{3}, 1\rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\rangle.$$
- The vector part of tangent line $\mathbf{T}$ is $\mathbf{r}'(\frac{\pi}{3})$ and a point on line is $\mathbf{r}(\frac{\pi}{3})$.
- The **vector equation** is: $\mathbf{T}(t) = \mathbf{r}(\frac{\pi}{3}) + t\mathbf{r}'(\frac{\pi}{3})$.

Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $\mathbf{T}$ tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $\mathbf{T}$ in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
$$\mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$
- At $t = \frac{\pi}{3}$,
$$\mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,$$
$$\mathbf{r}'(\frac{\pi}{3}) = \langle -\sin\frac{\pi}{3}, \cos\frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle.$$
- The vector part of tangent line $\mathbf{T}$ is $\mathbf{r}'(\frac{\pi}{3})$ and a point on line is $\mathbf{r}(\frac{\pi}{3})$.
- The **vector equation** is: $\mathbf{T}(t) = \mathbf{r}(\frac{\pi}{3}) + t\mathbf{r}'(\frac{\pi}{3})$.
- The **parametric equations** are:
$$\begin{aligned} x &= \tfrac{3}{2} - \tfrac{\sqrt{3}}{2}t \\ y &= \tfrac{\sqrt{3}}{2} + \tfrac{1}{2}t \\ z &= \tfrac{\pi}{3} + t. \end{aligned}$$

$\square$

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Determine $\mathbf{r}(t)$ for all t.

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Determine $\mathbf{r}(t)$ for all t.

Solution:

- Find $\mathbf{r}(t)$ by integration:

$$\mathbf{r}(t) = \int \mathbf{r}'(t)\, dt = \int \langle -\sin 2t, \cos 2t, 0\rangle\, dt$$

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Determine $\mathbf{r}(t)$ for all t.

Solution:

- Find $\mathbf{r}(t)$ by integration:

$$\mathbf{r}(t) = \int \mathbf{r}'(t)\, dt = \int \langle -\sin 2t, \cos 2t, 0\rangle\, dt$$

$$= \langle \frac{1}{2}\cos(2t) + x_0, \frac{1}{2}\sin(2t) + y_0, z_0\rangle.$$

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Determine $\mathbf{r}(t)$ for all t.

Solution:

- Find $\mathbf{r}(t)$ by integration:

$$\mathbf{r}(t) = \int \mathbf{r}'(t)\, dt = \int \langle -\sin 2t, \cos 2t, 0\rangle\, dt$$

$$= \langle \frac{1}{2}\cos(2t) + x_0, \frac{1}{2}\sin(2t) + y_0, z_0\rangle.$$

- Now solve for the point (x_0, y_0, z_0) using $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$:

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Determine $\mathbf{r}(t)$ for all t.

Solution:

- Find $\mathbf{r}(t)$ by integration:

$$\mathbf{r}(t) = \int \mathbf{r}'(t)\, dt = \int \langle -\sin 2t, \cos 2t, 0\rangle\, dt$$

$$= \langle \frac{1}{2}\cos(2t) + x_0, \frac{1}{2}\sin(2t) + y_0, z_0\rangle.$$

- Now solve for the point (x_0, y_0, z_0) using $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$: $(\frac{1}{2}\cos(0) + x_0, \frac{1}{2}\sin(0) + y_0, z_0) = (\frac{1}{2} + x_0, y_0, z_0) = (\frac{1}{2}, 0, 1)$.

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Determine $\mathbf{r}(t)$ for all t.

Solution:

- Find $\mathbf{r}(t)$ by integration:

$$\mathbf{r}(t) = \int \mathbf{r}'(t)\, dt = \int \langle -\sin 2t, \cos 2t, 0\rangle\, dt$$

$$= \langle \frac{1}{2}\cos(2t) + x_0, \frac{1}{2}\sin(2t) + y_0, z_0\rangle.$$

- Now solve for the point (x_0, y_0, z_0) using $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$: $(\frac{1}{2}\cos(0) + x_0, \frac{1}{2}\sin(0) + y_0, z_0) = (\frac{1}{2} + x_0, y_0, z_0) = (\frac{1}{2}, 0, 1)$. So $x_0 = 0,\ y_0 = 0,\ z_0 = 1$.

Problem 9(a) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Determine $\mathbf{r}(t)$ for all t.

Solution:

- Find $\mathbf{r}(t)$ by integration:
$$\mathbf{r}(t) = \int \mathbf{r}'(t)\, dt = \int \langle -\sin 2t, \cos 2t, 0\rangle\, dt$$
$$= \langle \frac{1}{2}\cos(2t) + x_0, \frac{1}{2}\sin(2t) + y_0, z_0\rangle.$$
- Now solve for the point (x_0, y_0, z_0) using $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$:
$(\frac{1}{2}\cos(0) + x_0, \frac{1}{2}\sin(0) + y_0, z_0) = (\frac{1}{2} + x_0, y_0, z_0) = (\frac{1}{2}, 0, 1)$.
So $x_0 = 0,\ y_0 = 0,\ z_0 = 1$.
- Thus,
$$\mathbf{r}(t) = \langle \frac{1}{2}\cos 2t, \frac{1}{2}\sin 2t, 1\rangle.$$

$\square$

Problem 9(b) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Show that $\mathbf{r}(t)$ is **orthogonal** to $\mathbf{r}'(t)$ for all t.

Problem 9(b) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Show that $\mathbf{r}(t)$ is **orthogonal** to $\mathbf{r}'(t)$ for all t.

Solution:

- By part (a),

$$\mathbf{r}(t) = \langle \frac{1}{2}\cos 2t, \frac{1}{2}\sin 2t, 1\rangle,$$

Problem 9(b) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Show that $\mathbf{r}(t)$ is **orthogonal** to $\mathbf{r}'(t)$ for all t.

Solution:

- By part (a),
$$\mathbf{r}(t) = \langle \frac{1}{2}\cos 2t, \frac{1}{2}\sin 2t, 1 \rangle,$$
- Taking dot products, we get:

$$\mathbf{r}(t) \cdot \mathbf{r}'(t) = -\frac{1}{2}\cos 2t \sin 2t + \frac{1}{2}\sin 2t \cos 2t + 0 = 0.$$

Problem 9(b) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Show that $\mathbf{r}(t)$ is **orthogonal** to $\mathbf{r}'(t)$ for all t.

Solution:

- By part (a),
$$\mathbf{r}(t) = \langle \frac{1}{2}\cos 2t, \frac{1}{2}\sin 2t, 1\rangle,$$
- Taking dot products, we get:
$$\mathbf{r}(t)\cdot\mathbf{r}'(t) = -\frac{1}{2}\cos 2t \sin 2t + \frac{1}{2}\sin 2t \cos 2t + 0 = 0.$$
- Since the dot product is zero, then for each t, $\mathbf{r}(t)$ is **orthogonal** to $\mathbf{r}'(t)$.

$\square$

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \leq t \leq 1$.

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \leq t \leq 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$\mathbf{L} = \int_0^1 |\mathbf{r}'(t)|\, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0\rangle|\ dt$$

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$\mathbf{L} = \int_0^1 |\mathbf{r}'(t)|\, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0\rangle|\ dt$$

$$= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t}\ dt$$

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0,1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$\mathbf{L} = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0\rangle|\; dt$$

$$= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t}\; dt = \int_0^1 |1|\,dt$$

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$\mathbf{L} = \int_0^1 |\mathbf{r}'(t)|\, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0\rangle \,|\ \ dt$$

$$= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t}\ dt = \int_0^1 |1|\, dt = \ t\Big|_0^1$$

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$\mathbf{L} = \int_0^1 |\mathbf{r}'(t)|\, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0\rangle|\ dt$$

$$= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t}\ dt = \int_0^1 |1|\, dt = \left. t\right|_0^1 = 1.$$

Problem 9(c) - Spring 2008

Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ on the interval $0 \le t \le 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1\rangle$. Find the arclength $\mathbf{L}$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \le t \le 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$\mathbf{L} = \int_0^1 |\mathbf{r}'(t)|\, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0\rangle|\ dt$$

$$= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t}\ dt = \int_0^1 |1|\, dt = \ t\Big|_0^1 = 1.$$

- Thus

$$\mathbf{L} = 1.$$

$\square$

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time t is $\mathbf{r}'(t)$ and the speed $\mathbf{s}(t)$ is its length $|\mathbf{r}'(t)|$.

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time t is $\mathbf{r}'(t)$ and the speed $\mathbf{s}(t)$ is its length $|\mathbf{r}'(t)|$.
- Calculating with $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle$:

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time t is $\mathbf{r}'(t)$ and the speed $\mathbf{s}(t)$ is its length $|\mathbf{r}'(t)|$.
- Calculating with $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle$:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,$$

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time t is $\mathbf{r}'(t)$ and the speed $\mathbf{s}(t)$ is its length $|\mathbf{r}'(t)|$.
- Calculating with $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle$:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,$$

$$\mathbf{s}(t) = |\mathbf{r}'(t)|$$

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time t is $\mathbf{r}'(t)$ and the speed $\mathbf{s}(t)$ is its length $|\mathbf{r}'(t)|$.
- Calculating with $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3\rangle$:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2\rangle,$$

$$\mathbf{s}(t) = |\mathbf{r}'(t)| = \sqrt{2^2 + (2t)^2 + (-t^2)^2}$$

Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
Find the speed $\mathbf{s}(t)$ and the velocity $\mathbf{v}(t)$ of the object at time t.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time t is $\mathbf{r}'(t)$ and the speed $\mathbf{s}(t)$ is its length $|\mathbf{r}'(t)|$.
- Calculating with $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle$:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,$$

$$\mathbf{s}(t) = |\mathbf{r}'(t)| = \sqrt{2^2 + (2t)^2 + (-t^2)^2} = \sqrt{4 + 4t^2 + t^4}.$$

□

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle$, show that the paths of the two objects **intersect** at a common point P.

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle$, show that the paths of the two objects **intersect** at a common point P.

Solution:

- Note that $\mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle$ and $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle$.

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1\rangle + s\langle 2, -1, -5\rangle$, show that the paths of the two objects **intersect** at a common point P.

Solution:

- Note that $\mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s\rangle$ and $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3\rangle$.
- Setting the x and y-coordinates of $\mathbf{w}(s)$ and $\mathbf{r}(t)$ equal, we obtain:
$$x = 2t = 2 + 2s \Longrightarrow t = s + 1$$

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1\rangle + s\langle 2, -1, -5\rangle$, show that the paths of the two objects **intersect** at a common point P.

Solution:

- Note that $\mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s\rangle$ and $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3\rangle$.
- Setting the x and y-coordinates of $\mathbf{w}(s)$ and $\mathbf{r}(t)$ equal, we obtain:

$$x = 2t = 2 + 2s \Longrightarrow t = s + 1$$

$$y = t^2 - 6 = 5 - s \Longrightarrow (s+1)^2 - 6 = s^2 + 2s - 5 = 5 - s$$

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1\rangle + s\langle 2, -1, -5\rangle$, show that the paths of the two objects **intersect** at a common point P.

Solution:

- Note that $\mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s\rangle$ and $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3\rangle$.
- Setting the x and y-coordinates of $\mathbf{w}(s)$ and $\mathbf{r}(t)$ equal, we obtain:

$$x = 2t = 2 + 2s \Longrightarrow t = s + 1$$

$$y = t^2 - 6 = 5 - s \Longrightarrow (s+1)^2 - 6 = s^2 + 2s - 5 = 5 - s$$

$$\Longrightarrow s^2 + 3s - 10 = 0 \Longrightarrow (s+5)(s-2) = 0.$$

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1\rangle + s\langle 2, -1, -5\rangle$, show that the paths of the two objects **intersect** at a common point P.

Solution:

- Note that $\mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s\rangle$ and $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3\rangle$.
- Setting the x and y-coordinates of $\mathbf{w}(s)$ and $\mathbf{r}(t)$ equal, we obtain:

$$x = 2t = 2 + 2s \Longrightarrow t = s + 1$$

$$y = t^2 - 6 = 5 - s \Longrightarrow (s+1)^2 - 6 = s^2 + 2s - 5 = 5 - s$$

$$\Longrightarrow s^2 + 3s - 10 = 0 \Longrightarrow (s+5)(s-2) = 0.$$

- So, ($s = 2$ and $t = 3$) or ($s = -5$ and $t = -4$).

Problem 10(b) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function $\mathbf{w}(s) = \langle 2, 5, 1\rangle + s\langle 2, -1, -5\rangle$, show that the paths of the two objects **intersect** at a common point P.

Solution:

- Note that $\mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s\rangle$ and $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3\rangle$.
- Setting the x and y-coordinates of $\mathbf{w}(s)$ and $\mathbf{r}(t)$ equal, we obtain:

$$x = 2t = 2 + 2s \Longrightarrow t = s + 1$$

$$y = t^2 - 6 = 5 - s \Longrightarrow (s+1)^2 - 6 = s^2 + 2s - 5 = 5 - s$$

$$\Longrightarrow s^2 + 3s - 10 = 0 \Longrightarrow (s+5)(s-2) = 0.$$

- So, $(s = 2$ and $t = 3)$ or $(s = -5$ and $t = -4)$.
- Since

$$\mathbf{r}(3) = \langle 6, 3, -9\rangle = \mathbf{w}(2),$$

  the paths **intersect** at $P = (6, 3, -9)$. $\square$

Problem 10(c) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
If $s = t$ in part **(b)**, (i.e. the position of the second object is $\mathbf{w}(t)$ when the first object is at position $\mathbf{r}(t)$), do the two objects ever collide?

Problem 10(c) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
If $s = t$ in part **(b)**, (i.e. the position of the second object is $\mathbf{w}(t)$ when the first object is at position $\mathbf{r}(t)$), do the two objects ever collide?

Solution:

- Set $t = s$ in part **(b)**.

Problem 10(c) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
If $s = t$ in part **(b)**, (i.e. the position of the second object is $\mathbf{w}(t)$ when the first object is at position $\mathbf{r}(t)$), do the two objects ever collide?

Solution:

- Set $t = s$ in part **(b)**.
- Then the x-coordinate of $\mathbf{r}(t)$ is $2t$ and the x-coordinate of $\mathbf{w}(t) = \langle 2 + 2t, 5 - t, 1 - 5t \rangle$ is $2 + 2t$, and $2t \neq 2 + 2t$ for all t.

Problem 10(c) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet),
If $s = t$ in part **(b)**, (i.e. the position of the second object is $\mathbf{w}(t)$ when the first object is at position $\mathbf{r}(t)$), do the two objects ever collide?

Solution:

- Set $t = s$ in part **(b)**.
- Then the x-coordinate of $\mathbf{r}(t)$ is $2t$ and the x-coordinate of $\mathbf{w}(t) = \langle 2 + 2t, 5 - t, 1 - 5t \rangle$ is $2 + 2t$, and $2t \neq 2 + 2t$ for all t.
- Since $\mathbf{r}(t)$ and $\mathbf{w}(t)$ have different x-coordinates for all values of t, then they **never collide**.

$\square$

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7,6,4)$ and $B(4,6,5)$.

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7, 6, 4)$ and $B(4, 6, 5)$.

Solution:

- A vector parallel to the line **L** is:

$$\mathbf{v} = \overrightarrow{AB} = \langle 4 - 7, 6 - 6, 5 - 4, \rangle = \langle -3, 0, 1 \rangle.$$

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7,6,4)$ and $B(4,6,5)$.

Solution:

- A vector parallel to the line **L** is:

$$\mathbf{v} = \overrightarrow{AB} = \langle 4-7, 6-6, 5-4, \rangle = \langle -3, 0, 1 \rangle.$$

- A point on the line is $A(7,6,4)$.

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7, 6, 4)$ and $B(4, 6, 5)$.

Solution:

- A vector parallel to the line **L** is:

$$\mathbf{v} = \overrightarrow{AB} = \langle 4 - 7, 6 - 6, 5 - 4, \rangle = \langle -3, 0, 1 \rangle.$$

- A point on the line is $A(7, 6, 4)$.
- Therefore **parametric equations** for the line **L** are:

$$\begin{aligned} x &= 7 - 3t \\ y &= 6 \\ z &= 4 + t. \end{aligned}$$

$\square$

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

  $$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle$$

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector **v** parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \mathbf{k}$$

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector **v** parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \mathbf{k} = \langle 1, 3, 5 \rangle.$$

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \mathbf{k} = \langle 1, 3, 5 \rangle.$$

- A point on **L** is any (x_0, y_0, z_0) that satisfies **both** of the plane equations.

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \mathbf{k} = \langle 1, 3, 5 \rangle.$$

- A point on **L** is any (x_0, y_0, z_0) that satisfies **both** of the plane equations. Setting $z = 0$, we obtain the equations $x - 2y = 5$ and $2x + y = 0$ and find such a point $(1, -2, 0)$.

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}\mathbf{k} = \langle 1, 3, 5\rangle.$$

- A point on **L** is any (x_0, y_0, z_0) that satisfies **both** of the plane equations. Setting $z = 0$, we obtain the equations $x - 2y = 5$ and $2x + y = 0$ and find such a point $(1, -2, 0)$.
- Therefore **parametric equations** for **L** are:

$$x = 1 + t$$
$$y = -2 + 3t$$
$$z = 5t. \qquad \square$$

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane.

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1,0,2)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2,-2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,0,-3\rangle$.

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2, -2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -3 \rangle$.
- So the normal vector is

$$\mathbf{n} = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} =$$

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1,0,2)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2,-2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,0,-3\rangle$.
- So the normal vector is

$$\mathbf{n} = \langle 2,-2,-1\rangle \times \langle 3,0,-3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} =$$

$$\begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix}\mathbf{k} = \langle 6,3,6\rangle.$$

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1,0,2)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2,-2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,0,-3\rangle$.
- So the normal vector is

$$\mathbf{n} = \langle 2,-2,-1\rangle \times \langle 3,0,-3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} =$$

$$\begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} \mathbf{k} = \langle 6,3,6\rangle.$$

- A point on the plane is $P(-1,0,2)$.

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1,0,2)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2,-2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,0,-3\rangle$.
- So the normal vector is

$$\mathbf{n} = \langle 2,-2,-1\rangle \times \langle 3,0,-3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} =$$

$$\begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} \mathbf{k} = \langle 6,3,6\rangle.$$

- A point on the plane is $P(-1,0,2)$. Therefore,

$$6(x-(-1)) + 3(y-0) + 6(z-2) = 0,$$

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2, -2, -1\rangle$ and $\overrightarrow{PR} = \langle 3, 0, -3\rangle$.
- So the normal vector is

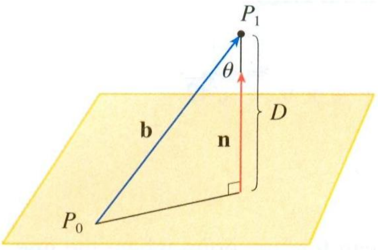
$$\mathbf{n} = \langle 2, -2, -1\rangle \times \langle 3, 0, -3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} =$$

$$\begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} \mathbf{k} = \langle 6, 3, 6\rangle.$$

- A point on the plane is $P(-1, 0, 2)$. Therefore,

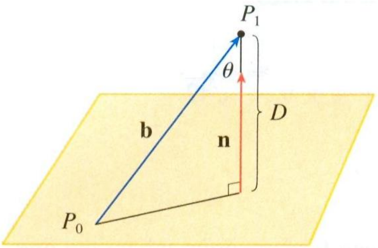
$$6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0,$$

or simplified, $$6x + 3y + 6z - 6 = 0.$$ $\square$



Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

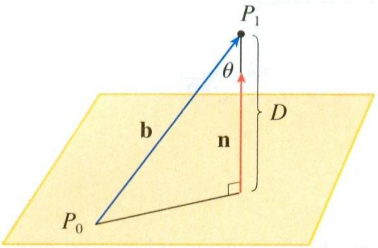


Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane.

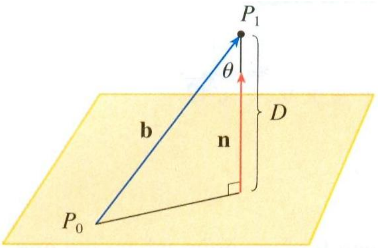


Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$.

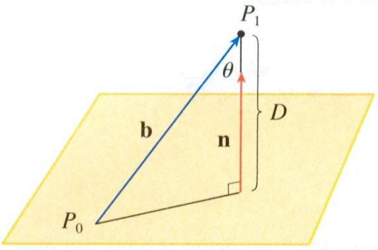


Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance **D** from $(1, 0, -1)$ to the plane is equal to:



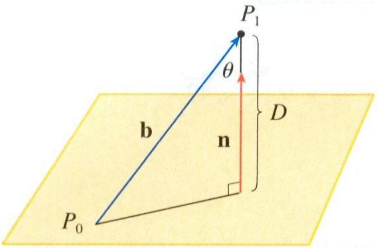
Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2\rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1\rangle$. The distance **D** from $(1, 0, -1)$ to the plane is equal to:

$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| =$$



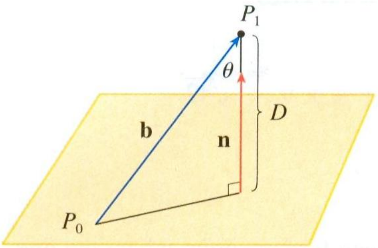
Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance **D** from $(1, 0, -1)$ to the plane is equal to:

$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$



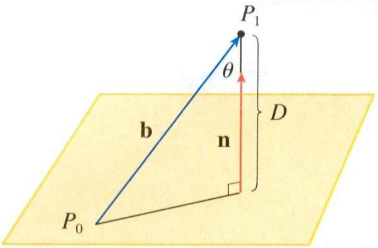
Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance **D** from $(1, 0, -1)$ to the plane is equal to:

$$|\mathbf{comp_n\ b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 1, -1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle|$$



Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2\rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1\rangle$. The distance **D** from $(1, 0, -1)$ to the plane is equal to:

$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| = \left|\mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}\right| = |\langle 1, -1, -1\rangle \cdot \frac{1}{3}\langle 2, 1, -2\rangle| = 1.$$

□

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$;

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point P in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point P in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation: $$2(1 + 2t) + (t) - 2(-1 - 2t) = 1.$$

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point P in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation:
$$2(1 + 2t) + (t) - 2(-1 - 2t) = 1.$$
Simplifying and solving for t,
$$9t + 4 = 1 \Longrightarrow t = -\frac{1}{3}.$$

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point P in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation:
$$2(1 + 2t) + (t) - 2(-1 - 2t) = 1.$$
Simplifying and solving for t,
$$9t + 4 = 1 \implies t = -\frac{1}{3}.$$
- Plugging this t-value into the **parametric equations**, we get the coordinates of the point of intersection: $x = 1 + 2(-\frac{1}{3}) = \frac{1}{3}$, $y = -\frac{1}{3}$, $z = -1 - 2(-\frac{1}{3}) = -\frac{1}{3}$.

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point P in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation:
$$2(1 + 2t) + (t) - 2(-1 - 2t) = 1.$$
Simplifying and solving for t,
$$9t + 4 = 1 \Longrightarrow t = -\frac{1}{3}.$$
- Plugging this t-value into the **parametric equations**, we get the coordinates of the point of intersection: $x = 1 + 2(-\frac{1}{3}) = \frac{1}{3}$, $y = -\frac{1}{3}$, $z = -1 - 2(-\frac{1}{3}) = -\frac{1}{3}$.
- So the point on the plane closest to $(1, 0, -1)$ is $P = (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$.

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle,$
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle$, $\mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle$, where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1,0,2)$.

Solution:

- The point $(1,0,2)$ corresponds to the t-value $t=1$ for $\mathbf{r}_1$ and s-value $s=1$ for $\mathbf{r}_2$.

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle$, $\mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle$,
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1, 0, 2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle,$
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1, 0, 2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t = 1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle$

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle,$
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1, 0, 2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t = 1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle$.

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle,$
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1, 0, 2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t = 1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle$.
- $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 4s \rangle$ is the tangent vector to $\mathbf{r}_2(s)$.

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle$, $\mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle$,
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1, 0, 2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t = 1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle$.
- $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 4s \rangle$ is the tangent vector to $\mathbf{r}_2(s)$.
- At $s = 1$, $\mathbf{r}_2'(1) = \langle \frac{1}{1}, 2(1)-2, 4(1) \rangle$

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle,$
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1,0,2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t = 1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle$.
- $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 4s \rangle$ is the tangent vector to $\mathbf{r}_2(s)$.
- At $s = 1$, $\quad \mathbf{r}_2'(1) = \langle \frac{1}{1}, 2(1)-2, 4(1) \rangle = \langle 1, 0, 4 \rangle$.

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle,$
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1, 0, 2)$.

Solution:

- The point $(1, 0, 2)$ corresponds to the t-value $t = 1$ for $\mathbf{r}_1$ and s-value $s = 1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t = 1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle$.
- $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 4s \rangle$ is the tangent vector to $\mathbf{r}_2(s)$.
- At $s = 1$, $\mathbf{r}_2'(1) = \langle \frac{1}{1}, 2(1)-2, 4(1) \rangle = \langle 1, 0, 4 \rangle$.
- Therefore,

$$\cos(\theta) = \frac{\langle 0, 2, 8 \rangle \cdot \langle 1, 0, 4 \rangle}{|\langle 0, 2, 8 \rangle||\langle 1, 0, 4 \rangle|}$$

Problem 13(a) - Spring 2007

Consider the two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, 2t^4\rangle$, $\mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2\rangle$,
where t and s are two independent real parameters. Find the cosine of the angle θ between the tangent vectors of the two curves at the intersection point $(1,0,2)$.

Solution:

- The point $(1,0,2)$ corresponds to the t-value $t=1$ for $\mathbf{r}_1$ and s-value $s=1$ for $\mathbf{r}_2$.
- $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3\rangle$ is the tangent vector to $\mathbf{r}_1(t)$.
- At $t=1$, $\mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3)\rangle = \langle 0,2,8\rangle$.
- $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 4s\rangle$ is the tangent vector to $\mathbf{r}_2(s)$.
- At $s=1$, $\mathbf{r}_2'(1) = \langle \frac{1}{1}, 2(1)-2, 4(1)\rangle = \langle 1,0,4\rangle$.
- Therefore,

$$\cos(\theta) = \frac{\langle 0,2,8\rangle \cdot \langle 1,0,4\rangle}{|\langle 0,2,8\rangle||\langle 1,0,4\rangle|} = \frac{32}{\sqrt{68}\sqrt{17}}.$$

$\square$

Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

$$x^2 + y^2 + 2y + z^2 + 4z = 20.$$

Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

$$x^2 + y^2 + 2y + z^2 + 4z = 20.$$

Solution:

- Completing the square in the y and z variables, we get

$$x^2 + (y^2 + 2y + 1) + (z^2 + 4z + 4) = 20 + 1 + 4.$$

Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

$$x^2 + y^2 + 2y + z^2 + 4z = 20.$$

Solution:

- Completing the square in the y and z variables, we get

$$x^2 + (y^2 + 2y + 1) + (z^2 + 4z + 4) = 20 + 1 + 4.$$

- Rewriting, we have

$$x^2 + (y + 1)^2 + (z + 2)^2 = 25 = 5^2.$$

Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

$$x^2 + y^2 + 2y + z^2 + 4z = 20.$$

Solution:

- Completing the square in the y and z variables, we get

$$x^2 + (y^2 + 2y + 1) + (z^2 + 4z + 4) = 20 + 1 + 4.$$

- Rewriting, we have

$$x^2 + (y + 1)^2 + (z + 2)^2 = 25 = 5^2.$$

- Hence, the **center** is $\mathbf{C} = (0, -1, -2)$ and the **radius** is $r = 5$.

$\square$

Problem 14(a) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
At the time $t = 4$, this particle is at the point $(0, 5, 4)$. Find an **equation of the tangent line T** to the position curve $\mathbf{r}(t)$at the time $t = 4$.

Problem 14(a) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
At the time $t = 4$, this particle is at the point $(0, 5, 4)$. Find an **equation of the tangent line T** to the position curve $\mathbf{r}(t)$at the time $t = 4$.

Solution:

- This line goes through the point $(0, 5, 4)$ and has vector part parallel to the tangent vector $\mathbf{v}(4) = \langle 8, -8, 4 \rangle$.

Problem 14(a) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
At the time $t = 4$, this particle is at the point $(0, 5, 4)$. Find an **equation of the tangent line T** to the position curve $\mathbf{r}(t)$at the time $t = 4$.

Solution:

- This line goes through the point $(0, 5, 4)$ and has vector part parallel to the tangent vector $\mathbf{v}(4) = \langle 8, -8, 4\rangle$.
- The **vector equation** is: $\mathbf{T}(t) = \langle 0, 5, 4\rangle + t\langle 8, -8, 4\rangle$

Problem 14(a) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
At the time $t = 4$, this particle is at the point $(0, 5, 4)$. Find an **equation of the tangent line T** to the position curve $\mathbf{r}(t)$at the time $t = 4$.

Solution:

- This line goes through the point $(0, 5, 4)$ and has vector part parallel to the tangent vector $\mathbf{v}(4) = \langle 8, -8, 4\rangle$.
- The **vector equation** is: $\mathbf{T}(t) = \langle 0, 5, 4\rangle + t\langle 8, -8, 4\rangle$
- So the line **T** has the **parametric equations**:

$$x = 8t$$
$$y = 5 - 8t$$
$$z = 4 + 4t.$$

$\square$

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$\mathbf{L} = \int_2^4 |\mathbf{v}(t)|\, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2}\, dt$$

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$\mathbf{L} = \int_2^4 |\mathbf{v}(t)|\, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2}\, dt$$

$$= \int_2^4 \sqrt{9t^2}\, dt$$

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$\mathbf{L} = \int_2^4 |\mathbf{v}(t)|\, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2}\, dt$$

$$= \int_2^4 \sqrt{9t^2}\, dt = \int_2^4 3t\, dt$$

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$\mathbf{L} = \int_2^4 |\mathbf{v}(t)|\, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2}\, dt$$

$$= \int_2^4 \sqrt{9t^2}\, dt = \int_2^4 3t\, dt$$

$$= \frac{3}{2}t^2\Big|_2^4$$

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$\mathbf{L} = \int_2^4 |\mathbf{v}(t)|\ dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2}\ dt$$

$$= \int_2^4 \sqrt{9t^2}\ dt = \int_2^4 3t\ dt$$

$$= \frac{3}{2}t^2\Big|_2^4 = \frac{3}{2}(16 - 4)$$

Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$.
Find the length $\mathbf{L}$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$\mathbf{L} = \int_2^4 |\mathbf{v}(t)|\, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2}\, dt$$

$$= \int_2^4 \sqrt{9t^2}\, dt = \int_2^4 3t\, dt$$

$$= \frac{3}{2}t^2\Big|_2^4 = \frac{3}{2}(16 - 4) = 18.$$

$\square$

Problem 14(c) - Spring 2007

Find a vector function $\mathbf{r}(t)$ which represents the **curve of intersection** of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

Problem 14(c) - Spring 2007

Find a vector function $\mathbf{r}(t)$ which represents the **curve of intersection** of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

Solution:

- Since the first equation is the equation of a circular cylinder, parametrize the x and y coordinates by setting $x = \cos(t)$ and $y = \sin(t)$.

Problem 14(c) - Spring 2007

Find a vector function $\mathbf{r}(t)$ which represents the **curve of intersection** of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

Solution:

- Since the first equation is the equation of a circular cylinder, parametrize the x and y coordinates by setting $x = \cos(t)$ and $y = \sin(t)$.
- Next use the second equation $z = 4 - x - 2y$ to solve for z in terms of t:

Problem 14(c) - Spring 2007

Find a vector function $\mathbf{r}(t)$ which represents the **curve of intersection** of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

Solution:

- Since the first equation is the equation of a circular cylinder, parametrize the x and y coordinates by setting $x = \cos(t)$ and $y = \sin(t)$.
- Next use the second equation $z = 4 - x - 2y$ to solve for z in terms of t:

$$z = 4 - x - 2y = 4 - \cos(t) - 2\sin(t).$$

Problem 14(c) - Spring 2007

Find a vector function $\mathbf{r}(t)$ which represents the **curve of intersection** of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

Solution:

- Since the first equation is the equation of a circular cylinder, parametrize the x and y coordinates by setting $x = \cos(t)$ and $y = \sin(t)$.
- Next use the second equation $z = 4 - x - 2y$ to solve for z in terms of t:

$$z = 4 - x - 2y = 4 - \cos(t) - 2\sin(t).$$

- Therefore,

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), 4 - \cos(t) - 2\sin(t) \rangle.$$

$\square$

Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area **A** of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area **A** of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \left| \begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{matrix} \right|$$

Problem 15(a) - Spring 2008

Consider the points $A(2, 1, 0)$, $B(1, 0, 2)$ and $C(0, 2, 1)$. Find the area **A** of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \left| \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} \mathbf{k} \right|$$

Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area **A** of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \left| \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} \mathbf{k} \right|$$

$$= |\langle -3, -3, -3 \rangle|$$

Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area **A** of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \left| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{array} \right|$$

$$= \left| \left| \begin{array}{cc} -1 & 2 \\ 1 & 1 \end{array} \right| \mathbf{i} - \left| \begin{array}{cc} -1 & 2 \\ -2 & 1 \end{array} \right| \mathbf{j} + \left| \begin{array}{cc} -1 & -1 \\ -2 & 1 \end{array} \right| \mathbf{k} \right|$$

$$= |\langle -3, -3, -3 \rangle| = \sqrt{27}.$$

Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area $\mathbf{A}$ of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \left| \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} \mathbf{k} \right|$$

$$= |\langle -3, -3, -3 \rangle| = \sqrt{27}.$$

- So the area of the triangle ABC is

$$\mathbf{A} = \frac{\sqrt{27}}{2}.$$

□

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- We find $\mathbf{r}(t)$ by integrating $\mathbf{r}'(t) = \mathbf{v}(t)$:

  $$\mathbf{r}(t) = \int_0^t \mathbf{v}(t)\,dt + \mathbf{r}(0)$$

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- We find $\mathbf{r}(t)$ by integrating $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(t)\,dt + \mathbf{r}(0) = \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle\Big|_0^t + \langle 1, 2, 0 \rangle$$

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- We find $\mathbf{r}(t)$ by integrating $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(t)\,dt + \mathbf{r}(0) = \langle -\cos t, \frac{1}{2}\sin 2t, e^t \rangle \Big|_0^t + \langle 1, 2, 0 \rangle$$

$$= \langle -\cos t, \frac{1}{2}\sin 2t, e^t \rangle - \langle -\cos 0, \frac{1}{2}\sin 0, e^0 \rangle + \langle 1, 2, 0 \rangle$$

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- We find $\mathbf{r}(t)$ by integrating $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(t)\, dt + \mathbf{r}(0) = \langle -\cos t, \frac{1}{2}\sin 2t, e^t \rangle \Big|_0^t + \langle 1, 2, 0 \rangle$$

$$= \langle -\cos t, \frac{1}{2}\sin 2t, e^t \rangle - \langle -\cos 0, \frac{1}{2}\sin 0, e^0 \rangle + \langle 1, 2, 0 \rangle$$

$$= \langle -\cos t, \frac{1}{2}\sin 2t, e^t \rangle - \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle$$

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- We find $\mathbf{r}(t)$ by integrating $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\begin{aligned}
\mathbf{r}(t) = \int_0^t \mathbf{v}(t)\,dt + \mathbf{r}(0) &= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle \Big|_0^t + \langle 1, 2, 0 \rangle \\
&= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle - \left\langle -\cos 0, \frac{1}{2}\sin 0, e^0 \right\rangle + \langle 1, 2, 0 \rangle \\
&= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle - \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle \\
&= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle + \langle 2, 2, -1 \rangle.
\end{aligned}$$

Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- We find $\mathbf{r}(t)$ by integrating $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\begin{aligned}\mathbf{r}(t) = \int_0^t \mathbf{v}(t)\,dt + \mathbf{r}(0) &= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle \Big|_0^t + \langle 1, 2, 0 \rangle \\ &= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle - \left\langle -\cos 0, \frac{1}{2}\sin 0, e^0 \right\rangle + \langle 1, 2, 0 \rangle \\ &= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle - \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle \\ &= \left\langle -\cos t, \frac{1}{2}\sin 2t, e^t \right\rangle + \langle 2, 2, -1 \rangle.\end{aligned}$$

- So, $$\mathbf{r}(t) = \left\langle 2 - \cos t,\ 2 + \frac{1}{2}\sin 2t,\ -1 + e^t \right\rangle.$$

$\square$

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Solution:

- To find the equation of a plane, we need to find its normal $\mathbf{n}$ and a point on it.

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Solution:

- To find the equation of a plane, we need to find its normal $\mathbf{n}$ and a point on it. Setting $t = 0$, we find the point $(4, 3, 1)$ on the first line.

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Solution:

- To find the equation of a plane, we need to find its normal $\mathbf{n}$ and a point on it. Setting $t = 0$, we find the point $(4, 3, 1)$ on the first line.
- The part vector $\mathbf{v}_1$ of the first line is $\langle -4, -1, 5 \rangle$ and the vector part $\mathbf{v}_2$ of the second line is $\langle -1, 2, 0 \rangle$.

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Solution:

- To find the equation of a plane, we need to find its normal $\mathbf{n}$ and a point on it. Setting $t = 0$, we find the point $(4, 3, 1)$ on the first line.
- The part vector $\mathbf{v}_1$ of the first line is $\langle -4, -1, 5 \rangle$ and the vector part $\mathbf{v}_2$ of the second line is $\langle -1, 2, 0 \rangle$.
- Since the vector

$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -1 & 5 \\ -1 & 2 & 0 \end{vmatrix} = \langle -10, -5, -9 \rangle,$$

is orthogonal to both $\mathbf{v}_1$ and $\mathbf{v}_2$, it is the normal to the plane.

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Solution:

- To find the equation of a plane, we need to find its normal $\mathbf{n}$ and a point on it. Setting $t = 0$, we find the point $(4, 3, 1)$ on the first line.
- The part vector $\mathbf{v}_1$ of the first line is $\langle -4, -1, 5 \rangle$ and the vector part $\mathbf{v}_2$ of the second line is $\langle -1, 2, 0 \rangle$.
- Since the vector

$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -1 & 5 \\ -1 & 2 & 0 \end{vmatrix} = \langle -10, -5, -9 \rangle,$$

  is orthogonal to both $\mathbf{v}_1$ and $\mathbf{v}_2$, it is the normal to the plane.
- The **equation of the plane** is:

$$\langle -10, -5, -9 \rangle \cdot \langle x - 4, y - 3, z - 1 \rangle$$

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

$$x = 4 - t, \quad y = 3 + 2t, \quad z = 1$$

Solution:

- To find the equation of a plane, we need to find its normal $\mathbf{n}$ and a point on it. Setting $t = 0$, we find the point $(4, 3, 1)$ on the first line.
- The part vector $\mathbf{v}_1$ of the first line is $\langle -4, -1, 5 \rangle$ and the vector part $\mathbf{v}_2$ of the second line is $\langle -1, 2, 0 \rangle$.
- Since the vector

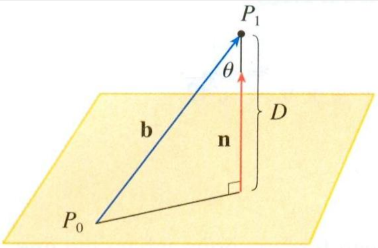
$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -1 & 5 \\ -1 & 2 & 0 \end{vmatrix} = \langle -10, -5, -9 \rangle,$$

  is orthogonal to both $\mathbf{v}_1$ and $\mathbf{v}_2$, it is the normal to the plane.
- The **equation of the plane** is:

$$\langle -10, -5, -9 \rangle \cdot \langle x - 4, y - 3, z - 1 \rangle$$

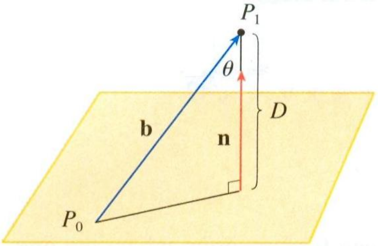
$$= -10(x - 4) - 5(y - 3) - 9(z - 1) = 0.$$

$\square$



Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.

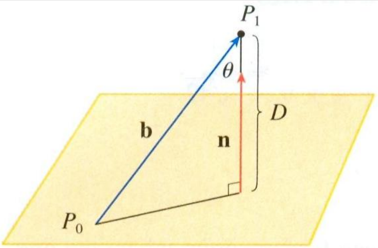


Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.

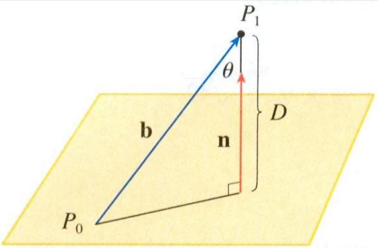


Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $4x - 6y - z - 5 = 0$.

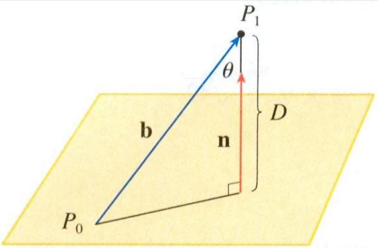


Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $4x - 6y - z - 5 = 0$.
- So, the distance from $(3, -2, 7)$ to the plane is:



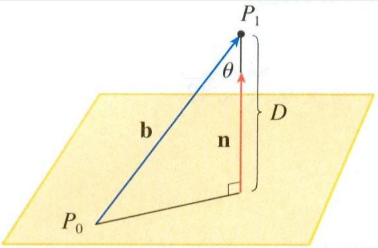
Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $4x - 6y - z - 5 = 0$.
- So, the distance from $(3, -2, 7)$ to the plane is:

$$\mathbf{D} = \frac{|(4 \cdot 3) + (-6 \cdot -2) + (-1 \cdot 7) - 5|}{\sqrt{4^2 + (-6)^2 + (-1)^2}}$$



Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $4x - 6y - z - 5 = 0$.
- So, the distance from $(3, -2, 7)$ to the plane is:

$$\mathbf{D} = \frac{|(4 \cdot 3) + (-6 \cdot -2) + (-1 \cdot 7) - 5|}{\sqrt{4^2 + (-6)^2 + (-1)^2}} = \frac{12}{\sqrt{53}}.$$

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$

- Equating x and y-coordinates:

$$x = t = -4s + 3$$
$$y = 2t + 1 = -3s + 2.$$

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$
$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$
$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$

- Equating x and y-coordinates:

$$x = t = -4s+3$$
$$y = 2t+1 = -3s+2.$$

- Solving gives $s = 1$ and $t = -1$.

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2\rangle$$

$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1\rangle$$

- Equating x and y-coordinates:

$$x = t = -4s + 3$$
$$y = 2t + 1 = -3s + 2.$$

- Solving gives $s = 1$ and $t = -1$.
- $\mathbf{L}_1(-1) = \langle -1, -1, -1\rangle \neq \langle -1, -1, 3\rangle = \mathbf{L}_2(1)$.

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$

- Equating x and y-coordinates:

$$\begin{aligned} x = t &= -4s+3 \\ y = 2t+1 &= -3s+2. \end{aligned}$$

- Solving gives $s = 1$ and $t = -1$.
- $\mathbf{L}_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = \mathbf{L}_2(1)$. So these lines do **not intersect**.

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$

- Equating x and y-coordinates:

$$x = t = -4s + 3$$
$$y = 2t + 1 = -3s + 2.$$

- Solving gives $s = 1$ and $t = -1$.
- $\mathbf{L}_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = \mathbf{L}_2(1)$. So these lines do **not intersect**.
- Since the lines are clearly **not parallel** (the direction vectors $\langle 1, 2, 3 \rangle$ and $\langle -4, -3, 2 \rangle$ are **not parallel**), the lines are **skew**. $\square$

Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

Find the **acceleration** of the particle. Write down a formula for the **speed** of the particle (you do not need to simplify the expression algebraically).

Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

Find the **acceleration** of the particle. Write down a formula for the **speed** of the particle (you do not need to simplify the expression algebraically).

Solution:

- Recall the **acceleration vector** $\mathbf{a}(t) = \mathbf{v}'(t)$.

Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

Find the **acceleration** of the particle. Write down a formula for the **speed** of the particle (you do not need to simplify the expression algebraically).

Solution:

- Recall the **acceleration vector** $\mathbf{a}(t) = \mathbf{v}'(t)$. Hence,

$$\mathbf{a}(t) = \langle 6t, 4\cos(2t), e^t \rangle.$$

Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

Find the **acceleration** of the particle. Write down a formula for the **speed** of the particle (you do not need to simplify the expression algebraically).

Solution:

- Recall the **acceleration vector** $\mathbf{a}(t) = \mathbf{v}'(t)$. Hence,

$$\mathbf{a}(t) = \langle 6t, 4\cos(2t), e^t \rangle.$$

- Recall that the **speed**(t) is the length of the velocity vector.

Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

Find the **acceleration** of the particle. Write down a formula for the **speed** of the particle (you do not need to simplify the expression algebraically).

Solution:

- Recall the **acceleration vector** $\mathbf{a}(t) = \mathbf{v}'(t)$. Hence,

$$\mathbf{a}(t) = \langle 6t, 4\cos(2t), e^t \rangle.$$

- Recall that the **speed**(t) is the length of the velocity vector. Hence,

$$\textbf{speed}(t) = \sqrt{9t^4 + 4\sin^2(2t) + e^{2t}}.$$

$\square$

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we first integrate the velocity $\mathbf{v}(t)$ and second use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we first integrate the velocity $\mathbf{v}(t)$ and second use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

$$\mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt$$

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we first integrate the velocity $\mathbf{v}(t)$ and second use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

$$\mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle.$$

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we first integrate the velocity $\mathbf{v}(t)$ and second use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

  $$\mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle.$$

- Plugging in the position at $t = 0$, we get:

  $$\langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle$$

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we first integrate the velocity $\mathbf{v}(t)$ and second use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

  $$\mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle.$$

- Plugging in the position at $t = 0$, we get:

  $$\langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle = \langle 0, -1, 2 \rangle.$$

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we first integrate the velocity $\mathbf{v}(t)$ and second use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

  $$\mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle.$$

- Plugging in the position at $t = 0$, we get:

  $$\langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle = \langle 0, -1, 2 \rangle.$$

  Thus, $x_0 = 0$, $y_0 = 0$ and $z_0 = 1$.

Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

$$\mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle.$$

If initially the particle has the position $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$, what is the position at time t?

Solution:

- To find the position $\mathbf{r}(t)$, we <u>first</u> integrate the velocity $\mathbf{v}(t)$ and <u>second</u> use the initial position value $\mathbf{r}(0) = \langle 0, -1, 2 \rangle$ to solve for the **constants of integration**.

  $$\mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle.$$

- Plugging in the position at $t = 0$, we get:

  $$\langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle = \langle 0, -1, 2 \rangle.$$

  Thus, $x_0 = 0$, $y_0 = 0$ and $z_0 = 1$.

- Hence,

  $$\mathbf{r}(t) = \langle t^3, -\cos 2t, e^t + 1 \rangle. \quad \square$$

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1\rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0\rangle$.

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1\rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0\rangle$. Then the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is:

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0,2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,2,0\rangle$. Then the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is:

$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0,2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,2,0\rangle$. Then the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is:

$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{vmatrix} \right\|$$

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0,2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,2,0\rangle$. Then the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is:

$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \left\| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{array} \right\|$$

$$= |\langle 2,-3,-6\rangle|$$

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0,2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,2,0\rangle$. Then the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is:

$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{vmatrix} \right\|$$

$$= |\langle 2,-3,-6\rangle| = \sqrt{4+9+36} = 7$$

□

Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

Solution:

- Note that:

$$\overrightarrow{PQ} = \langle 0, 2, -1 \rangle \qquad \overrightarrow{PR} = \langle 3, 2, 0 \rangle.$$

Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

Solution:

- Note that:

$$\overrightarrow{PQ} = \langle 0, 2, -1 \rangle \qquad \overrightarrow{PR} = \langle 3, 2, 0 \rangle.$$

- By our formula for dot products:

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}$$

Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

Solution:

- Note that:

$$\overrightarrow{PQ} = \langle 0, 2, -1 \rangle \qquad \overrightarrow{PR} = \langle 3, 2, 0 \rangle.$$

- By our formula for dot products:

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{\langle 0, 2, -1 \rangle \cdot \langle 3, 2, 0 \rangle}{\sqrt{5}\sqrt{13}}$$

Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

Solution:

- Note that:

$$\overrightarrow{PQ} = \langle 0,2,-1\rangle \qquad \overrightarrow{PR} = \langle 3,2,0\rangle.$$

- By our formula for dot products:

$$\cos\theta = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{\langle 0,2,-1\rangle\cdot\langle 3,2,0\rangle}{\sqrt{5}\sqrt{13}} = \frac{4}{\sqrt{5}\sqrt{13}}.$$

$\square$

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $\mathbf{S}$.

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by **S**. Then

$$\overrightarrow{O\mathbf{S}} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0,1,0\rangle + \langle 3,2,0\rangle = \langle 3,3,0\rangle\,,$$

where O is the origin.

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $\mathbf{S}$. Then

$$\overrightarrow{O\mathbf{S}} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0,1,0\rangle + \langle 3,2,0\rangle = \langle 3,3,0\rangle\,,$$

where O is the origin. That is,

$$\mathbf{S} = (3,3,0).$$

$\square$

Problem 21(a) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine the point(s) of **intersection** of **C** with the *xz*-plane.

Problem 21(a) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine the point(s) of **intersection** of **C** with the *xz*-plane.

Solution:

- The points of intersection of **C** with the *xz*-plane correspond to the points where the *y*-coordinate of **C** is 0.

Problem 21(a) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine the point(s) of **intersection** of **C** with the *xz*-plane.

Solution:

- The points of intersection of **C** with the *xz*-plane correspond to the points where the y-coordinate of **C** is 0.
- When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$.

Problem 21(a) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine the point(s) of **intersection** of **C** with the *xz*-plane.

Solution:

- The points of intersection of **C** with the *xz*-plane correspond to the points where the *y*-coordinate of **C** is 0.
- When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$.
- Hence,

$$\langle 2 - (\frac{1}{2})^2, 2 \cdot \frac{1}{2} - 1, \ln \frac{1}{2} \rangle = \langle 1\frac{3}{4}, 0, -\ln 2 \rangle$$

is the unique point of the **intersection** of **C** with *xz*-plane.

$\square$

Problem 21(b) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine **parametric equations** of tangent line to **C** at $(1, 1, 0)$.

Problem 21(b) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine **parametric equations** of tangent line to **C** at $(1, 1, 0)$.

Solution:

- Using the y-coordinate of **C**, note that $t = 1$ when $(1, 1, 0) \in$ **C**.

Problem 21(b) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$

Determine **parametric equations** of tangent line to **C** at $(1, 1, 0)$.

Solution:

- Using the y-coordinate of **C**, note that $t = 1$ when $(1, 1, 0) \in$ **C**.
- The velocity vector to

$$\mathbf{C}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$$

is:

Problem 21(b) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine **parametric equations** of tangent line to **C** at $(1, 1, 0)$.

Solution:

- Using the y-coordinate of **C**, note that $t = 1$ when $(1, 1, 0) \in$ **C**.
- The velocity vector to

$$\mathbf{C}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$$

is:

$$\mathbf{C}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$

Problem 21(b) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$

Determine **parametric equations** of tangent line to **C** at $(1, 1, 0)$.

Solution:

- Using the y-coordinate of **C**, note that $t = 1$ when $(1, 1, 0) \in \mathbf{C}$.
- The velocity vector to
$$\mathbf{C}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$$
is:
$$\mathbf{C}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$
- Thus,
$$\mathbf{C}'(1) = \langle -2, 2, 1 \rangle$$
is the vector part of the tangent line to **C** at $(1, 1, 0)$.

Problem 21(b) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$

Determine **parametric equations** of tangent line to **C** at $(1, 1, 0)$.

Solution:

- Using the y-coordinate of **C**, note that $t = 1$ when $(1, 1, 0) \in \mathbf{C}$.
- The velocity vector to

$$\mathbf{C}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$$

  is:

$$\mathbf{C}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$

- Thus,

$$\mathbf{C}'(1) = \langle -2, 2, 1 \rangle$$

  is the vector part of the tangent line to **C** at $(1, 1, 0)$.
- The **parametric equations** are:

$$x = 1 - 2t$$
$$y = 1 + 2t$$
$$z = t.$$

□

Problem 21(c) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Set up, but not solve, a formula that will determine the length **L** of **C** for $1 \leq t \leq 2$.

Problem 21(c) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$

Set up, but not solve, a formula that will determine the length **L** of **C** for $1 \leq t \leq 2$.

Solution:

- The vector equation of **C** is $\mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$ with velocity vector

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$

Problem 21(c) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Set up, but not solve, a formula that will determine the length **L** of **C** for $1 \le t \le 2$.

Solution:

- The vector equation of **C** is $\mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$ with velocity vector

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$

- Since the length of **L** is the integral of the speed $|\mathbf{r}'(t)|$,

$$\mathbf{L} = \int_1^2 |\langle -2t, 2, \frac{1}{t} \rangle| \, dt$$

Problem 21(c) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$

Set up, but not solve, a formula that will determine the length **L** of **C** for $1 \leq t \leq 2$.

Solution:

- The vector equation of **C** is $\mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$ with velocity vector

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$

- Since the length of **L** is the integral of the speed $|\mathbf{r}'(t)|$,

$$\mathbf{L} = \int_1^2 |\langle -2t, 2, \frac{1}{t} \rangle| \, dt = \int_1^2 \sqrt{4t^2 + 4 + \frac{1}{t^2}} \, dt.$$

$\square$

Problem 22(a) - Fall 2006

Find **parametric equations** for the line **r** which contains $A(2, 0, 1)$ and $B(-1, 1, -1)$.

Problem 22(a) - Fall 2006

Find **parametric equations** for the line **r** which contains $A(2, 0, 1)$ and $B(-1, 1, -1)$.

Solution:

- Note that $\overrightarrow{AB} = \langle -3, 1, -2 \rangle$ and the **vector equation** is:

  $\mathbf{r}(t) = \vec{A} + t\overrightarrow{AB}$

Problem 22(a) - Fall 2006

Find **parametric equations** for the line **r** which contains $A(2,0,1)$ and $B(-1,1,-1)$.

Solution:

- Note that $\overrightarrow{AB} = \langle -3, 1, -2\rangle$ and the **vector equation** is:

$$\mathbf{r}(t) = \vec{A} + t\overrightarrow{AB} = \langle 2,0,1\rangle + t\langle -3,1,-2\rangle$$

Problem 22(a) - Fall 2006

Find **parametric equations** for the line **r** which contains $A(2,0,1)$ and $B(-1,1,-1)$.

Solution:

- Note that $\overrightarrow{AB} = \langle -3, 1, -2 \rangle$ and the **vector equation** is:

$$\mathbf{r}(t) = \vec{A} + t\overrightarrow{AB} = \langle 2,0,1 \rangle + t\langle -3,1,-2 \rangle = \langle 2-3t, t, 1-2t \rangle.$$

Problem 22(a) - Fall 2006

Find **parametric equations** for the line **r** which contains $A(2, 0, 1)$ and $B(-1, 1, -1)$.

Solution:

- Note that $\overrightarrow{AB} = \langle -3, 1, -2 \rangle$ and the **vector equation** is:

  $$\mathbf{r}(t) = \vec{A} + t\overrightarrow{AB} = \langle 2, 0, 1 \rangle + t\langle -3, 1, -2 \rangle = \langle 2 - 3t, t, 1 - 2t \rangle.$$

- The **parametric equations** are:

  $$x = 2 - 3t$$
  $$y = t$$
  $$z = 1 - 2t.$$

$\square$

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$.

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s$$

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1\rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3\rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

$$y = 3t = 4 + s$$

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

$$y = 3t = 4 + s \implies 3t = 4 + s.$$

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

$$y = 3t = 4 + s \implies 3t = 4 + s.$$

Solving yields: $$t = 6 \ \textbf{ and } \ s = 14.$$

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

$$y = 3t = 4 + s \implies 3t = 4 + s.$$

Solving yields: $$t = 6 \ \textbf{ and } \ s = 14.$$

Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means the z-coordinates are never equal and the lines do **not intersect**.

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

$$y = 3t = 4 + s \implies 3t = 4 + s.$$

  Solving yields: $$t = 6 \ \textbf{and} \ s = 14.$$

  Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means the z-coordinates are never equal and the lines do **not intersect**.
- Thus, the lines are **skew**.

$\square$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix}$$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

- Since $P(-1,2,1)$ lies on the plane, the **equation of the plane** is:

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Solution:

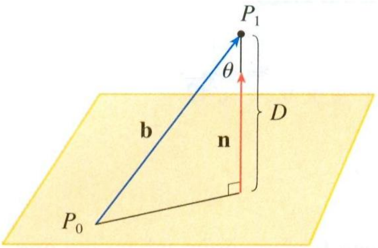
- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

- Since $P(-1,2,1)$ lies on the plane, the **equation of the plane** is:

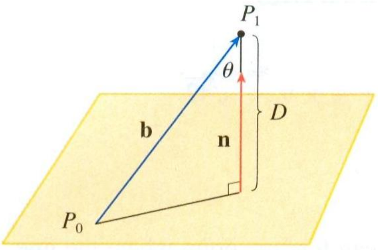
$$\langle 8,4,6\rangle \cdot \langle x+1, y-2, z-1\rangle = 8(x+1)+4(y-2)+6(z-1) = 0.$$

□



Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

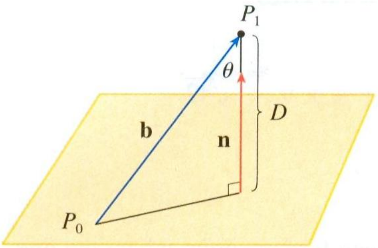


Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane.

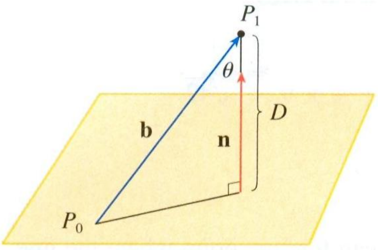


Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$.

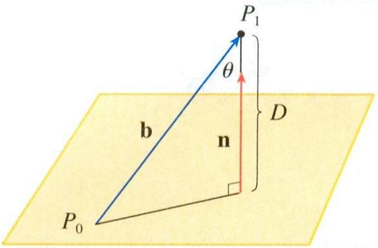


Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance **D** from $(1, 2, -1)$ to the plane is equal to:



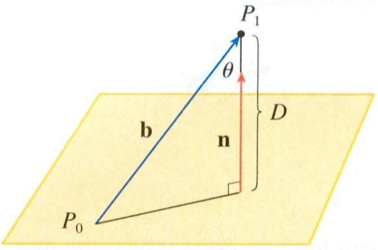
Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance **D** from $(1, 2, -1)$ to the plane is equal to:

$$|\mathbf{comp_n\ b}| =$$



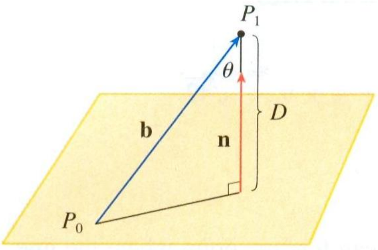
Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance **D** from $(1, 2, -1)$ to the plane is equal to:

$$|\mathbf{comp_n}\ \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$



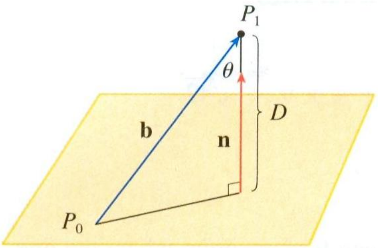
Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2\rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1\rangle$. The distance **D** from $(1, 2, -1)$ to the plane is equal to:

$$|\mathbf{comp_n\ b}| = \left|\mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}\right| = |\langle 1, 1, -1\rangle \cdot \frac{1}{3}\langle 2, 1, -2\rangle|$$



Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance **D** from $(1, 2, -1)$ to the plane is equal to:

$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 1, 1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle| = \frac{5}{3}.$$

□

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1, 0, 1 \rangle$, then $t = 1$.

Problem 24(a) - Fall 2006

Let two space curves

$\mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle,$

be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1, 0, 1 \rangle$, then $t = 1$.
- When $\mathbf{r}_2(s) = \langle 1, 0, 1 \rangle$, then $s = 1$.

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1, 0, 1 \rangle$, then $t = 1$.
- When $\mathbf{r}_2(s) = \langle 1, 0, 1 \rangle$, then $s = 1$.
- Calculating derivatives, we obtain:

  $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle$
  $\mathbf{r}_1'(1) = \langle 0, 2, 4 \rangle$
  $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 2s \rangle$
  $\mathbf{r}_2'(1) = \langle 1, 0, 2 \rangle.$

Problem 24(a) - Fall 2006

Let two space curves

$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, s^2 \rangle,$

be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1,0,1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1,0,1 \rangle$, then $t=1$.
- When $\mathbf{r}_2(s) = \langle 1,0,1 \rangle$, then $s=1$.
- Calculating derivatives, we obtain:

  $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle$

  $\mathbf{r}_1'(1) = \langle 0,2,4 \rangle$

  $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 2s \rangle$

  $\mathbf{r}_2'(1) = \langle 1,0,2 \rangle.$
- Hence,

$$\cos\theta = \frac{\mathbf{r}_1'(1)\cdot\mathbf{r}_2'(1)}{|\mathbf{r}_1'(1)||\mathbf{r}_2'(1)|}$$

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1, 0, 1 \rangle$, then $t = 1$.
- When $\mathbf{r}_2(s) = \langle 1, 0, 1 \rangle$, then $s = 1$.
- Calculating derivatives, we obtain:
  $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle$
  $\mathbf{r}_1'(1) = \langle 0, 2, 4 \rangle$
  $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s - 2, 2s \rangle$
  $\mathbf{r}_2'(1) = \langle 1, 0, 2 \rangle.$
- Hence,
$$\cos\theta = \frac{\mathbf{r}_1'(1) \cdot \mathbf{r}_2'(1)}{|\mathbf{r}_1'(1)||\mathbf{r}_2'(1)|} = \frac{\langle 0, 2, 4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{20}\sqrt{5}}$$

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1,0,1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1,0,1 \rangle$, then $t = 1$.
- When $\mathbf{r}_2(s) = \langle 1,0,1 \rangle$, then $s = 1$.
- Calculating derivatives, we obtain:
  $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle$
  $\mathbf{r}_1'(1) = \langle 0,2,4 \rangle$
  $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 2s \rangle$
  $\mathbf{r}_2'(1) = \langle 1,0,2 \rangle$.
- Hence,

$$\cos\theta = \frac{\mathbf{r}_1'(1)\cdot\mathbf{r}_2'(1)}{|\mathbf{r}_1'(1)||\mathbf{r}_2'(1)|} = \frac{\langle 0,2,4\rangle \cdot \langle 1,0,2\rangle}{\sqrt{20}\sqrt{5}}$$
$$= \frac{1}{\sqrt{100}}(0\cdot 1 + 2\cdot 0 + 4\cdot 2)$$

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1+\ln s, s^2-2s+1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1,0,1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1,0,1 \rangle$, then $t = 1$.
- When $\mathbf{r}_2(s) = \langle 1,0,1 \rangle$, then $s = 1$.
- Calculating derivatives, we obtain:
  $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle$
  $\mathbf{r}_1'(1) = \langle 0,2,4 \rangle$
  $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 2s \rangle$
  $\mathbf{r}_2'(1) = \langle 1,0,2 \rangle.$
- Hence,

$$\cos\theta = \frac{\mathbf{r}_1'(1)\cdot\mathbf{r}_2'(1)}{|\mathbf{r}_1'(1)||\mathbf{r}_2'(1)|} = \frac{\langle 0,2,4\rangle\cdot\langle 1,0,2\rangle}{\sqrt{20}\sqrt{5}}$$
$$= \frac{1}{\sqrt{100}}(0\cdot 1 + 2\cdot 0 + 4\cdot 2) = \frac{8}{10}$$

Problem 24(a) - Fall 2006

Let two space curves
$\mathbf{r}_1(t) = \langle \cos(t-1), t^2-1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle,$
be given where t and s are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point $(1, 0, 1)$.

Solution:

- When $\mathbf{r}_1(t) = \langle 1, 0, 1 \rangle$, then $t = 1$.
- When $\mathbf{r}_2(s) = \langle 1, 0, 1 \rangle$, then $s = 1$.
- Calculating derivatives, we obtain:

  $\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle$
  $\mathbf{r}_1'(1) = \langle 0, 2, 4 \rangle$
  $\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 2s \rangle$
  $\mathbf{r}_2'(1) = \langle 1, 0, 2 \rangle$.
- Hence,

$$\cos\theta = \frac{\mathbf{r}_1'(1) \cdot \mathbf{r}_2'(1)}{|\mathbf{r}_1'(1)||\mathbf{r}_2'(1)|} = \frac{\langle 0, 2, 4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{20}\sqrt{5}}$$

$$= \frac{1}{\sqrt{100}}(0 \cdot 1 + 2 \cdot 0 + 4 \cdot 2) = \frac{8}{10} = \frac{4}{5}.$$

$\square$

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

$$= \int \langle \sin t, \cos 2t, e^t \rangle\, dt = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle.$$

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

$$= \int \langle \sin t, \cos 2t, e^t \rangle \, dt = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle.$$

- Now use the initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$ to solve for x_0, y_0, z_0.

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$
and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:
$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$
$$= \int \langle \sin t, \cos 2t, e^t \rangle\, dt = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle.$$
- Now use the initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$ to solve for x_0, y_0, z_0.
$$-\cos(0) + x_0 = -1 + x_0 = 1 \implies x_0 = 2.$$

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$
and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:
$$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt$$
$$= \int \langle \sin t, \cos 2t, e^t \rangle\,dt = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle.$$
- Now use the initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$ to solve for x_0, y_0, z_0.
$$-\cos(0) + x_0 = -1 + x_0 = 1 \Longrightarrow x_0 = 2.$$
$$\frac{1}{2}\sin(0) + y_0 = 0 + y_0 = 2 \Longrightarrow y_0 = 2.$$

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

$$= \int \langle \sin t, \cos 2t, e^t \rangle\, dt = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle.$$

- Now use the initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$ to solve for x_0, y_0, z_0.

$$-\cos(0) + x_0 = -1 + x_0 = 1 \Longrightarrow x_0 = 2.$$

$$\frac{1}{2}\sin(0) + y_0 = 0 + y_0 = 2 \Longrightarrow y_0 = 2.$$

$$e^0 + z_0 = 1 + z_0 = 0 \Longrightarrow z_0 = -1.$$

Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

$$\mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle$$

and initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$. Find the position vector function $\mathbf{r}(t)$.

Solution:

- The **position vector function** $\mathbf{r}(t)$ is the integral of its derivative $\mathbf{r}'(t) = \mathbf{v}(t)$:

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

$$= \int \langle \sin t, \cos 2t, e^t \rangle\, dt = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle.$$

- Now use the initial position $\mathbf{r}(0) = \langle 1, 2, 0 \rangle$ to solve for x_0, y_0, z_0.

$$-\cos(0) + x_0 = -1 + x_0 = 1 \Longrightarrow x_0 = 2.$$

$$\frac{1}{2}\sin(0) + y_0 = 0 + y_0 = 2 \Longrightarrow y_0 = 2.$$

$$e^0 + z_0 = 1 + z_0 = 0 \Longrightarrow z_0 = -1.$$

- Hence, $$\mathbf{r}(t) = \langle -\cos(t) + 2, \frac{1}{2}\sin(2t) + 2, e^t - 1. \rangle$$

□

Problem 25(a) - Fall 2006

Let $f(x, y) = e^{x^2-y} + x\sqrt{4 - y^2}$. Find partial derivatives f_x, f_y and f_{xy}.

Problem 25(b) - Fall 2006

Find an equation for the tangent plane of the graph of

$$f(x, y) = \sin(2x + y) + 1$$

at the point $(0, 0, 1)$.

Problem 26(a) - Fall 2006

Let $g(x, y) = ye^x$. Estimate $g(0.1, 1.9)$ using the linear approximation of $g(x, y)$ at $(x, y) = (0, 2)$.

Solutions to these problems:

These types of problems will not be on this exam. $\square$

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Solution:

- Complete the square in order to put the equation in the form:

$$(x - x_0)^2 + (y - y_0) + (z - z_0)^2 = r^2.$$

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Solution:

- Complete the square in order to put the equation in the form:

$$(x - x_0)^2 + (y - y_0) + (z - z_0)^2 = r^2.$$

- We get:

$$x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.$$

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Solution:

- Complete the square in order to put the equation in the form:

$$(x - x_0)^2 + (y - y_0) + (z - z_0)^2 = r^2.$$

- We get:

$$x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.$$

- This gives the equation

$$(x - 0)^2 + (y - 0)^2 + (z + 3)^2 = 25 = 5^2.$$

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Solution:

- Complete the square in order to put the equation in the form:

$$(x - x_0)^2 + (y - y_0) + (z - z_0)^2 = r^2.$$

- We get:

$$x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.$$

- This gives the equation

$$(x - 0)^2 + (y - 0)^2 + (z + 3)^2 = 25 = 5^2.$$

Hence, the **center** is $\mathbf{C} = (0, 0, -3)$ and the **radius** is $r = 5$.

□

Problem 26(c) - Fall 2006

Let $f(x, y) = \sqrt{16 - x^2 - y^2}$. Draw a contour map of level curves $f(x, y) = k$ with $k = 1,\ 2,\ 3.$ Label the level curves by the corresponding values of k.

Solution:

A problem of this type will not be on this exam. $\square$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

  $$\mathbf{L} = \vec{A} + t\overrightarrow{AB}$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1\rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1\rangle + t\langle 3, 2, -1\rangle = \langle 2+3t, 1+2t, -1-t\rangle.$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

  $$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$
- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the **equation of the plane** gives:

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

  $$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$

- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the **equation of the plane** gives:

  $$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$

- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the **equation of the plane** gives:

$$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$

$$\implies -4t = 16$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$

- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the **equation of the plane** gives:

$$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$

$$\implies -4t = 16 \implies t = -4.$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1\rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1\rangle + t\langle 3, 2, -1\rangle = \langle 2+3t, 1+2t, -1-t\rangle.$$

- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the **equation of the plane** gives:

$$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$

$$\implies -4t = 16 \implies t = -4.$$

- So, the **point of intersection** is:

$$\mathbf{L}(-4) = \langle 2 - 12, 1 - 8, -1 - (-4)\rangle$$

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.
- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$

- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the **equation of the plane** gives:

$$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$

$$\implies -4t = 16 \implies t = -4.$$

- So, the **point of intersection** is:

$$\mathbf{L}(-4) = \langle 2 - 12, 1 - 8, -1 - (-4) \rangle = \langle -10, -7, 3 \rangle.$$

$\square$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

$$z = 3 + t^2 = 3 + (3-s)^2$$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

$$z = 3 + t^2 = 3 + (3-s)^2 = 3 + 9 - 6s + s^2$$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2\rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2\rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

$$z = 3 + t^2 = 3 + (3-s)^2 = 3 + 9 - 6s + s^2 = s^2$$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

$$z = 3 + t^2 = 3 + (3-s)^2 = 3 + 9 - 6s + s^2 = s^2$$

- Thus, the **parameter values** are:

$$12 - 6s = 0$$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

$$z = 3 + t^2 = 3 + (3-s)^2 = 3 + 9 - 6s + s^2 = s^2$$

- Thus, the **parameter values** are:

$$12 - 6s = 0 \Longrightarrow (s = 2 \text{ and } t = 1).$$

Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Show that the two space curves **intersect** by finding the point of intersection and the **parameter values** where this occurs.

Solution:

- Equate the x and z-coordinates:

$$x = t = 3 - s$$

$$z = 3 + t^2 = 3 + (3-s)^2 = 3 + 9 - 6s + s^2 = s^2$$

- Thus, the **parameter values** are:

$$12 - 6s = 0 \Longrightarrow (s = 2 \text{ and } t = 1).$$

- So, $\mathbf{r}_1(1) = \langle 1, 0, 4 \rangle = \mathbf{r}_2(2)$ is the desired **intersection point**. $\square$

Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \qquad \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.

Find **parametric equation** for the tangent line to the space curve $\mathbf{r}_1(t)$ at the intersection point. (Use the value $t = 1$ in part **(a)**).

Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \qquad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle$$

where t and s are two independent real parameters.

Find **parametric equation** for the tangent line to the space curve $\mathbf{r}_1(t)$ at the intersection point. (Use the value $t = 1$ in part **(a)**).

Solution:

- The velocity vector of $\mathbf{r}_1(t)$ at the intersection point is $\mathbf{r}_1'(1)$.

Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \qquad \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Find **parametric equation** for the tangent line to the space curve $\mathbf{r}_1(t)$ at the intersection point. (Use the value $t = 1$ in part **(a)**).

Solution:

- The velocity vector of $\mathbf{r}_1(t)$ at the intersection point is $\mathbf{r}_1'(1)$.
- Since

$$\mathbf{r}_1'(t) = \langle 1, -1, 2t \rangle,$$

$$\mathbf{r}_1'(1) = \langle 1, -1, 2 \rangle.$$

Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \qquad \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.
Find **parametric equation** for the tangent line to the space curve $\mathbf{r}_1(t)$ at the intersection point. (Use the value $t = 1$ in part **(a)**).

Solution:

- The velocity vector of $\mathbf{r}_1(t)$ at the intersection point is $\mathbf{r}_1'(1)$.
- Since

$$\mathbf{r}_1'(t) = \langle 1, -1, 2t \rangle,$$

$$\mathbf{r}_1'(1) = \langle 1, -1, 2 \rangle.$$

- The **vector equation** of the tangent line is:
  $\mathbf{T}(t) = \mathbf{r}_1(1) + t\langle 1, -1, 2 \rangle = \langle 1, 0, 4 \rangle + t\langle 1, -1, 2 \rangle$

Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2\rangle, \qquad \mathbf{r}_2(s) = \langle 3-s, s-2, s^2\rangle$$

where t and s are two independent real parameters.
Find **parametric equation** for the tangent line to the space curve $\mathbf{r}_1(t)$ at the intersection point. (Use the value $t = 1$ in part **(a)**).

Solution:

- The velocity vector of $\mathbf{r}_1(t)$ at the intersection point is $\mathbf{r}_1'(1)$.
- Since
$$\mathbf{r}_1'(t) = \langle 1, -1, 2t\rangle,$$
$$\mathbf{r}_1'(1) = \langle 1, -1, 2\rangle.$$
- The **vector equation** of the tangent line is:
$\mathbf{T}(t) = \mathbf{r}_1(1) + t\langle 1, -1, 2\rangle = \langle 1, 0, 4\rangle + t\langle 1, -1, 2\rangle = \langle 1+t, -t, 4+2t\rangle.$

Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

$$\mathbf{r}_1(t) = \langle t, 1-t, 3+t^2 \rangle, \qquad \mathbf{r}_2(s) = \langle 3-s, s-2, s^2 \rangle$$

where t and s are two independent real parameters.

Find **parametric equation** for the tangent line to the space curve $\mathbf{r}_1(t)$ at the intersection point. (Use the value $t = 1$ in part **(a)**).

Solution:

- The velocity vector of $\mathbf{r}_1(t)$ at the intersection point is $\mathbf{r}_1'(1)$.
- Since
$$\mathbf{r}_1'(t) = \langle 1, -1, 2t \rangle,$$
$$\mathbf{r}_1'(1) = \langle 1, -1, 2 \rangle.$$
- The **vector equation** of the tangent line is:
$\mathbf{T}(t) = \mathbf{r}_1(1) + t\langle 1, -1, 2 \rangle = \langle 1, 0, 4 \rangle + t\langle 1, -1, 2 \rangle = \langle 1+t, -t, 4+2t \rangle.$
- The **parametric equations** are:
$$\begin{aligned} x &= 1+t \\ y &= -t \\ z &= 4+2t \end{aligned} \quad .$$

□

Problem 29

Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point C.

Problem 29

Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point C.

Solution:

After drawing a picture, the point C is easily seen to be:

$$\overrightarrow{OA} + \overrightarrow{BD}$$

Problem 29

Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point C.

Solution:

After drawing a picture, the point C is easily seen to be:

$$\overrightarrow{OA} + \overrightarrow{BD} = \langle 2, 5, 1 \rangle + \langle 2, 1, -7 \rangle$$

Problem 29

Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point C.

Solution:

After drawing a picture, the point C is easily seen to be:

$$\overrightarrow{OA} + \overrightarrow{BD} = \langle 2, 5, 1\rangle + \langle 2, 1, -7\rangle = \langle 4, 6, -6\rangle,$$

where O is the origin. $\square$

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1 \rangle$ into the formula:

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1 \rangle$ into the formula:

$$\mathbf{proj_a b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \mathbf{a}.$$

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1 \rangle$ into the formula:

$$\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}}\mathbf{a}.$$

- Plugging in, we get:

$$\mathbf{proj}_{\overrightarrow{AB}}(\overrightarrow{AC})$$

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2\rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1\rangle$ into the formula:

$$\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\,\mathbf{a}.$$

- Plugging in, we get:

$$\mathbf{proj}_{\overrightarrow{AB}}(\overrightarrow{AC}) = \frac{\langle -1, -1, 2\rangle \cdot \langle -2, 1, 1\rangle}{\langle -1, -1, 2\rangle \cdot \langle -1, -1, 2\rangle}\,\langle -1, -1, 2\rangle$$

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2\rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1\rangle$ into the formula:

$$\mathbf{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\,\mathbf{a}.$$

- Plugging in, we get:

$$\mathbf{proj}_{\overrightarrow{AB}}(\overrightarrow{AC}) = \frac{\langle -1, -1, 2\rangle \cdot \langle -2, 1, 1\rangle}{\langle -1, -1, 2\rangle \cdot \langle -1, -1, 2\rangle}\,\langle -1, -1, 2\rangle = \frac{1}{2}\langle -1, -1, 2\rangle.$$

$\square$

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$.

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2.

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2}$$

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left\| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{array} \right\|$$

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle **Δ** with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left\| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{array} \right\|$$

$$= \frac{1}{2} |\langle -3, -3, -3 \rangle|$$

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2} |\langle -3, -3, -3 \rangle| = \frac{1}{2} \sqrt{9 + 9 + 9}$$

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2}\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2}|\langle -3, -3, -3\rangle| = \frac{1}{2}\sqrt{9+9+9} = \frac{1}{2}\sqrt{27}.$$

□

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Problem 30(c)

Consider the points $A = (2,1,0)$, $B = (1,0,2)$ and $C = (0,2,1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Solution:

- From the figure drawn on the blackboard, we see that the distance **d** from C to **L** is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction

$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}}\, \overrightarrow{AC}.$$

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Solution:

- From the figure drawn on the blackboard, we see that the distance **d** from C to **L** is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction

$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}}\, \overrightarrow{AC}.$$

- The vector **v** lies in the plane containing A, B, C and is perpendicular to $\overrightarrow{AB}$.

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Solution:

- From the figure drawn on the blackboard, we see that the distance **d** from C to **L** is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction

$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}} \overrightarrow{AC}.$$

- The vector **v** lies in the plane containing A, B, C and is perpendicular to $\overrightarrow{AB}$.
- Hence,

$$\mathbf{d} = |\mathbf{comp}_{\mathbf{v}} \overrightarrow{AC}|.$$

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Solution:

- From the figure drawn on the blackboard, we see that the distance **d** from C to **L** is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction

$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}} \overrightarrow{AC}.$$

- The vector $\mathbf{v}$ lies in the plane containing A, B, C and is perpendicular to $\overrightarrow{AB}$.
- Hence,

$$\mathbf{d} = |\mathbf{comp}_{\mathbf{v}} \overrightarrow{AC}|.$$

- Next, you the student, do the algebraic calculation of **d**. □

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$.

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$. Hence $\mathbf{v}$ can be taken to be:

  $$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle$$

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part **v** of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$. Hence **v** can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of $\mathbf{P}$ is zero.

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line $\mathbf{L}$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of $\mathbf{P}$ is zero. Setting $z = 0$, we get:

$$\begin{aligned} x - 2y &= 1 \\ 2x + y &= 1. \end{aligned}$$

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part **v** of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence **v** can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of **P** is zero. Setting $z = 0$, we get:

$$\begin{aligned} x - 2y &= 1 \\ 2x + y &= 1. \end{aligned}$$

  Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$.

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line $\mathbf{L}$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of $\mathbf{P}$ is zero. Setting $z = 0$, we get:

$$\begin{aligned} x - 2y &= 1 \\ 2x + y &= 1. \end{aligned}$$

  Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $\mathbf{P} = \langle \frac{3}{5}, -\frac{1}{5}, 0\rangle$ lies on the line $\mathbf{L}$.

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part **v** of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence **v** can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of **P** is zero. Setting $z = 0$, we get:

$$\begin{aligned} x - 2y &= 1 \\ 2x + y &= 1. \end{aligned}$$

  Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $\mathbf{P} = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle$ lies on the line **L**.

- The **parametric equations** are:

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1\rangle$ and $\langle 2, 1, 1\rangle$. Hence $\mathbf{v}$ can be taken to be:

$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$

- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of $\mathbf{P}$ is zero. Setting $z = 0$, we get:

$$\begin{aligned} x - 2y &= 1 \\ 2x + y &= 1. \end{aligned}$$

  Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $\mathbf{P} = \langle \frac{3}{5}, -\frac{1}{5}, 0\rangle$ lies on the line **L**.

- The **parametric equations** are:

$$\begin{aligned} x &= \tfrac{3}{5} - 3t \\ y &= -\tfrac{1}{5} + t \\ z &= 5t. \end{aligned}$$

□

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $\mathbf{L}_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $\mathbf{L}_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:

$$\mathbf{L}_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle$$

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $\mathbf{L}_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:

$$\mathbf{L}_1(t) = \langle 1, 0, 1\rangle + t\langle -2, 4, 0\rangle = \langle 1 - 2t, 4t, 1\rangle$$

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $\mathbf{L}_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:

$$\mathbf{L}_1(t) = \langle 1, 0, 1\rangle + t\langle -2, 4, 0\rangle = \langle 1 - 2t, 4t, 1\rangle$$
$$\mathbf{L}_2(s) = \langle 2, 3, -1\rangle + s\langle 2, 1, -2\rangle = \langle 2 + 2s, 3 + s, -1 - 2s\rangle$$

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1,0,1)$ and $(-1,4,1)$ and let $\mathbf{L}_2$ denote the line through the points $(2,3,-1)$ and $(4,4,-3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:
$$\mathbf{L}_1(t) = \langle 1,0,1\rangle + t\langle -2,4,0\rangle = \langle 1-2t, 4t, 1\rangle$$
$$\mathbf{L}_2(s) = \langle 2,3,-1\rangle + s\langle 2,1,-2\rangle = \langle 2+2s, 3+s, -1-2s\rangle$$
- Equating z-coordinates, we find $1 = -1-2s \implies s = -1$.

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $\mathbf{L}_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:

$$\mathbf{L}_1(t) = \langle 1, 0, 1\rangle + t\langle -2, 4, 0\rangle = \langle 1 - 2t, 4t, 1\rangle$$

$$\mathbf{L}_2(s) = \langle 2, 3, -1\rangle + s\langle 2, 1, -2\rangle = \langle 2 + 2s, 3 + s, -1 - 2s\rangle$$

- Equating z-coordinates, we find $1 = -1 - 2s \implies s = -1$.
- Equating y-coordinates with $s = -1$, we find $4t = 3 - 1 \implies t = \frac{1}{2}$.

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1,0,1)$ and $(-1,4,1)$ and let $\mathbf{L}_2$ denote the line through the points $(2,3,-1)$ and $(4,4,-3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:
$$\mathbf{L}_1(t) = \langle 1,0,1\rangle + t\langle -2,4,0\rangle = \langle 1-2t, 4t, 1\rangle$$
$$\mathbf{L}_2(s) = \langle 2,3,-1\rangle + s\langle 2,1,-2\rangle = \langle 2+2s, 3+s, -1-2s\rangle$$
- Equating z-coordinates, we find $1 = -1-2s \implies s = -1$.
- Equating y-coordinates with $s=-1$, we find $4t = 3-1 \implies t = \frac{1}{2}$.
- Equating x-coordinates with $s=-1$ and $t=\frac{1}{2}$, we find:
$$\mathbf{L}_1(\frac{1}{2}) = \langle 0,2,1\rangle = \mathbf{L}_2(-1).$$

Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1,0,1)$ and $(-1,4,1)$ and let $\mathbf{L}_2$ denote the line through the points $(2,3,-1)$ and $(4,4,-3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ **intersect**? If not, are they **skew** or **parallel**?

Solution:

- The vector equations of the lines are:

$$\mathbf{L}_1(t) = \langle 1,0,1\rangle + t\langle -2,4,0\rangle = \langle 1-2t, 4t, 1\rangle$$

$$\mathbf{L}_2(s) = \langle 2,3,-1\rangle + s\langle 2,1,-2\rangle = \langle 2+2s, 3+s, -1-2s\rangle$$

- Equating z-coordinates, we find $1 = -1-2s \implies s = -1$.
- Equating y-coordinates with $s=-1$, we find $4t = 3-1 \implies t = \frac{1}{2}$.
- Equating x-coordinates with $s=-1$ and $t=\frac{1}{2}$, we find:

$$\mathbf{L}_1(\frac{1}{2}) = \langle 0,2,1\rangle = \mathbf{L}_2(-1).$$

- Hence, the lines **intersect**.

$\square$

Problem 33(a)

Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Problem 33(a)

Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The volume **V** is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

Problem 33(a)

Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The volume **V** is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

$$\mathbf{V} = \left\| \begin{matrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{matrix} \right\|$$

Problem 33(a)

Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The volume **V** is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4\rangle$, $\overrightarrow{AC} = \langle 3, -1, -5\rangle$, $\overrightarrow{AD} = \langle 0, -4, -3\rangle$.

$$\mathbf{V} = \left\| \begin{matrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{matrix} \right\|$$

$$= |2 \cdot (-17) + -(-3) \cdot (-9) + (-4) \cdot (-12)|$$

Problem 33(a)

Find the volume $\mathbf{V}$ of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The volume $\mathbf{V}$ is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

$$\mathbf{V} = \left\| \begin{array}{ccc} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{array} \right\|$$

$$= |2 \cdot (-17) + -(-3) \cdot (-9) + (-4) \cdot (-12)| = |-13|$$

Problem 33(a)

Find the volume $\mathbf{V}$ of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The volume $\mathbf{V}$ is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

$$\mathbf{V} = \left\| \begin{matrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{matrix} \right\|$$

$$= |2 \cdot (-17) + -(-3) \cdot (-9) + (-4) \cdot (-12)| = |-13| = 13.$$

□

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

- Consider the vectors $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

- Consider the vectors $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.
- The normal vector is:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

- Consider the vectors $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.
- The normal vector is:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix}$$

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

- Consider the vectors $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.
- The normal vector is:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix} = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}.$$

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

- Consider the vectors $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.
- The normal vector is:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix} = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}.$$

- Since $A = (1, 4, 2)$, is on the plane, then the **equation of the plane** is given by:

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

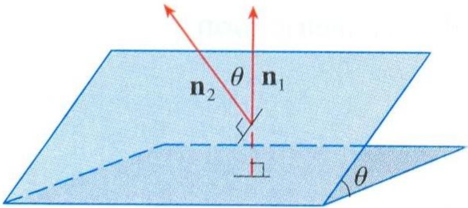
- Consider the vectors $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$ and $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.
- The normal vector is:

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix} = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}.$$

- Since $A = (1, 4, 2)$, is on the plane, then the **equation of the plane** is given by:

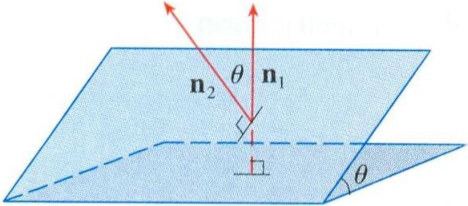
$$11(x-1) - 2(y-4) + 7(z-2) = 0.$$

□



Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2),\ B = (3, 1, -2),\ C = (4, 3 - 3)$ and the xy-plane.

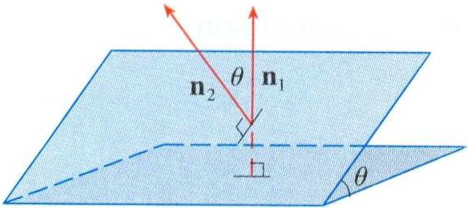


Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3 - 3)$ and the xy-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.

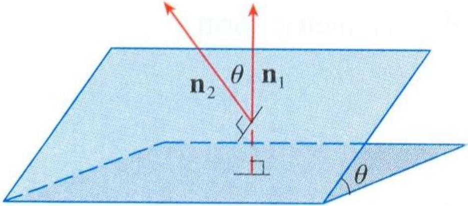


Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3 - 3)$ and the xy-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If θ is the angle between the planes, then:



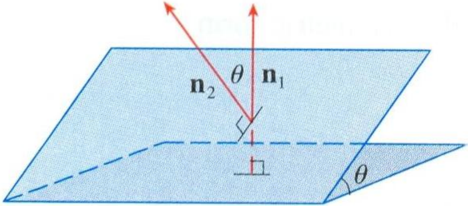
Problem 33(c)

Find the angle between the plane through $A = (1, 4, 2),\ B = (3, 1, -2),\ C = (4, 3 - 3)$ and the xy-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If θ is the angle between the planes, then:

$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$$



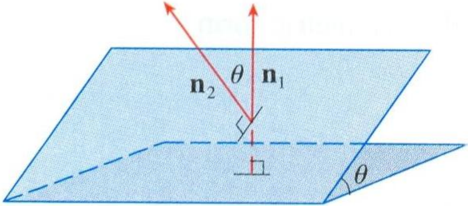
Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3 - 3)$ and the xy-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If θ is the angle between the planes, then:

$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} = \frac{7}{\sqrt{11^2 + (-2)^2 + 7^2}}$$



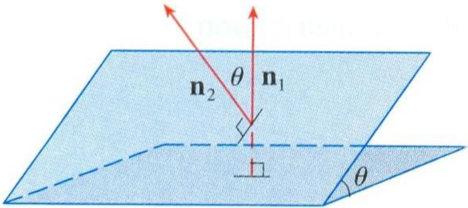
Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3-3)$ and the xy-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If θ is the angle between the planes, then:

$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} = \frac{7}{\sqrt{11^2 + (-2)^2 + 7^2}} = \frac{7}{\sqrt{174}}.$$



Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2),\ B = (3, 1, -2),\ C = (4, 3 - 3)$ and the xy-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If θ is the angle between the planes, then:
$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} = \frac{7}{\sqrt{11^2 + (-2)^2 + 7^2}} = \frac{7}{\sqrt{174}}.$$
- $$\theta = \cos^{-1}\left(\frac{1}{\sqrt{174}}\right).$$

□

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.

$$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int \langle 2t, 2t^{\frac{1}{2}}, 1\rangle\,dt$$

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt = \int \langle 2t, 2t^{\frac{1}{2}}, 1\rangle\, dt$$

$$= \langle t^2 + x_0, \frac{4}{3}t^{\frac{3}{2}} + y_0, t + z_0\rangle.$$

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.
$$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int \langle 2t, 2t^{\frac{1}{2}}, 1\rangle\,dt$$
$$= \langle t^2 + x_0, \frac{4}{3}t^{\frac{3}{2}} + y_0, t + z_0\rangle.$$
- Now use the initial position $\mathbf{r}(0) = \langle -1, 5, 4\rangle$ to find $x_0 = -1$; $y_0 = 5$; $z_0 = 4$.

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.

$$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int \langle 2t, 2t^{\frac{1}{2}}, 1\rangle\,dt$$
$$= \langle t^2 + x_0, \frac{4}{3}t^{\frac{3}{2}} + y_0, t + z_0\rangle.$$

- Now use the initial position $\mathbf{r}(0) = \langle -1, 5, 4\rangle$ to find $x_0 = -1;\ y_0 = 5;\ z_0 = 4$.
- Thus,

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3}t^{\frac{3}{2}} + 5, t + 4\rangle$$

Problem 34(a)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. At the time $t = 0$ this particle is at the point $(-1, 5, 4)$. Find the position vector $\mathbf{r}(t)$ of the particle at the time $t = 4$.

Solution:

- To find the position $\mathbf{r}(t)$, integrate the velocity vector field $\mathbf{r}'(t) = \mathbf{v}(t)$.
$$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int \langle 2t, 2t^{\frac{1}{2}}, 1\rangle\,dt$$
$$= \langle t^2 + x_0, \frac{4}{3}t^{\frac{3}{2}} + y_0, t + z_0\rangle.$$
- Now use the initial position $\mathbf{r}(0) = \langle -1, 5, 4\rangle$ to find $x_0 = -1;\ y_0 = 5;\ z_0 = 4.$
- Thus,
$$\mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3}t^{\frac{3}{2}} + 5, t + 4\rangle$$
$$\mathbf{r}(4) = \langle 15, \frac{32}{3} + 5, 8\rangle.$$

□

Problem 34(b)

The velocity vector of a particle moving in space equals
$\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find an **equation of the tangent line** $\mathbf{T}$ to the curve at the time $t = 4$.

Problem 34(b)

The velocity vector of a particle moving in space equals
$\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find an **equation of the tangent line** $\mathbf{T}$ to the curve at the time $t = 4$.

Solution:

- Vector equation of the tangent line $\mathbf{T}$ to $\mathbf{r}(t)$ at $t = 4$ is:

$$\mathbf{T}(s) = \mathbf{r}(4) + s\mathbf{r}'(4) = \mathbf{r}(4) + s\mathbf{v}(4).$$

Problem 34(b)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find an **equation of the tangent line T** to the curve at the time $t = 4$.

Solution:

- Vector equation of the tangent line $\mathbf{T}$ to $\mathbf{r}(t)$ at $t = 4$ is:

$$\mathbf{T}(s) = \mathbf{r}(4) + s\mathbf{r}'(4) = \mathbf{r}(4) + s\mathbf{v}(4).$$

- By part (a), $\mathbf{r}(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle$.

Problem 34(b)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find an **equation of the tangent line T** to the curve at the time $t = 4$.

Solution:

- Vector equation of the tangent line $\mathbf{T}$ to $\mathbf{r}(t)$ at $t = 4$ is:

$$\mathbf{T}(s) = \mathbf{r}(4) + s\mathbf{r}'(4) = \mathbf{r}(4) + s\mathbf{v}(4).$$

- By part (a), $\mathbf{r}(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle$.
- Since

$$\mathbf{v}(4) = 8\mathbf{i} + 4\mathbf{j} + \mathbf{k} = \langle 8, 4, 1 \rangle,$$

Problem 34(b)

The velocity vector of a particle moving in space equals
$\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find an **equation of the tangent line T** to the curve at the time $t = 4$.

Solution:

- Vector equation of the tangent line $\mathbf{T}$ to $\mathbf{r}(t)$ at $t = 4$ is:

$$\mathbf{T}(s) = \mathbf{r}(4) + s\mathbf{r}'(4) = \mathbf{r}(4) + s\mathbf{v}(4).$$

- By part (a), $\mathbf{r}(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle$.
- Since

$$\mathbf{v}(4) = 8\mathbf{i} + 4\mathbf{j} + \mathbf{k} = \langle 8, 4, 1 \rangle,$$

then

$$\mathbf{T}(s) = \langle 15, \frac{32}{3} + 5, 8 \rangle + s\langle 8, 4, 1 \rangle.$$

□

Problem 34(c)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Does the particle ever pass through the point $P = (80, 41, 13)$?

Problem 34(c)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Does the particle ever pass through the point $P = (80, 41, 13)$?

Solution:

- From part **(a)**, we have

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3}t^{\frac{3}{2}} + 5, t + 4\rangle.$$

Problem 34(c)

The velocity vector of a particle moving in space equals
$\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Does the particle ever pass through the point $P = (80, 41, 13)$?

Solution:

- From part **(a)**, we have

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3}t^{\frac{3}{2}} + 5, t + 4\rangle.$$

- If $\mathbf{r}(t) = \langle 80, 41, 13\rangle$, then $\quad t + 4 = 13 \Longrightarrow t = 9.$

Problem 34(c)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Does the particle ever pass through the point $P = (80, 41, 13)$?

Solution:

- From part **(a)**, we have

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3}t^{\frac{3}{2}} + 5, t + 4\rangle.$$

- If $\mathbf{r}(t) = \langle 80, 41, 13\rangle$, then $t + 4 = 13 \Longrightarrow t = 9.$
- Hence the point

$$\mathbf{r}(9) = \langle 80, 41, 13\rangle$$

  is on the curve $\mathbf{r}(t)$.

$\square$

Problem 34(d)

The velocity vector of a particle moving in space equals $\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find the length of the arc traveled from time $t = 1$ to time $t = 2$.

Problem 34(d)

The velocity vector of a particle moving in space equals
$\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find the length of the arc traveled from time $t = 1$ to time $t = 2$.

Solution:

$$\textbf{Length} = \int_1^2 |\mathbf{v}(t)|\, dt = \int_1^2 \sqrt{4t^2 + 4t + 1}\, dt.$$

Problem 34(d)

The velocity vector of a particle moving in space equals
$\mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k}$ at any time $t \geq 0$.
Find the length of the arc traveled from time $t = 1$ to time $t = 2$.

Solution:

$$\textbf{Length} = \int_1^2 |\mathbf{v}(t)|\, dt = \int_1^2 \sqrt{4t^2 + 4t + 1}\, dt.$$

Since we are not using calculators on our exam, then this is the final answer. $\square$

Problem 35(a)

Consider the surface $x^2 + 3y^2 - 2z^2 = 1$.
What are the traces in $x = k, y = k, z = k$? Sketch a few.

Problem 35(a)

Consider the surface $x^2 + 3y^2 - 2z^2 = 1$.
What are the traces in $x = k, y = k, z = k$? Sketch a few.

Solution:

- For $x = k \neq 1$, we get the hyperbolas $3y^2 - 2z^2 = k$.
- For $x = 1$, we get the 2 lines $y = \pm\frac{3}{2}z$.
- For $z = 0$, we get the ellipse $x^2 + 3y^2 = 1$.
- For $z = 1$, we get the ellipse $x^2 + 3y^2 = 3$.
- I am leaving it to you to do the sketches!

$\square$

Problem 35(b)

Consider the surface $x^2 + 3y^2 - 2z^2 = 1$.
Sketch the surface in the space.

Solution:

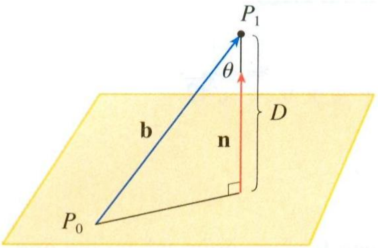
Sorry, you need to do the sketch. $\square$

Problem 36

Find an equation for the tangent plane to the graph of $f(x, y) = y \ln x$ at $(1, 4, 0)$.

Solution:

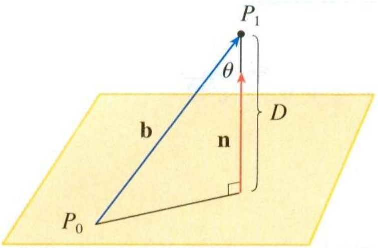
A problem of this type will not be on this exam. $\square$



Problem 37

Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$



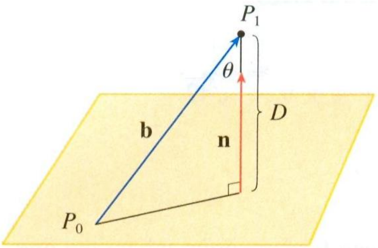
Problem 37

Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane.



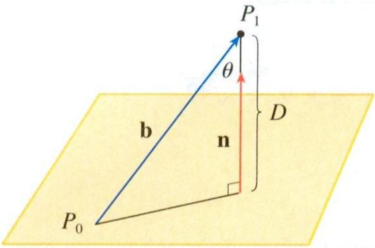
Problem 37

Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from P_0 to P_1 which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$.



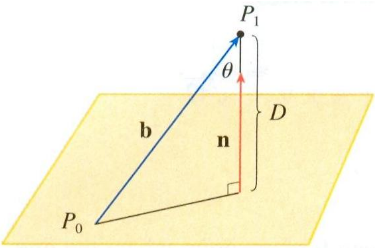
Problem 37

Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from P_0 to P_1 which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance **D** from P_1 to the first plane is equal to:



Problem 37

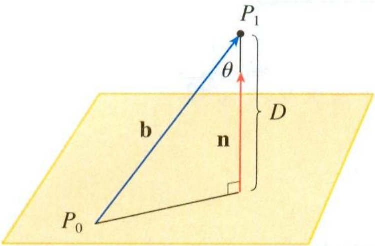
Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from P_0 to P_1 which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance **D** from P_1 to the first plane is equal to:

$$|\mathbf{comp_n}\ \mathbf{b}| =$$



Problem 37

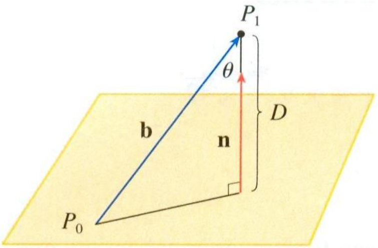
Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from P_0 to P_1 which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance **D** from P_1 to the first plane is equal to:

$$|\mathbf{comp_n\ b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$



Problem 37

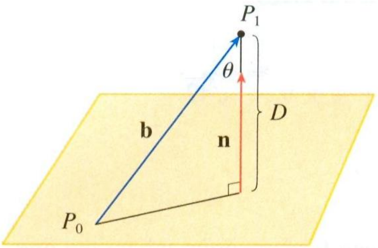
Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from P_0 to P_1 which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance **D** from P_1 to the first plane is equal to:

$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 0, 0, \frac{5}{2} \rangle \cdot \frac{1}{\sqrt{6}} \langle 2, 1, -1 \rangle|$$



Problem 37

Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from P_0 to P_1 which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance **D** from P_1 to the first plane is equal to:

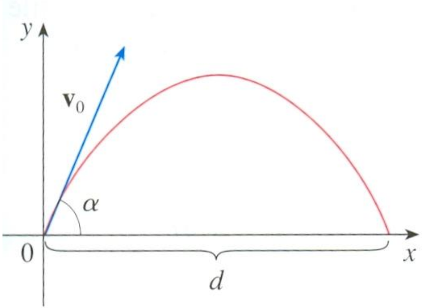
$$|\mathbf{comp_n\ b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 0, 0, \frac{5}{2} \rangle \cdot \frac{1}{\sqrt{6}} \langle 2, 1, -1 \rangle| = \frac{5}{2\sqrt{6}}. \qquad \square$$

Problem 38

Identify the surface given by the equation $4x^2 + 4y^2 - 8y - z^2 = 0$. Draw the traces and sketch the curve.

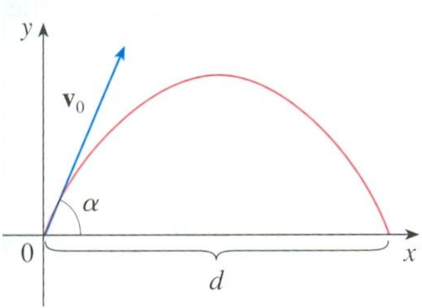
Solution:

Sorry, no sketch given. $\square$



Problem 39(a)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
Write an equation for the acceleration vector.



Problem 39(a)

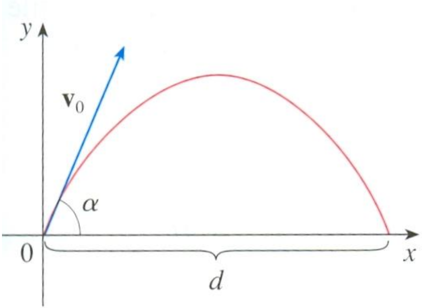
A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
Write an equation for the acceleration vector.

Solution:

Since the force due to gravity acts downward, we have

$$\mathbf{F} = m\mathbf{a} = -mg\mathbf{j},$$

where $g = |a| \approx 9.8\ \mathbf{m}/\mathbf{s}^2$.



Problem 39(a)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
Write an equation for the acceleration vector.

Solution:

Since the force due to gravity acts downward, we have

$$\mathbf{F} = m\mathbf{a} = -mg\mathbf{j},$$

where $g = |a| \approx 9.8 \ \mathbf{m}/\mathbf{s}^2$. Thus $\mathbf{a} = -g\mathbf{j}$. □

Problem 39(b) and 34(c)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
(b) Write a vector for initial velocity $\mathbf{v}(0)$.
(c) Write a vector for the initial position $\mathbf{r}(0)$

Problem 39(b) and 34(c)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
(b) Write a vector for initial velocity $\mathbf{v}(0)$.
(c) Write a vector for the initial position $\mathbf{r}(0)$

Solution:

- Initial velocity is:

$$\mathbf{v}(0) = 100(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = 50\sqrt{3}\mathbf{i} + 50\mathbf{j},$$

in units of m/s.

Problem 39(b) and 34(c)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
(b) Write a vector for initial velocity $\mathbf{v}(0)$.
(c) Write a vector for the initial position $\mathbf{r}(0)$

Solution:

- Initial velocity is:

$$\mathbf{v}(0) = 100(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = 50\sqrt{3}\mathbf{i} + 50\mathbf{j},$$

  in units of m/s.
- The initial position is:

$$\mathbf{r}(0) = 5\mathbf{j},$$

  in units of meters m.

$\square$

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
At what time does the projectile hit the ground?

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
At what time does the projectile hit the ground?

Solution:

- We first find the velocity $\mathbf{r}(t)$ and position $\mathbf{r}(t)$ functions.

$$\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}(0)$$

$$\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.$$

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
At what time does the projectile hit the ground?

Solution:

- We first find the velocity $\mathbf{r}(t)$ and position $\mathbf{r}(t)$ functions.

$$\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}(0)$$

$$\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.$$

Since $\mathbf{D} = \mathbf{r}(0) = 5\mathbf{j}$, then $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + 5\mathbf{j}$.

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
At what time does the projectile hit the ground?

Solution:

- We first find the velocity $\mathbf{r}(t)$ and position $\mathbf{r}(t)$ functions.
$$\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}(0)$$
$$\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.$$
Since $\mathbf{D} = \mathbf{r}(0) = 5\mathbf{j}$, then $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + 5\mathbf{j}$.
- Hence,
$$\mathbf{r}(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}.$$

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
At what time does the projectile hit the ground?

Solution:

- We first find the velocity $\mathbf{r}(t)$ and position $\mathbf{r}(t)$ functions.
$$\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}(0)$$
$$\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.$$
Since $\mathbf{D} = \mathbf{r}(0) = 5\mathbf{j}$, then $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + 5\mathbf{j}$.
- Hence,
$$\mathbf{r}(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}.$$
- The projectile hits the ground when $50t - \frac{1}{2}gt^2 + 5 = 0$.

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
At what time does the projectile hit the ground?

Solution:

- We first find the velocity $\mathbf{r}(t)$ and position $\mathbf{r}(t)$ functions.
$$\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}(0)$$
$$\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.$$
Since $\mathbf{D} = \mathbf{r}(0) = 5\mathbf{j}$, then $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + 5\mathbf{j}$.
- Hence,
$$\mathbf{r}(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}.$$
- The projectile hits the ground when $50t - \frac{1}{2}gt^2 + 5 = 0$. Applying the quadratic formula, we find
$$t = \frac{100 + \sqrt{100^2 + 40g}}{2g}.$$

$\square$

Problem 39(e)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
How far did it travel, horizontally, before it hit the ground?

Problem 39(e)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
How far did it travel, horizontally, before it hit the ground?

Solution:

- Recall $\mathbf{r}(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}$ and the projectile hits the ground when $t = \frac{100+\sqrt{100^2+40g}}{2g}$.

Problem 39(e)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
How far did it travel, horizontally, before it hit the ground?

Solution:

- Recall $\mathbf{r}(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}$ and the projectile hits the ground when $t = \frac{100+\sqrt{100^2+40g}}{2g}$.
- The horizontal distance $\mathbf{d}$ traveled is the value of the x-coordinate of $\mathbf{r}(t)$ at $t = \frac{100+\sqrt{100^2+40g}}{2g}$:

Problem 39(e)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.
How far did it travel, horizontally, before it hit the ground?

Solution:

- Recall $\mathbf{r}(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}$ and the projectile hits the ground when $t = \frac{100+\sqrt{100^2+40g}}{2g}$.
- The horizontal distance $\mathbf{d}$ traveled is the value of the x-coordinate of $\mathbf{r}(t)$ at $t = \frac{100+\sqrt{100^2+40g}}{2g}$:

$$\mathbf{d} = 50\sqrt{3}\left(\frac{100+\sqrt{100^2+40g}}{2g}\right).$$

$\square$

Problem 40

Explain why the limit of $f(x, y) = (3x^2y^2)/(2x^4 + y^4)$ does not exist as (x, y) approaches $(0, 0)$.

Solution:

A problem of this type will not be on this exam. $\square$

Problem 41

Find an **equation of the plane** that passes through the point $P(1,1,0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

Problem 41

Find an **equation of the plane** that passes through the point $P(1,1,0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

Solution:

- The direction vector $\mathbf{a} = \langle 3, -1, 2 \rangle$ of the line is parallel to the plane.

Problem 41

Find an **equation of the plane** that passes through the point $P(1, 1, 0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

Solution:

- The direction vector $\mathbf{a} = \langle 3, -1, 2\rangle$ of the line is parallel to the plane.
- For $t = 0$, the point $Q = \langle 2, 1, 2\rangle$ on the line and the plane.

Problem 41

Find an **equation of the plane** that passes through the point $P(1,1,0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

Solution:

- The direction vector $\mathbf{a} = \langle 3, -1, 2 \rangle$ of the line is parallel to the plane.
- For $t = 0$, the point $Q = \langle 2, 1, 2 \rangle$ on the line and the plane.
- So $\mathbf{b} = \overrightarrow{PQ} = \langle 1, 0, 2 \rangle$ is also parallel to the plane.

Problem 41

Find an **equation of the plane** that passes through the point $P(1,1,0)$ and contains the line given by **parametric equations** $x = 2+3t$, $y = 1-t$, $z = 2+2t$.

Solution:

- The direction vector $\mathbf{a} = \langle 3, -1, 2 \rangle$ of the line is parallel to the plane.
- For $t = 0$, the point $Q = \langle 2, 1, 2 \rangle$ on the line and the plane.
- So $\mathbf{b} = \overrightarrow{PQ} = \langle 1, 0, 2 \rangle$ is also parallel to the plane.
- To find a normal vector to the plane, take cross products:

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = \langle -2, -4, 1 \rangle.$$

Problem 41

Find an **equation of the plane** that passes through the point $P(1,1,0)$ and contains the line given by **parametric equations** $x = 2+3t$, $y = 1-t$, $z = 2+2t$.

Solution:

- The direction vector $\mathbf{a} = \langle 3,-1,2\rangle$ of the line is parallel to the plane.
- For $t = 0$, the point $Q = \langle 2,1,2\rangle$ on the line and the plane.
- So $\mathbf{b} = \overrightarrow{PQ} = \langle 1,0,2\rangle$ is also parallel to the plane.
- To find a normal vector to the plane, take cross products:

$$\mathbf{n} = \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = \langle -2,-4,1\rangle.$$

- Since $(1,1,0)$ is on the plane, the **equation of the plane** is:

$$\langle -2,-4,1\rangle \cdot \langle x-1, y-1, z\rangle = -2(x-1)-4(y-1)+z = 0.$$

□

Problem 42(a)

Find all of the first order and second order partial derivatives of the function. $f(x, y) = x^3 - xy^2 + y$

Solution:

There is no problem of this type on this exam. □

Problem 42(b)

Find all of the first order and second order partial derivatives of the function. $f(x, y) = \ln(x + \sqrt{x^2 + y^2})$

Solution:

There is no problem of this type on this exam. □

Problem 43

Find the linear approximation of the function $f(x, y) = xye^x$ at $(x, y) = (1, 1)$, and use it to estimate $f(1.1, 0.9)$.

Solution:

There is no problem of this type on this exam. □

Problem 44

Find a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection** of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.

Problem 44

Find a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection** of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.

Solution:

- Set $t = x$.

Problem 44

Find a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection** of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.

Solution:

- Set $t = x$.
- Since $y = x^2 = t^2$, we get from the equation of the paraboloid a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection**:

Problem 44

Find a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection** of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.

Solution:

- Set $t = x$.
- Since $y = x^2 = t^2$, we get from the equation of the paraboloid a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection**:

$$\mathbf{r}(t) = \langle t, t^2, 2t^2 + (t^2)^2 \rangle$$

Problem 44

Find a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection** of the paraboloid $z = 2x^2 + y^2$ and the parabolic cylinder $y = x^2$.

Solution:

- Set $t = x$.
- Since $y = x^2 = t^2$, we get from the equation of the paraboloid a **vector function** $\mathbf{r}(t)$ which represents the **curve of intersection**:

$$\mathbf{r}(t) = \langle t, t^2, 2t^2 + (t^2)^2 \rangle = \langle t, t^2, 2t^2 + t^4 \rangle.$$

$\square$