Hi students!

I am putting this version of my review for this Tuesday and Wednesday nights here on the website. **DO NOT PRINT!!**; it is very long!! **Enjoy!!**

Your course chair, **Bill**

PS. There are probably errors in some of the solutions presented here and for a few problems you need to complete them or simplify the answers; some questions are left to you the student. Also you might need to add more detailed explanations or justifications on the actual similar problems on your exam.

After our exam, I have added the solutions right after this slide.
Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} \).
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} \).
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \).
Problem 1(a) - Spring 2009

Given $a = \langle 3, 6, -2 \rangle$, $b = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $b$ along $a$. (Hint: Use projections.)

Solution:

- We have $|a| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$n = \frac{a}{|a|}$$
Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:
- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \).
- Then

\[
\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7} \mathbf{a}
\]
Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$. Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{unit vector parallel to } \mathbf{a}.$$
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \).
- Then
  \[
  \mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7} \mathbf{a} = \text{unit vector parallel to } \mathbf{a}.
  \]
- So,
  \[
  \text{proj}_{\mathbf{a}} \mathbf{b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n}
  \]
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \).
- Then
  \[
  \mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7} \mathbf{a} = \text{unit vector parallel to } \mathbf{a}.
  \]
- So,
  \[
  \text{proj}_a \mathbf{b} = (\mathbf{b} \cdot \mathbf{n}) \mathbf{n} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2} \mathbf{a}
  \]
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \).
- Then
\[
\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7} \mathbf{a} = \text{unit vector parallel to } \mathbf{a}.
\]
- So,
\[
\text{proj}_{\mathbf{a}} \mathbf{b} = (\mathbf{b} \cdot \mathbf{n}) \mathbf{n} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2} \mathbf{a} = \frac{1}{49} \langle 1, 2, 3 \rangle \cdot \langle 3, 6, -2 \rangle \langle 3, 6, -2 \rangle.
\]
Problem 1(a) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write down the vector projection of \( \mathbf{b} \) along \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have \( |\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7 \).
- Then
  \[
  \mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7} \mathbf{a} = \text{unit vector parallel to } \mathbf{a}.
  \]
- So,
  \[
  \text{proj}_{\mathbf{a}} \mathbf{b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2} \mathbf{a} = \frac{1}{49} \langle 1, 2, 3 \rangle \cdot \langle 3, 6, -2 \rangle \langle 3, 6, -2 \rangle = \frac{9}{49} \langle 3, 6, -2 \rangle.
  \]
Problem 1(b) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)
Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

**Solution:**

- We have

\[
\mathbf{b} = \langle 1, 2, 3 \rangle
\]
Problem 1(b) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- We have

\[
\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49} \langle 3, 6, -2 \rangle + \frac{9}{49} \langle 3, 6, -2 \rangle
\]
Problem 1(b) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

We have

\[
\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49} \langle 3, 6, -2 \rangle + \frac{9}{49} \langle 3, 6, -2 \rangle
\]

\[=
\frac{1}{49} \langle 22, 44, 165 \rangle + \frac{9}{49} \langle 3, 6, -2 \rangle.
\]
Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

  $$\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49} \langle 3, 6, -2 \rangle + \frac{9}{49} \langle 3, 6, -2 \rangle$$

  $$= \frac{1}{49} \langle 22, 44, 165 \rangle + \frac{9}{49} \langle 3, 6, -2 \rangle.$$

- Here

  $$\frac{9}{49} \langle 3, 6, -2 \rangle \text{ parallel to } \mathbf{a} = \langle 3, 6, -2 \rangle$$

  and

  $$\frac{1}{49} \langle 22, 44, 165 \rangle \text{ orthogonal to } \mathbf{a} = \langle 3, 6, -2 \rangle.$$
Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)
Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).

Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

Why so? All we did was to write

\[
\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n}) \mathbf{n} + (\mathbf{b} \cdot \mathbf{n}) \mathbf{n}
\]

where \( \mathbf{n} = \frac{\mathbf{a}}{7}, \mathbf{n}^2 = 1. \)
Problem 1(b) Continuation - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- Why so? All we did was to write
  \[
  \mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}
  \]
  where \( \mathbf{n} = \frac{\mathbf{a}}{7} \), \( \mathbf{n}^2 = 1 \).
- Of course this is the same as
  \[
  \mathbf{b} = (\mathbf{b} - \text{proj}_\mathbf{a} \mathbf{b}) + \text{proj}_\mathbf{a} \mathbf{b}.
  \]
Problem 1(b) Continuation - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- Why so? All we did was to write
  \[
  \mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}
  \]
  where \( \mathbf{n} = \frac{\mathbf{a}}{7}, \mathbf{n}^2 = 1 \).
- Of course this is the same as
  \[
  \mathbf{b} = (\mathbf{b} - \text{proj}_a \mathbf{b}) + \text{proj}_a \mathbf{b}.
  \]
  That is, we write \( \mathbf{b} \) as \( \text{proj}_a \mathbf{b} \) plus “the rest”. 
Given \( \mathbf{a} = \langle 3, 6, -2 \rangle, \quad \mathbf{b} = \langle 1, 2, 3 \rangle. \)
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- Why so? All we did was to write

\[
\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}
\]

where \( \mathbf{n} = \frac{\mathbf{a}}{7}, \quad \mathbf{n}^2 = 1. \)

- Of course this is the same as

\[
\mathbf{b} = (\mathbf{b} - \operatorname{proj}_a \mathbf{b}) + \operatorname{proj}_a \mathbf{b}.
\]

That is, we write \( \mathbf{b} \) as \( \operatorname{proj}_a \mathbf{b} \) plus “the rest”. But “the rest” is orthogonal to \( \mathbf{n} \) (and to \( \mathbf{a} \)), since

\[
(\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n}) \cdot \mathbf{n}
\]
Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write
  $$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$
  where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.
- Of course this is the same as
  $$\mathbf{b} = (\mathbf{b} - \text{proj}_\mathbf{a}\mathbf{b}) + \text{proj}_\mathbf{a}\mathbf{b}.$$

That is, we write $\mathbf{b}$ as $\text{proj}_\mathbf{a}\mathbf{b}$ plus “the rest”. But “the rest” is orthogonal to $\mathbf{n}$ (and to $\mathbf{a}$), since
$$\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} - (\mathbf{b} \cdot \mathbf{n})(\mathbf{n} \cdot \mathbf{n})$$
Problem 1(b) Continuation - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Write \( \mathbf{b} \) as a sum of a vector parallel to \( \mathbf{a} \) and a vector orthogonal to \( \mathbf{a} \). (Hint: Use projections.)

Solution:

- Why so? All we did was to write
  \[
  \mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}
  \]
  where \( \mathbf{n} = \frac{\mathbf{a}}{7}, \mathbf{n}^2 = 1 \).
- Of course this is the same as
  \[
  \mathbf{b} = (\mathbf{b} - \text{proj}_\mathbf{a} \mathbf{b}) + \text{proj}_\mathbf{a} \mathbf{b}.
  \]
  That is, we write \( \mathbf{b} \) as \( \text{proj}_\mathbf{a} \mathbf{b} \) plus “the rest”. But “the rest” is orthogonal to \( \mathbf{n} \) (and to \( \mathbf{a} \)), since
  \[
  (\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n}) \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} - (\mathbf{b} \cdot \mathbf{n})(\mathbf{n} \cdot \mathbf{n}) = 0, \text{ as } \mathbf{n} \cdot \mathbf{n} = 1.
  \]
Problem 1(c) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Let \( \theta \) be the angle between \( \mathbf{a} \) and \( \mathbf{b} \). Find \( \cos \theta \).
Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \). Let \( \theta \) be the angle between \( \mathbf{a} \) and \( \mathbf{b} \). Find \( \cos \theta \).

Solution:

\[
\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}
\]
Problem 1(c) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle, \quad \mathbf{b} = \langle 1, 2, 3 \rangle \).

Let \( \theta \) be the angle between \( \mathbf{a} \) and \( \mathbf{b} \). Find \( \cos \theta \).

Solution:

\[
\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}|| \cdot ||\mathbf{b}||} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{||\langle 3, 6, -2 \rangle|| \cdot ||\langle 1, 2, 3 \rangle||}
\]
Problem 1(c) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \).
Let \( \theta \) be the angle between \( \mathbf{a} \) and \( \mathbf{b} \). Find \( \cos \theta \).

Solution:

\[
\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{|\langle 3, 6, -2 \rangle| |\langle 1, 2, 3 \rangle|} = \frac{9}{\sqrt{49} \sqrt{14}}
\]
Problem 1(c) - Spring 2009

Given \( \mathbf{a} = \langle 3, 6, -2 \rangle \), \( \mathbf{b} = \langle 1, 2, 3 \rangle \). Let \( \theta \) be the angle between \( \mathbf{a} \) and \( \mathbf{b} \). Find \( \cos \theta \).

Solution:

\[
\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{\|\langle 3, 6, -2 \rangle\|\|\langle 1, 2, 3 \rangle\|} = \frac{9}{\sqrt{49} \sqrt{14}} = \frac{9}{7 \sqrt{14}}.
\]
Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$. Let $L$ be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for $L$. 

Solution:

To get parametric equations for $L$, you need a point through which the line passes and a vector parallel to the line. For example, take the point to be $A$ and the vector to be $\vec{AB}$. The vector equation of $L$ is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5 \rangle + t\langle 4, -5, -3 \rangle = \langle -1 + 4t, 7 - 5t, 5 - 3t \rangle,$$

where $O$ is the origin.

The parametric equations are:

$$\begin{align*}
x &= -1 + 4t \\
y &= 7 - 5t \\
z &= 5 - 3t
\end{align*}$$

$t \in \mathbb{R}$. 

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$. Let $L$ be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for $L$.

Solution:

- To get **parametric equations** for $L$ you need a point through which the line passes and a vector parallel to the line.
Problem 2(a) - Spring 2009

Given \( A = (-1, 7, 5) \), \( B = (3, 2, 2) \) and \( C = (1, 2, 3) \). Let \( L \) be the line which passes through the points \( A = (-1, 7, 5) \) and \( B = (3, 2, 2) \). Find the parametric equations for \( L \).

Solution:

- To get **parametric equations** for \( L \) you need a point through which the line passes and a vector parallel to the line. For example, take the point to be \( A \) and the vector to be \( \overrightarrow{AB} \).
Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$. Let $L$ be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for $L$.

Solution:

- To get **parametric equations** for $L$ you need a point through which the line passes and a vector parallel to the line. For example, take the point to be $A$ and the vector to be $\overrightarrow{AB}$.

- The vector equation of $L$ is

  $$r(t) = \overrightarrow{OA} + t\overrightarrow{AB}$$
Problem 2(a) - Spring 2009

Given \( A = (-1, 7, 5), \ B = (3, 2, 2) \) and \( C = (1, 2, 3) \).
Let \( L \) be the line which passes through the points \( A = (-1, 7, 5) \) and \( B = (3, 2, 2) \). Find the parametric equations for \( L \).

Solution:

- To get **parametric equations** for \( L \) you need a point through which the line passes and a vector parallel to the line. For example, take the point to be \( A \) and the vector to be \( \overrightarrow{AB} \).
- The vector equation of \( L \) is

\[
r(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5 \rangle + t \langle 4, -5, -3 \rangle
\]
Problem 2(a) - Spring 2009

Given \( A = (-1, 7, 5), \ B = (3, 2, 2) \) and \( C = (1, 2, 3) \). Let \( L \) be the line which passes through the points \( A = (-1, 7, 5) \) and \( B = (3, 2, 2) \). Find the parametric equations for \( L \).

Solution:

- To get **parametric equations** for \( L \) you need a point through which the line passes and a vector parallel to the line. For example, take the point to be \( A \) and the vector to be \( \overrightarrow{AB} \).
- The vector equation of \( L \) is

\[
\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5 \rangle + t \langle 4, -5, -3 \rangle = \langle -1 + 4t, 7 - 5t, 5 - 3t \rangle,
\]

where \( O \) is the origin.
Problem 2(a) - Spring 2009

Given \( A = (-1, 7, 5) \), \( B = (3, 2, 2) \) and \( C = (1, 2, 3) \).
Let \( L \) be the line which passes through the points \( A = (-1, 7, 5) \) and \( B = (3, 2, 2) \). Find the parametric equations for \( L \).

Solution:

- To get **parametric equations** for \( L \) you need a point through which the line passes and a vector parallel to the line. For example, take the point to be \( A \) and the vector to be \( \overrightarrow{AB} \).
- The vector equation of \( L \) is
  \[ r(t) = \overrightarrow{OA} + t \overrightarrow{AB} = \langle -1, 7, 5 \rangle + t \langle 4, -5, -3 \rangle = \langle -1 + 4t, 7 - 5t, 5 - 3t \rangle, \]
  where \( O \) is the origin.
- The **parametric equations** are:
  \[
  \begin{align*}
  x &= -1 + 4t \\
  y &= 7 - 5t, \quad t \in \mathbb{R} \\
  z &= 5 - 3t
  \end{align*}
  \]
Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$. $A$, $B$ and $C$ are three of the four vertices of a parallelogram, while $CA$ and $CB$ are two of the four edges. Find the fourth vertex.
Problem 2(b) - Spring 2009

Given \( A = (-1, 7, 5) \), \( B = (3, 2, 2) \) and \( C = (1, 2, 3) \). 
\( A \), \( B \) and \( C \) are three of the four vertices of a parallelogram, 
while \( CA \) and \( CB \) are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by \( D \).
Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$. $A$, $B$ and $C$ are three of the four vertices of a parallelogram, while $CA$ and $CB$ are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by $D$. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB}$$
Problem 2(b) - Spring 2009

Given \( A = (-1, 7, 5), \ B = (3, 2, 2) \) and \( C = (1, 2, 3) \). \( A, B \) and \( C \) are three of the four vertices of a parallelogram, while \( CA \) and \( CB \) are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by \( D \). Then

\[
\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle
\]
Problem 2(b) - Spring 2009

Given \( A = (-1, 7, 5), \) \( B = (3, 2, 2) \) and \( C = (1, 2, 3) \). \( A, B \) and \( C \) are three of the four vertices of a parallelogram, while \( CA \) and \( CB \) are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by \( D \). Then

\[
\vec{OD} = \vec{OA} + \vec{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle = \langle 1, 7, 4 \rangle,
\]

where \( O \) is the origin.
Problem 2(b) - Spring 2009

Given \( A = (-1, 7, 5) \), \( B = (3, 2, 2) \) and \( C = (1, 2, 3) \). \( A \), \( B \) and \( C \) are three of the four vertices of a parallelogram, while \( CA \) and \( CB \) are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by \( D \). Then

\[
\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle = \langle 1, 7, 4 \rangle,
\]

where \( O \) is the origin. That is,

\[ D = (1, 7, 4). \]
Consider the points $P(1, 3, 5), Q(-2, 1, 2), R(1, 1, 1)$ in $\mathbb{R}^3$. Find an equation for the plane containing $P, Q$ and $R$. 

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point $P$, it suffices to find $\mathbf{n}$.

Note that:

$$\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle = 2 \langle 1, -6, 3 \rangle.$$

So the equation of the plane is:

$$(x - 1) - 6(y - 3) + 3(z - 5) = 0.$$
Consider the points $P(1, 3, 5), Q(-2, 1, 2), R(1, 1, 1)$ in $\mathbb{R}^3$. Find an equation for the plane containing $P, Q$ and $R$.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point $P$, it suffices to find $\mathbf{n}$. 

Note that:

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So the equation of the plane is:

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Problem 3(a) - Spring 2009

Consider the points \( P(1, 3, 5), \ Q(-2, 1, 2), \ R(1, 1, 1) \) in \( \mathbb{R}^3 \).
Find an equation for the plane containing \( P, \ Q \) and \( R \).

Solution:

Since a plane is determined by its normal vector \( \mathbf{n} \) and a point on it, say the point \( P \), it suffices to find \( \mathbf{n} \). Note that:

\[
\mathbf{n} = \mathbf{PQ} \times \mathbf{PR}
\]
Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find an equation for the plane containing $P$, $Q$ and $R$.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point $P$, it suffices to find $\mathbf{n}$. Note that:

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$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle = 2\langle 1, -6, 3 \rangle.$$
Consider the points \( P(1, 3, 5), \ Q(-2, 1, 2), \ R(1, 1, 1) \) in \( \mathbb{R}^3 \). Find an equation for the plane containing \( P, \ Q \) and \( R \).

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\[
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\]

So the equation of the plane is:

\[
(x - 1) - 6(y - 3) + 3(z - 5) = 0.
\]
Consider the points $P(1, 3, 5), Q(-2, 1, 2), R(1, 1, 1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices $P, Q, R$. 

Solution: The area of the triangle $\Delta$ with vertices $P, Q, R$ can be found by taking the area of the parallelogram spanned by $\vec{PQ}$ and $\vec{PR}$ and dividing it by 2. Thus, using \text{(a)}, we have:

$$\text{Area}(\Delta) = \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} |\langle 1, -6, 3 \rangle| = \sqrt{1 + 36 + 9} = \sqrt{46}.$$
Problem 3(b) - Spring 2009

Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices $P$, $Q$, $R$.

Solution:

The area of the triangle $\Delta$ with vertices $P$, $Q$, $R$ can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2.
Problem 3(b) - Spring 2009

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\[
\text{Area}(\Delta) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2}
\]
Problem 3(b) - Spring 2009

Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices $P$, $Q$, $R$.

Solution:

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$$\text{Area}(\Delta) = \frac{|\vec{PQ} \times \vec{PR}|}{2} = \frac{1}{2} |2\langle 1, -6, 3 \rangle|$$
Problem 3(b) - Spring 2009

Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices $P$, $Q$, $R$.

Solution:

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$$= \sqrt{1 + 36 + 9}$$
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\[
\text{Area}(\Delta) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2} = \frac{1}{2} |2\langle 1, -6, 3 \rangle|
\]

\[
= \sqrt{1 + 36 + 9} = \sqrt{46}.
\]
Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes 
\( x + y + 3z = 1 \) and \( x - y + 2z = 0 \).
Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $v$ parallel to the line is the cross product of the normal vectors of the planes:

$$v = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle$$
Find parametric equations for the line of intersection of the planes
\( x + y + 3z = 1 \) and \( x - y + 2z = 0 \).

Solution:
A vector \( \mathbf{v} \) parallel to the line is the cross product of the
normal vectors of the planes:

\[
\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix}
i & j & k \\
1 & 1 & 3 \\
1 & -1 & 2 \\
\end{vmatrix}
\]

A point on \( L \) is any \((x_0, y_0, z_0)\) that satisfies the equations of
both planes. Setting \( z = 0 \), we obtain the equations
\( x + y = 1 \) and \( x - y = 1 \) and find such a point \((\frac{1}{2}, \frac{1}{2}, 0)\).

Therefore parametric equations for \( L \) are:

\[
\begin{align*}
x &= \frac{1}{2} + 5t \\
y &= \frac{1}{2} + t \\
z &= -2t
\end{align*}
\]
Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:
  
  $\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix} = \langle 5, 1, -2 \rangle$.

- A point on $L$ is any $(x_0, y_0, z_0)$ that satisfies the equations of both planes.
Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$\mathbf{v} = \langle 1, 1, 3 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix} = \langle 5, 1, -2 \rangle.$$

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Setting $z = 0$, we obtain the equations $x + y = 1$ and $x - y = 1$ and find such a point $(\frac{1}{2}, \frac{1}{2}, 0)$. Therefore parametric equations for $L$ are:

$$\begin{align*}
x &= \frac{1}{2} + 5t \\
y &= \frac{1}{2} + t \\
z &= -2t.
\end{align*}$$
Problem 5(a) - Spring 2009

Consider the parametrised curve

\[ r(t) = \langle t, t^2, t^3 \rangle, \quad t \in \mathbb{R}. \]

Set up an integral for the length of the arc between \( t = 0 \) and \( t = 1 \). Do \textbf{not} attempt to evaluate the integral.

\[ L = \int_{0}^{1} \sqrt{1 + 4t^2 + 9t^4} \, dt. \]
Consider the parametrised curve

\[ \mathbf{r}(t) = \langle t, t^2, t^3 \rangle, \quad t \in \mathbb{R}. \]

Set up an integral for the length of the arc between \( t = 0 \) and \( t = 1 \). Do not attempt to evaluate the integral.

Solution:

- The velocity field is:

\[ \mathbf{v}(t) = \mathbf{r}'(t) \]
Consider the parametrised curve

\[ \mathbf{r}(t) = \langle t, t^2, t^3 \rangle, \quad t \in \mathbb{R}. \]

Set up an integral for the length of the arc between \( t = 0 \) and \( t = 1 \). Do **not** attempt to evaluate the integral.

**Solution:**

- The **velocity field** is:

  \[ \mathbf{v}(t) = \mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle. \]
Problem 5(a) - Spring 2009

Consider the parametrised curve

\[ r(t) = \langle t, t^2, t^3 \rangle, \ t \in \mathbb{R}. \]

Set up an integral for the length of the arc between \( t = 0 \) and \( t = 1 \). Do **not** attempt to evaluate the integral.

Solution:

- The velocity field is:
  \[ v(t) = r'(t) = \langle 1, 2t, 3t^2 \rangle. \]

- Then the speed is
  \[ |r'(t)| = \sqrt{1 + 4t^2 + 9t^4}. \]
Problem 5(a) - Spring 2009

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Set up an integral for the length of the arc between \( t = 0 \) and \( t = 1 \). Do not attempt to evaluate the integral.

Solution:

- The velocity field is:
  \[ v(t) = r'(t) = \langle 1, 2t, 3t^2 \rangle. \]

- Then the speed is
  \[ |r'(t)| = \sqrt{1 + 4t^2 + 9t^4}. \]

- Therefore, the length of the arc is:
  \[ L = \int_0^1 \sqrt{1 + 4t^2 + 9t^4} \, dt. \]
Problem 5(b) - Spring 2009

Consider the parametrised curve

\[ \mathbf{r}(t) = \langle t, t^2, t^3 \rangle, \ t \in \mathbb{R}. \]

Write down the parametric equations of tangent line to \( \mathbf{r}(t) \) at \((2, 4, 8)\).
Problem 5(b) - Spring 2009

Consider the parametrised curve
\[ r(t) = \langle t, t^2, t^3 \rangle, \quad t \in \mathbb{R}. \]

Write down the parametric equations of tangent line to \( r(t) \) at \((2, 4, 8)\).

Solution:
- The parametrized curve passes through the point \((2, 4, 8)\) if and only if
  \[ t = 2, \quad t^2 = 4, \quad t^3 = 8 \iff t = 2. \]
Consider the parametrised curve
\[ r(t) = \langle t, t^2, t^3 \rangle, \ t \in \mathbb{R}. \]

Write down the parametric equations of tangent line to \( r(t) \) at \((2, 4, 8)\).

Solution:

- The parametrized curve passes through the point \((2, 4, 8)\) if and only if \( t = 2, \ t^2 = 4, \ t^3 = 8 \iff t = 2 \).
- The \textbf{velocity vector field} to the curve is given by
  \[ r'(t) = \langle 1, 2t, 3t^2 \rangle \text{ hence } r'(2) = \langle 1, 4, 12 \rangle. \]
Consider the parametrised curve
\[ \mathbf{r}(t) = \langle t, t^2, t^3 \rangle, \quad t \in \mathbb{R}. \]
Write down the parametric equations of tangent line to \( \mathbf{r}(t) \) at \( (2, 4, 8) \).

Solution:

- The parametrized curve passes through the point \((2, 4, 8)\) if and only if
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- The velocity vector field to the curve is given by
  \[ \mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle \text{ hence } \mathbf{r}'(2) = \langle 1, 4, 12 \rangle. \]
- The equation of the tangent line in question is:
  \[
  \begin{align*}
  x &= 2 + \tau \\
  y &= 4 + 4\tau, \quad \tau \in \mathbb{R} \\
  z &= 8 + 12\tau
  \end{align*}
  \]
Problem 5(b) - Spring 2009

Consider the parametrised curve
\[ r(t) = \langle t, t^2, t^3 \rangle, \ t \in \mathbb{R}. \]

Write down the parametric equations of tangent line to \( r(t) \) at \( (2, 4, 8) \).

Solution:

- The parametrized curve passes through the point \( (2, 4, 8) \) if and only if
  \[ t = 2, \ t^2 = 4, \ t^3 = 8 \iff t = 2. \]
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  \[ r'(t) = \langle 1, 2t, 3t^2 \rangle \text{ hence } r'(2) = \langle 1, 4, 12 \rangle. \]
- The \textbf{equation of the tangent line} in question is:
  \[
  \begin{align*}
  x &= 2 + \tau \\
  y &= 4 + 4\tau, \quad \tau \in \mathbb{R} \\
  z &= 8 + 12\tau
  \end{align*}
  \]

\textbf{Caution:} The parameter along the line, \( \tau \), has nothing to do with the parameter along the curve, \( t \).
Consider the sphere $S$ in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$ 

Find its center $C$ and its radius $R$. 

Solution:

Completing the square we get

$$(x - 2)^2 + y^2 + (z - 3)^2 = 16.$$ 

This gives:

$C = (2, 0, 3)$

$R = 4$.
Problem 6(a) - Spring 2009

Consider the sphere $S$ in $\mathbb{R}^3$ given by the equation

$$\begin{align*}
x^2 + y^2 + z^2 - 4x - 6z - 3 &= 0.
\end{align*}$$

Find its center $C$ and its radius $R$.

Solution:

- Completing the square we get

$$\begin{align*}
(x - 2)^2 - 4 + y^2 + (z - 3)^2 - 9 - 3 &= 0 \\
\iff \\
(x - 2)^2 + y^2 + (z - 3)^2 &= 16.
\end{align*}$$
Consider the sphere $S$ in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

Find its center $C$ and its radius $R$.

**Solution:**

- Completing the square we get

$$(x - 2)^2 - 4 + y^2 + (z - 3)^2 - 9 - 3 = 0$$

$$\iff$$

$$(x - 2)^2 + y^2 + (z - 3)^2 = 16.$$

- This gives:

$$C = (2, 0, 3) \quad R = 4$$
What does the equation $x^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.
Problem 6(b) - Spring 2009

What does the equation $x^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.

Solution:

This a (straight, circular) **cylinder** determined by the circle in the $xz$-plane of radius 2 and center $(0, 0)$ and parallel to the $y$-axis.
Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of $45^\circ$ to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s$^2$ and neglect air friction.

(a) Find the velocity function $v(t)$ and the position function $r(t)$ of the ball. Use coordinates in the $xy$-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and $y$ represents the height.

Solution:

Acceleration due to gravity is $\mathbf{a} = \langle 0, -10 \rangle$.

Initial velocity is $v(0) = 12 \langle \cos \pi/4, \sin \pi/4 \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle$.

So the velocity function is $v(t) = v(0) + \mathbf{a}t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle$.

One can recover the position by integrating the velocity:

$r(t) = \int_0^t v(\tau)d\tau + r(0)$.

Notice the initial position is $r(0) = \langle 0, 2 \rangle$.

This integral yields:

$r(t) = r(0) + v(0)t + \mathbf{a}t^2/2 = \langle 6\sqrt{2}t, 2 + 6\sqrt{2}t - 5t^2 \rangle$. 

Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(a) Find the velocity function \( v(t) \) and the position function \( r(t) \) of the ball. Use coordinates in the \( xy \)-plane to describe what is happening; assume Jane is standing with her feet at the point \((0, 0)\) and \( y \) represents the height.

Solution:

- Acceleration due to gravity is \( a = \langle 0, -g \rangle \)
Jane throws a basketball at an angle of $45^\circ$ to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s$^2$ and neglect air friction.

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Solution:

- Acceleration due to gravity is $a = \langle 0, -g \rangle = \langle 0, -10 \rangle$. 
Problem 7(a) - Spring 2009

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Solution:

- Acceleration due to gravity is \( \mathbf{a} = \langle 0, -g \rangle = \langle 0, -10 \rangle \). Initial velocity is \( \mathbf{v}(0) = 12 \langle \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \rangle \)
Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

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(a) Find the velocity function \( \mathbf{v}(t) \) and the position function \( \mathbf{r}(t) \) of the ball. Use coordinates in the \( xy \)-plane to describe what is happening; assume Jane is standing with her feet at the point \((0, 0)\) and \(y\) represents the height.

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  \[
  \mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a} \, d\tau
  \]
Problem 7(a) - Spring 2009

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So the velocity function is

$$v(t) = v(0) + \int_0^t a \, d\tau = v(0) + at$$
Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

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So the velocity function is

\[
\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a} \, d\tau = \mathbf{v}(0) + \mathbf{a} \, t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle.
\]
Jane throws a basketball at an angle of $45^\circ$ to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude $10 \text{ m/s}^2$ and neglect air friction.

(a) Find the velocity function $v(t)$ and the position function $r(t)$ of the ball. Use coordinates in the $xy$-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and $y$ represents the height.

Solution:

- Acceleration due to gravity is $a = \langle 0, -g \rangle = \langle 0, -10 \rangle$. Initial velocity is
  $$v(0) = 12 \langle \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle.$$ 
  So the velocity function is
  $$v(t) = v(0) + \int_0^t a \, d\tau = v(0) + at = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle.$$

  One can recover the position by integrating the velocity:
  $$r(t) = \int_0^t v(\tau) \, d\tau + r(0).$$
Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(a) Find the velocity function \( v(t) \) and the position function \( r(t) \) of the ball. Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point \((0, 0)\) and \( y \) represents the height.

Solution:

- Acceleration due to gravity is \( \mathbf{a} = \langle 0, -g \rangle = \langle 0, -10 \rangle \). Initial velocity is \( \mathbf{v}(0) = 12\langle \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle \).

So the velocity function is

\[
\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a} \, d\tau = \mathbf{v}(0) + \mathbf{a} t = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle.
\]

One can recover the position by integrating the velocity:

\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) \, d\tau + \mathbf{r}(0).
\]

Notice the initial position is \( \mathbf{r}(0) = \langle 0, 2 \rangle \).
Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of $45^\circ$ to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s$^2$ and neglect air friction.

(a) Find the velocity function $v(t)$ and the position function $r(t)$ of the ball. Use coordinates in the $xy$-plane to describe what is happening; assume Jane is standing with her feet at the point $(0, 0)$ and $y$ represents the height.

Solution:

- Acceleration due to gravity is $a = \langle 0, -g \rangle = \langle 0, -10 \rangle$. Initial velocity is $v(0) = 12 \langle \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle$.
  So the velocity function is

$$v(t) = v(0) + \int_0^t a \, d\tau = v(0) + at = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle.$$ 

One can recover the position by integrating the velocity:

$$r(t) = \int_0^t v(\tau) \, d\tau + r(0).$$

Notice the initial position is $r(0) = \langle 0, 2 \rangle$. This integral yields:

$$r(t) = r(0) + v(0)t + a \frac{t^2}{2}.$$
Problem 7(a) - Spring 2009

Jane throws a basketball at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(a) Find the velocity function \( v(t) \) and the position function \( r(t) \) of the ball.

Use coordinates in the xy-plane to describe what is happening; assume Jane is standing with her feet at the point \((0, 0)\) and \(y\) represents the height.

Solution:

- Acceleration due to gravity is \( a = \langle 0, -g \rangle = \langle 0, -10 \rangle \). Initial velocity is \( v(0) = 12 \langle \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \rangle = \langle 6\sqrt{2}, 6\sqrt{2} \rangle \).

So the velocity function is

\[
 v(t) = v(0) + \int_0^t a \, d\tau = v(0) + at = \langle 6\sqrt{2}, 6\sqrt{2} - 10t \rangle.
\]

One can recover the position by integrating the velocity:

\[
 r(t) = \int_0^t v(\tau) \, d\tau + r(0).
\]

Notice the initial position is \( r(0) = \langle 0, 2 \rangle \). This integral yields:

\[
 r(t) = r(0) + v(0)t + a \frac{t^2}{2} = \langle 6\sqrt{2}t, 2 + 6\sqrt{2}t - 5t^2 \rangle.
\]
Problem 7(b) - Spring 2009

Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s^2 and neglect air friction.

(b) Find the speed of the ball at its highest point.

Solution:
At the highest point, the vertical component of the velocity is zero, so we only need to calculate the horizontal component which is $6\sqrt{2}$. Thus the speed at the highest point is $6\sqrt{2}$. 
Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction. 

(b) Find the speed of the ball at its highest point.

Solution:

At the highest point, the vertical component of the velocity is zero, so we only need to calculate the horizontal component which is $6\sqrt{2}$.
Problem 7(b) - Spring 2009

Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(b) Find the speed of the ball at its highest point.

Solution:

At the highest point, the vertical component of the velocity is zero, so we only need to calculate the horizontal component which is \(6\sqrt{2}\). Thus the speed at the highest point is \(6\sqrt{2}\).
Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(c) At what time T does the ball reach its highest point.
Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(c) At what time $T$ does the ball reach its highest point.

Solution:

When the ball reaches its highest point, the vertical component of its velocity is zero.
Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(c) At what time $T$ does the ball reach its highest point.

Solution:

When the ball reaches its highest point, the vertical component of its velocity is zero. That is,

$$6\sqrt{2} - 10t = 0,$$
Problem 7(c) - Spring 2009

Jane throws a basketball from the ground at an angle of 45° to the horizontal at an initial speed of 12 m/s, where m denotes meters. It leaves her hand 2 m above the ground. Assume the acceleration of the ball due to gravity is downward with magnitude 10 m/s² and neglect air friction.

(c) At what time T does the ball reach its highest point.

Solution:

When the ball reaches its highest point, the vertical component of its velocity is zero. That is,

\[ 6\sqrt{2} - 10t = 0, \]

so \( T = \frac{3\sqrt{2}}{5} \).
Find **parametric equations** for the line $L$ which contains $A(1,2,3)$ and $B(4,6,5)$.

Solution: To get the **parametric equations** of $L$, you need a point through which the line passes and a vector parallel to the line. Take the point to be $A$ and the vector to be the $\vec{AB}$.

The vector equation of $L$ is $\mathbf{r}(t) = \vec{OA} + t \vec{AB} = \langle 1, 2, 3 \rangle + t \langle 3, 4, 2 \rangle = \langle 1 + 3t, 2 + 4t, 3 + 2t \rangle$.

The **parametric equations** are:

$$x = 1 + 3t, \quad y = 2 + 4t, \quad z = 3 + 2t, \quad t \in \mathbb{R}.$$
Problem 1(a) - Fall 2008

Find **parametric equations** for the line \( L \) which contains \( A(1, 2, 3) \) and \( B(4, 6, 5) \).

**Solution:**

- To get the **parametric equations** of \( L \) you need a point through which the line passes and a vector parallel to the line.

\[
\begin{align*}
\text{To get the parametric equations of } L & \text{ you need a point through which the line passes and a vector parallel to the line.}
\end{align*}
\]
Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

**Solution:**

- To get the **parametric equations** of **L** you need a point through which the line passes and a vector parallel to the line.
- Take the point to be $A$ and the vector to be the $\overrightarrow{AB}$. 

The vector equation of $L$ is

$$r(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle 1, 2, 3 \rangle + t\langle 3, 4, 2 \rangle = \langle 1 + 3t, 2 + 4t, 3 + 2t \rangle,$$

where $O$ is the origin.

The **parametric equations** are:

$$x = 1 + 3t$$
$$y = 2 + 4t$$
$$z = 3 + 2t$$

$t \in \mathbb{R}$. 


Find **parametric equations** for the line $L$ which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

**Solution:**

- To get the **parametric equations** of $L$ you need a point through which the line passes and a vector parallel to the line. $\overrightarrow{AB}$.
- Take the point to be $A$ and the vector to be $\overrightarrow{AB}$.
- The vector equation of $L$ is

$$r(t) = \overrightarrow{OA} + t\overrightarrow{AB}$$
Problem 1(a) - Fall 2008

Find **parametric equations** for the line \( L \) which contains \( A(1, 2, 3) \) and \( B(4, 6, 5) \).

**Solution:**

- To get the **parametric equations** of \( L \) you need a point through which the line passes and a vector parallel to the line. Take the point to be \( A \) and the vector to be the \( \overrightarrow{AB} \).
- The vector equation of \( L \) is

\[
\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle 1, 2, 3 \rangle + t \langle 3, 4, 2 \rangle
\]
Problem 1(a) - Fall 2008

Find **parametric equations** for the line $L$ which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

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- To get the **parametric equations** of $L$ you need a point through which the line passes and a vector parallel to the line. 
- Take the point to be $A$ and the vector to be the $\vec{AB}$.
- The vector equation of $L$ is

$$
\mathbf{r}(t) = \vec{OA} + t\vec{AB} = \langle 1, 2, 3 \rangle + t \langle 3, 4, 2 \rangle = \langle 1 + 3t, 2 + 4t, 3 + 2t \rangle,
$$

where $O$ is the origin.
Problem 1(a) - Fall 2008

Find **parametric equations** for the line \( L \) which contains \( A(1, 2, 3) \) and \( B(4, 6, 5) \).

**Solution:**
- To get the **parametric equations** of \( L \) you need a point through which the line passes and a vector parallel to the line. 
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  \[
  \mathbf{r}(t) = \vec{OA} + t\vec{AB} = \langle 1, 2, 3 \rangle + t\langle 3, 4, 2 \rangle = \langle 1 + 3t, 2 + 4t, 3 + 2t \rangle,
  \]
  where \( O \) is the origin.
- The **parametric equations** are:
  \[
  x = 1 + 3t \\
  y = 2 + 4t, \quad t \in \mathbb{R} \\
  z = 3 + 2t
  \]
Problem 1(b) - Fall 2008

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$. 

**Solution:**

The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $v$ can be taken to be: 

$$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = i + 3j + 5k.$$

Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we obtain:

$$x - 2y = 10 \\ 2x + y = 0.$$ 

Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0 \rangle$ lies on the line $L$. The parametric equations are:

$$x = 2 + t \quad y = -4 + 3t \quad z = 5t.$$
Find **parametric equations** for the line **L** of intersection of the planes \( x - 2y + z = 10 \) and \( 2x + y - z = 0 \).

**Solution:**

1. The vector part \( \mathbf{v} \) of the line **L** of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, -1 \rangle \).

2. Setting \( z = 0 \), we obtain:
   
   \[
   \begin{align*}
   x - 2y &= 10 \\
   2x + y &= 0
   \end{align*}
   \]

   Solving, we find that \( x = 2 \) and \( y = -4 \).

   Hence, \( P = \langle 2, -4, 0 \rangle \) lies on the line **L**.

   The parametric equations are:

   \[
   \begin{align*}
   x &= 2 + t \\
   y &= -4 + 3t \\
   z &= 0 + 5t = 5t
   \end{align*}
   \]
Problem 1(b) - Fall 2008

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

**Solution:**

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $v$ can be taken to be:

$$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle$$
Problem 1(b) - Fall 2008

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 10 \) and \( 2x + y - z = 0 \).

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  \[
  \mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  1 & -2 & 1 \\
  2 & 1 & -1 
\end{vmatrix}
  \]

- Choose \( P \in L \) so the \( z \)-coordinate of \( P \) is zero. Setting \( z = 0 \), we obtain:
  \[
  x - 2y = 10 \\
  2x + y = 0
  \]
  Solving, we find that \( x = 2 \) and \( y = -4 \).
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The parametric equations are:

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- \( y = -4 + 3t \)
- \( z = 5t \)
Problem 1(b) - Fall 2008

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

**Solution:**

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $v$ can be taken to be:
  $$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\hat{i} + 3\hat{j} + 5\hat{k}.$$

Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we obtain:

$$x - 2y = 10 \quad 2x + y = 0.$$

Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0 \rangle$ lies on the line $L$.

The parametric equations are:

$$x = 2 + t, \quad y = -4 + 3t, \quad z = 5t.$$
Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 10 \) and \( 2x + y - z = 0 \).

**Solution:**

- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, -1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:
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  \mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.
  \]
- Choose \( P \in L \) so the z-coordinate of \( P \) is zero.

Problem 1(b) - Fall 2008

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 10 \) and \( 2x + y - z = 0 \).

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  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  1 & -2 & 1 \\
  2 & 1 & -1 \\
  \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.
  \]

- Choose \( P \in L \) so the \( z \)-coordinate of \( P \) is zero. Setting \( z = 0 \), we obtain:
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  x - 2y = 10 \\
  2x + y = 0.
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Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 10 \) and \( 2x + y - z = 0 \).

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- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, -1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:
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  \]

- Choose \( P \in L \) so the \( z \)-coordinate of \( P \) is zero. Setting \( z = 0 \), we obtain:
  \[
  \begin{align*}
  x - 2y &= 10 \\
  2x + y &= 0.
  \end{align*}
  \]
  Solving, we find that \( x = 2 \) and \( y = -4 \).
Problem 1(b) - Fall 2008

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

**Solution:**

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $v$ can be taken to be:
  \[
  v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1i + 3j + 5k.
  \]

- Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we obtain:
  \[
  x - 2y = 10 \\
  2x + y = 0.
  \]
  Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0 \rangle$ lies on the line $L$. The parametric equations are:
  \[
  x = 2 + t \\
  y = -4 + 3t \\
  z = 5t.
  \]
Problem 1(b) - Fall 2008

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

**Solution:**

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $v$ can be taken to be:
  
  \[ v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}. \]

- Choose $P \in L$ so the z-coordinate of $P$ is zero. Setting $z = 0$, we obtain:
  
  \[ x - 2y = 10 \]
  
  \[ 2x + y = 0. \]

  Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0 \rangle$ lies on the line $L$.

- The **parametric equations** are:
Find parametric equations for the line $L$ of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

Solution:

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$. Hence $v$ can be taken to be:
  
  $$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1i + 3j + 5k.$$

- Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we obtain:
  
  $$x - 2y = 10$$
  $$2x + y = 0.$$

  Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0 \rangle$ lies on the line $L$.

- The parametric equations are:
  
  $$x = 2 + t$$
  $$y = -4 + 3t$$
  $$z = 0 + 5t = 5t.$$
Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$. 
Problem 2(a) - Fall 2008

Find an equation of the plane which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

Method 1

Consider the vectors $\vec{PQ} = \langle 2, -2, 0 \rangle$ and $\vec{PR} = \langle 3, 0, -2 \rangle$ which lie parallel to the plane.
Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

**Solution:**

**Method 1**

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -2, 0 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -2 \rangle$ which lie parallel to the plane.
- Then consider the normal vector:

$$n = \overrightarrow{PQ} \times \overrightarrow{PR}$$
Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points \( P(-1, 0, 1), \) \( Q(1, -2, 1) \) and \( R(2, 0, -1) \).

**Solution:**

**Method 1**

- Consider the vectors \( \overrightarrow{PQ} = \langle 2, -2, 0 \rangle \) and \( \overrightarrow{PR} = \langle 3, 0, -2 \rangle \) which lie parallel to the plane.

- Then consider the normal vector:

\[
\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & -2 & 0 \\
3 & 0 & -2
\end{vmatrix}
\]

So the equation of the plane is given by:

\[
\langle 4, 4, 6 \rangle \cdot \langle x + 1, y, z - 1 \rangle = 4(x + 1) + 4y + 6(z - 1) = 0.
\]
Find an **equation of the plane** which contains the points $P(-1, 0, 1), Q(1, -2, 1)$ and $R(2, 0, -1)$.

**Solution:**

**Method 1**

- Consider the vectors $\vec{PQ} = \langle 2, -2, 0 \rangle$ and $\vec{PR} = \langle 3, 0, -2 \rangle$ which lie parallel to the plane.
- Then consider the normal vector:

$$\mathbf{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$
Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points \( P(-1, 0, 1), \ Q(1, -2, 1) \) and \( R(2, 0, -1) \).

**Solution:**

**Method 1**

- Consider the vectors \( \overrightarrow{PQ} = \langle 2, -2, 0 \rangle \) and \( \overrightarrow{PR} = \langle 3, 0, -2 \rangle \) which lie parallel to the plane.

- Then consider the normal vector:

\[
\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & -2 & 0 \\
3 & 0 & -2 \\
\end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.
\]

- So the **equation of the plane** is given by:

\[
\langle 4, 4, 6 \rangle \cdot \langle x + 1, y, z - 1 \rangle = 4(x + 1) + 4y + 6(z - 1) = 0.
\]
Problem 2(a) - Fall 2008

Find an equation of the plane which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.
Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

**Solution:**

**Method 2**

- The plane consists of all the points $S(x, y, z) \in \mathbb{R}^3$, such that \( \overrightarrow{PS}, \overrightarrow{PQ} \) and \( \overrightarrow{PR} \) are in the same plane (coplanar).
Problem 2(a) - Fall 2008

Find an equation of the plane which contains the points \( P(-1, 0, 1) \), \( Q(1, -2, 1) \) and \( R(2, 0, -1) \).

Solution:

**Method 2**

- The plane consists of all the points \( S(x, y, z) \in \mathbb{R}^3 \), such that \( \vec{PS}, \vec{PQ} \) and \( \vec{PR} \) are in the same plane (coplanar).
- But this happens if and only if their box product is zero.
Problem 2(a) - Fall 2008

Find an equation of the plane which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

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- So the equation of the plane is:

\[
\begin{vmatrix}
  x + 1 & y & z - 1 \\
  2 & -2 & 0 \\
  3 & 0 & -2
\end{vmatrix} = 4(x + 1) + 4y + 6(z - 1) = 0.
\]
Problem 2(a) - Fall 2008

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$$
Problem 2(b) - Fall 2008

Find the distance $D$ from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$. 

Solution:

Recall the distance formula $D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.

In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.

So, the distance from $(1, 2, -1)$ to the plane is:

$$D = \frac{|2 \cdot 1 + 1 \cdot 6 + (-2) \cdot (-1) - 19|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{|-9|}{\sqrt{9}} = 3.$$
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$$D = \frac{|(2 \cdot 1) + (1 \cdot 6) + (-2 \cdot -1) - 19|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{|-9|}{\sqrt{9}} = 3.$$
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So, the distance from $(1, 6, -1)$ to the plane is:

$D = \frac{|2 \cdot 1 + 1 \cdot 6 - 2 \cdot (-1) - 19|}{\sqrt{2^2+1^2+(-2)^2}} = \frac{|-9|}{\sqrt{9}} = 3$. 

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Problem 2(b) - Fall 2008

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$$D = \frac{|(2 \cdot 1) + (1 \cdot 6) + (-2 \cdot -1) - 19|}{\sqrt{2^2 + 1^2 + (-2)^2}}$$
Problem 2(b) - Fall 2008

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Problem 2(c) - Fall 2008

Find the point $Q$ in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find $Q$ or first find the equation of the line $L$ passing through $Q$ and the point $(1, 6, -1)$ and then solve for $Q$.)

Solution:

The line $L$ in the Hint passes through $(1, 6, -1)$ and is parallel to $n = \langle 2, 1, -2 \rangle$. So, $L$ has parametric equations:

$$
\begin{align*}
  x &= 1 + 2t \\
y &= 6 + t \\
z &= -1 - 2t
\end{align*}
$$

$L$ intersects the plane $2x + y - 2z = 19$ if and only if

$$
2(1 + 2t) + (6 + t) - 2(-1 - 2t) = 19 \iff 9t = 9 \iff t = 1.
$$

Substituting $t = 1$ in the parametric equations of $L$ gives the point $Q = (3, 7, -3)$. 

Problem 2(c) - Fall 2008

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- So, $L$ has **parametric equations**:

  $$
x = 1 + 2t
  \quad y = 6 + t, \quad t \in \mathbb{R}.
  \quad z = -1 - 2t
  $$
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Find the point \( Q \) in the plane \( 2x + y - 2z = 19 \) which is closest to the point \((1, 6, -1)\). (Hint: You can use part b) of this problem to help find \( Q \) or first find the equation of the line \( L \) passing through \( Q \) and the point \((1, 6, -1)\) and then solve for \( Q \).)

Solution:

- The line \( L \) in the Hint passes through \((1, 6, -1)\) and is parallel to \( \mathbf{n} = \langle 2, 1, -2 \rangle \).
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- Substituting $t = 1$ in the **parametric equations** of $L$ gives the point $Q = (3, 7, -3)$. 
Problem 3(a) - Fall 2008

Find the volume $V$ of the parallelepiped such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices $B, C, D$ are all adjacent to the vertex $A$. 

Solution: The parallelepiped is determined by its edges $\mathbf{AB} = \langle 0, -3, -2 \rangle$, $\mathbf{AC} = \langle 1, 1, -3 \rangle$, $\mathbf{AD} = \langle -2, -4, -1 \rangle$. Its volume can be computed as the absolute value of the box product $\mathbf{AB} \cdot (\mathbf{AC} \times \mathbf{AD})$, i.e.,

$$V = \left| \begin{vmatrix} 0 & -3 & -2 \\ 1 & 1 & -3 \\ -2 & -4 & -1 \end{vmatrix} \right| = | -17 | = 17.$$
Problem 3(a) - Fall 2008

Find the volume \( V \) of the parallelepiped such that the following four points \( A = (3, 4, 0) \), \( B = (3, 1, -2) \), \( C = (4, 5, -3) \), \( D = (1, 0, -1) \) are vertices and the vertices \( B, C, D \) are all adjacent to the vertex \( A \).

Solution:

The parallelepiped is determined by its edges

\[
\vec{AB} = \langle 0, -3, -2 \rangle, \quad \vec{AC} = \langle 1, 1, -3 \rangle, \quad \vec{AD} = \langle -2, -4, -1 \rangle.
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Solution:

The parallelepiped is determined by its edges

$$\overrightarrow{AB} = \langle 0, −3, −2 \rangle, \quad \overrightarrow{AC} = \langle 1, 1, −3 \rangle, \quad \overrightarrow{AD} = \langle −2, −4, −1 \rangle.$$

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Problem 3(a) - Fall 2008

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\[
V = \left| \begin{array}{ccc}
0 & -3 & -2 \\
1 & 1 & -3 \\
-2 & -4 & -1 \\
\end{array} \right| = |3(-1 - 6) - 2(-4 + 2)|
\]

\[
= |3(-7) - 2(-2)| = |-21 + 4| = 17
\]
Problem 3(a) - Fall 2008

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Find the center and radius of the sphere
\[ x^2 - 4x + y^2 + 4y + z^2 = 8. \]
Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere

\[ x^2 - 4x + y^2 + 4y + z^2 = 8. \]

**Solution:**

- Completing the square we get

\[
x^2 - 4x + y^2 + 4y + z^2 = (x^2 - 4x + 4) - 4 + (y^2 + 4y + 4) - 4 + (z^2)
\]

This gives:

- **Center** = \((2, -2, 0)\)
- **Radius** = 4
Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere

\[ x^2 - 4x + y^2 + 4y + z^2 = 8. \]

**Solution:**

- Completing the square we get

\[
\begin{align*}
    x^2 - 4x + y^2 + 4y + z^2 &= (x^2 - 4x + 4) - 4 + (y^2 + 4y + 4) - 4 + z^2 \\
    &= (x - 2)^2 - 4 + (y + 2)^2 - 4 + z^2 = 8 \\
    \iff (x - 2)^2 + (y + 2)^2 + z^2 &= 16.
\end{align*}
\]
Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere

\[ x^2 - 4x + y^2 + 4y + z^2 = 8. \]

**Solution:**

- Completing the square we get

\[
x^2 - 4x + y^2 + 4y + z^2 = (x^2 - 4x + 4) - 4 + (y^2 + 4y + 4) - 4 + (z^2)
\]

\[
= (x - 2)^2 - 4 + (y + 2)^2 - 4 + z^2 = 8
\]

\[
\iff
\]

\[
(x - 2)^2 + (y + 2)^2 + z^2 = 16.
\]

- This gives:

**Center** = \((2, -2, 0)\) \hspace{1cm} **Radius** = 4
The position vector of a particle moving in space equals
\[ \mathbf{r}(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \] at any time \( t \geq 0 \).

a) Find an equation of the tangent line to the curve at the point \((4, -4, 2)\).
Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals
\[ r(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \] at any time \( t \geq 0 \).

a) Find an equation of the tangent line to the curve at the point \((4, -4, 2)\).

Solution:

- The parametrized curve passes through the point \((4, -4, 2)\) if and only if
  \[ t^2 = 4, \quad -t^2 = -4, \quad \frac{1}{2} t^2 = 2 \iff t^2 = 4 \iff t = \pm 2. \]
Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals 
\[ \mathbf{r}(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \] 
at any time \( t \geq 0 \).

a) Find an **equation of the tangent line** to the curve at the point \((4, -4, 2)\).

Solution:

- The parametrized curve passes through the point \((4, -4, 2)\) if and only if 
  \[ t^2 = 4, \quad -t^2 = -4, \quad \frac{1}{2} t^2 = 2 \iff t^2 = 4 \iff t = \pm 2. \]

- Since we have that \( t \geq 0 \), we are left with the choice \( t_0 = 2 \).
The position vector of a particle moving in space equals $r(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k}$ at any time $t \geq 0$.

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**Solution:**

- The parametrized curve passes through the point $(4, -4, 2)$ if and only if
  
  $$t^2 = 4, \quad -t^2 = -4, \quad \frac{1}{2} t^2 = 2 \iff t^2 = 4 \iff t = \pm 2.$$

- Since we have that $t \geq 0$, we are left with the choice $t_0 = 2$.

- The **velocity vector field** to the curve is given by
  
  $$r'(t) = \langle 2t, -2t, t \rangle \text{ hence } r'(2) = \langle 4, -4, 2 \rangle.$$
Problem 4(a) - Fall 2008

The position vector of a particle moving in space equals \( r(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \) at any time \( t \geq 0 \).

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Solution:

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  \[
  t^2 = 4, \quad -t^2 = -4, \quad \frac{1}{2} t^2 = 2 \quad \iff \quad t^2 = 4 \quad \iff \quad t = \pm 2.
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- The **velocity vector field** to the curve is given by \( r'(t) = \langle 2t, -2t, t \rangle \) hence \( r'(2) = \langle 4, -4, 2 \rangle \).
- The **equation of the tangent line** in question is:
  \[
  x = 4 + 4t \\
  y = -4 - 4t, \quad t \geq 0. \\
  z = 2 + 2t
  \]
Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals
\( \mathbf{r}(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \) at any time \( t \geq 0 \).

(b) Find the length \( L \) of the arc traveled from time \( t = 1 \) to time \( t = 4 \).
The position vector of a particle moving in space equals 

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(b) Find the length \( L \) of the arc traveled from time \( t = 1 \) to time \( t = 4 \).

Solution:

The velocity field is:

\[ \mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle. \]
Problem 4(b) - Fall 2008

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- The velocity field is:

  \[
  \mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle.
  \]

- Since \( t \geq 0 \), the speed is:

  \[
  |\mathbf{r}'(t)| = \sqrt{9t^2} \implies \mathbf{r}'(t) = 3t.
  \]
The position vector of a particle moving in space equals 
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- Therefore, the length is:
  \[ L = \int_{1}^{4} 3t \, dt \]
Problem 4(b) - Fall 2008

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\[ r(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \]
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- Since \( t \geq 0 \), the speed is:
  \[ |r'(t)| = \sqrt{9t^2} \Rightarrow |r'(t)| = 3t. \]

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  \[ L = \int_{1}^{4} 3t \, dt = \frac{3}{2} t^2 \bigg|_{1}^{4} \]
The position vector of a particle moving in space equals 
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Solution:

- The velocity field is:
  \[ v(t) = r'(t) = \langle 2t, -2t, t \rangle. \]

- Since \( t \geq 0 \), the speed is:
  \[ |r'(t)| = \sqrt{9t^2} \implies |r'(t)| = 3t. \]

- Therefore, the length is:
  \[ L = \int_{1}^{4} 3t \, dt = \frac{3}{2} t^2 \bigg|_{1}^{4} = \frac{3}{2} \cdot 16 - \frac{3}{2} \cdot 1 = 45 \].
Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals 
\( \mathbf{r}(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \) at any time \( t \geq 0 \).

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  \[ |\mathbf{r}'(t)| = \sqrt{9t^2} \implies |\mathbf{r}'(t)| = 3t. \]

- Therefore, the length is:
  \[ L = \int_1^4 3t \, dt = \frac{3}{2} t^2 \bigg|_1^4 = \frac{3}{2} \cdot 16 - \frac{3}{2} \cdot 1 = \frac{3}{2} \cdot 15. \]
Problem 4(b) - Fall 2008

The position vector of a particle moving in space equals 
\[ \mathbf{r}(t) = t^2 \mathbf{i} - t^2 \mathbf{j} + \frac{1}{2} t^2 \mathbf{k} \] 
at any time \( t \geq 0 \).

(b) Find the length \( L \) of the arc traveled from time \( t = 1 \) to time \( t = 4 \).

Solution:

- The velocity field is:
  \[ \mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t, -2t, t \rangle. \]

- Since \( t \geq 0 \), the speed is:
  \[ |\mathbf{r}'(t)| = \sqrt{9t^2} \implies |\mathbf{r}'(t)| = 3t. \]

- Therefore, the length is:
  \[ L = \int_{1}^{4} 3t \, dt = \frac{3}{2} t^2 \bigg|_{1}^{4} = \frac{3}{2} \cdot 16 - \frac{3}{2} \cdot 1 = \frac{3}{2} \cdot 15 = \frac{45}{2}. \]
Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, 2 \cos 2t, 3e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).
Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, 2 \cos 2t, 3e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

**Solution:**

- One can recover the position, by integrating the velocity:
Suppose a particle moving in space has velocity
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Solution:

- One can recover the position, by integrating the velocity:
\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) d\tau + \mathbf{r}(0)
\]
Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, 2 \cos 2t, 3e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- One can recover the position, by integrating the velocity:

\[ \mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) d\tau + \mathbf{r}(0) \]

- Carrying out this integral yields:

\[ \mathbf{r}(t) = \left\langle -\cos \tau \bigg|_0^t, \sin 2\tau \bigg|_0^t, 3e^t \bigg|_0^t \right\rangle + \langle 1, 2, 0 \rangle \]
Problem 4(c) - Fall 2008

Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, 2 \cos 2t, 3e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- One can recover the position, by integrating the velocity:
\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(\tau) d\tau + \mathbf{r}(0)
\]
- Carrying out this integral yields:
\[
\mathbf{r}(t) = \left[ -\cos \tau \right]_0^t , \left[ \sin 2\tau \right]_0^t , \left[ 3e^t \right]_0^t + \langle 1, 2, 0 \rangle
\]
\[
= \langle 2 - \cos t, 2 + \sin 2t, 3e^t - 3 \rangle.
\]
Problem 5(a) - Fall 2008

Consider the points $A(2, 1, 0)$, $B(3, 0, 2)$ and $C(0, 2, 1)$. Find the area of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)
Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\text{Area} = \frac{1}{2} \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|.$$
Problem 5(a) - Fall 2008

Consider the points $A(2, 1, 0), B(3, 0, 2)$ and $C(0, 2, 1)$. Find the area of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

\[
\text{Area} = \frac{1}{2} \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|
\]

- Since $\overrightarrow{AB} = \langle 1, -1, 2 \rangle$ and $\overrightarrow{AC} = \langle -2, 1, 1 \rangle$, \[
\left| \overrightarrow{AB} \times \overrightarrow{AC} \right| = \sqrt{35}.
\]
Consider the points $A(2,1,0)$, $B(3,0,2)$ and $C(0,2,1)$. Find the area of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\text{Area} = \frac{1}{2} \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|.$$ 

- Since $\overrightarrow{AB} = \langle 1, -1, 2 \rangle$ and $\overrightarrow{AC} = \langle -2, 1, 1 \rangle$,

$$\text{Area} = \frac{1}{2} \left| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ -2 & 1 & 1 \end{array} \right| = \frac{1}{2} |\langle -3, -5, -1 \rangle|.$$
Problem 5(a) - Fall 2008

Consider the points $A(2, 1, 0)$, $B(3, 0, 2)$ and $C(0, 2, 1)$. Find the area of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- Recall that:

$$\text{Area} = \frac{1}{2} \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|.$$ 

- Since $\overrightarrow{AB} = \langle 1, -1, 2 \rangle$ and $\overrightarrow{AC} = \langle -2, 1, 1 \rangle$,

$$\text{Area} = \frac{1}{2} \left| \begin{array}{ccc} i & j & k \\ 1 & -1 & 2 \\ -2 & 1 & 1 \end{array} \right| = \frac{1}{2} \left| \langle -3, -5, -1 \rangle \right| = \frac{1}{2} \sqrt{35}. $$
Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(2, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the coordinates of the fourth vertex.

Solution:
Denote the fourth vertex by $S$. Then $\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0, 1, 0 \rangle + \langle 2, 2, 0 \rangle = \langle 2, 3, 0 \rangle$, where $O$ is the origin. That is, $S = (2, 3, 0)$. 
Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are \( P(0, -1, 1) \), \( Q(0, 1, 0) \) and \( R(2, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by \( S \).
Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are \( P(0, -1, 1) \), \( Q(0, 1, 0) \) and \( R(2, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by \( S \). Then

\[
\vec{OS} = \vec{OQ} + \vec{PR} = \langle 0, 1, 0 \rangle + \langle 2, 2, 0 \rangle = \langle 2, 3, 0 \rangle ,
\]

where \( O \) is the origin.
Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(2, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $S$. Then

$$\vec{OS} = \vec{OQ} + \vec{PR} = \langle 0, 1, 0 \rangle + \langle 2, 2, 0 \rangle = \langle 2, 3, 0 \rangle,$$

where $O$ is the origin. That is,

$$S = (2, 3, 0).$$
Find an equation of the plane through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$. 

Solution: 

Since a plane is determined by its normal vector $n$ and a point on it, say the point $A$, it suffices to find $n$. Note that:

$$n = \vec{AB} \times \vec{AC} = \left| \begin{array}{ccc} i & j & k \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{array} \right| = \langle -1, 1, 2 \rangle.$$

So the equation of the plane is:

$$-(x - 1) + (y - 2) + 2(z - 3) = 0.$$
Find an equation of the plane through the points \( A = (1, 2, 3) \), \( B = (0, 1, 3) \), and \( C = (2, 1, 4) \).

Solution:

Since a plane is determined by its normal vector \( \mathbf{n} \) and a point on it, say the point \( A \), it suffices to find \( \mathbf{n} \).
Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points \( A = (1, 2, 3) \), \( B = (0, 1, 3) \), and \( C = (2, 1, 4) \).

**Solution:**

Since a plane is determined by its normal vector \( \mathbf{n} \) and a point on it, say the point \( A \), it suffices to find \( \mathbf{n} \). Note that:

\[
\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix}
i & j & k \\
-1 & -1 & 0 \\
1 & -1 & 1 \\
\end{vmatrix} = \langle -1, 1, 2 \rangle.
\]

So the equation of the plane is:

\[
-1(x - 1) + (y - 2) + 2(z - 3) = 0.
\]
Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

**Solution:**

Since a plane is determined by its normal vector $n$ and a point on it, say the point $A$, it suffices to find $n$. Note that:

$$n = AB \times AC = \begin{vmatrix} i & j & k \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \langle -1, 1, 2 \rangle.$$

So the **equation of the plane** is:

$$-(x - 1) + (y - 2) + 2(z - 3) = 0.$$
Problem 6(b) - Spring 2008

Find the area of the triangle \( \triangle \) with vertices at the points 
\( A = (1, 2, 3), \ B = (0, 1, 3), \) and \( C = (2, 1, 4). \)

Hint: the area of this triangle is related to the area of a certain parallelogram
Problem 6(b) - Spring 2008

Find the area of the triangle $\triangle$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

*Hint: the area of this triangle is related to the area of a certain parallelogram*

**Solution:**

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. 
Problem 6(b) - Spring 2008

Find the area of the triangle $\triangle$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

*Hint: the area of this triangle is related to the area of a certain parallelogram*

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\triangle$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2.

Thus:

\[
\text{Area}\left(\triangle\right) = \frac{1}{2} \left|\overrightarrow{AB} \times \overrightarrow{AC}\right|
\]

\[
= \frac{1}{2} \left|\begin{vmatrix}
1 & 1 & 2 \\
-1 & 2 & 0 \\
1 & 0 & 1 \\
\end{vmatrix}\right|
\]

\[
= \frac{1}{2} \sqrt{1 + 1 + 4}
\]

\[
= \frac{1}{2} \sqrt{6}
\]
Problem 6(b) - Spring 2008

Find the area of the triangle $\Delta$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

*Hint: the area of this triangle is related to the area of a certain parallelogram*

Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2}$$
Problem 6(b) - Spring 2008

Find the area of the triangle $\Delta$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

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Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\vec{AB}$ and $\vec{AC}$ and dividing by 2. Thus:

\[
\text{Area}(\Delta) = \frac{\left| \vec{AB} \times \vec{AC} \right|}{2} = \frac{1}{2} \left| \begin{array}{ccc}
\vec{i} & \vec{j} & \vec{k} \\
-1 & -1 & 0 \\
1 & -1 & 1
\end{array} \right|
\]
Problem 6(b) - Spring 2008

Find the area of the triangle \( \Delta \) with vertices at the points
\[ A = (1, 2, 3), \quad B = (0, 1, 3), \quad \text{and} \quad C = (2, 1, 4). \]

*Hint: the area of this triangle is related to the area of a certain parallelogram*

Solution:

Consider the points \( A = (1, 2, 3), \quad B = (0, 1, 3) \) and \( C = (2, 1, 4) \). Then the area of the triangle \( \Delta \) with these vertices can be found by taking the area of the parallelogram spanned by \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) and dividing by 2. Thus:

\[
\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left| \begin{array}{ccc}
  i & j & k \\
-1 & -1 & 0 \\
 1 & -1 & 1 \\
\end{array} \right|
\]

\[
= \frac{1}{2} |\langle-1, 1, 2\rangle|
\]
Problem 6(b) - Spring 2008

Find the area of the triangle $\Delta$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$. 

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\[
\text{Area}(\Delta) = \frac{\left| \overrightarrow{AB} \times \overrightarrow{AC} \right|}{2} = \frac{1}{2} \left| \begin{array}{ccc}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  -1 & -1 & 0 \\
  1 & -1 & 1 \\
\end{array} \right| \\
= \frac{1}{2} |\langle -1, 1, 2 \rangle| = \frac{1}{2} \sqrt{1 + 1 + 4} = \frac{1}{2} \sqrt{6}
\]
Problem 6(b) - Spring 2008

Find the area of the triangle $\Delta$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

*Hint: the area of this triangle is related to the area of a certain parallelogram*

**Solution:**

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \begin{vmatrix} i & j & k \\ -1 & -1 & 0 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2} |\langle-1, 1, 2\rangle| = \frac{1}{2} \sqrt{1 + 1 + 4} = \frac{1}{2} \sqrt{6}.$$
Find the **parametric equations** of the line passing through the point \((2, 4, 1)\) that is perpendicular to the plane \(3x - y + 5z = 77\).
Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point \((2, 4, 1)\) that is perpendicular to the plane \(3x - y + 5z = 77\).

**Solution:**

- The vector part of the line \(L\) is the normal vector \( \mathbf{n} = \langle 3, -1, 5 \rangle \) to the plane.
Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point \((2, 4, 1)\) that is perpendicular to the plane \(3x - y + 5z = 77\).

**Solution:**

- The vector part of the line \(L\) is the normal vector \(\mathbf{n} = \langle 3, -1, 5 \rangle\) to the plane.
- The **vector equation** of \(L\) is:
  \[
  \mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}
  \]
Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point \((2, 4, 1)\) that is perpendicular to the plane \(3x - y + 5z = 77\).

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- The **vector equation** of \(L\) is:
  \[
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  \]
  \[
  = \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle
  \]
Find the **parametric equations** of the line passing through the point \((2, 4, 1)\) that is perpendicular to the plane \(3x - y + 5z = 77\).

**Solution:**

- The vector part of the line \(L\) is the normal vector \(\mathbf{n} = \langle 3, -1, 5 \rangle\) to the plane.
- The **vector equation** of \(L\) is:
  \[
  \mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}
  \]
  \[
  = \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle = \langle 2 + 3t, 4 - t, 1 + 5t \rangle.
  \]
Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

**Solution:**

- The vector part of the line $L$ is the normal vector $n = \langle 3, -1, 5 \rangle$ to the plane.
- The **vector equation** of $L$ is:
  
  $$r(t) = \langle 2, 4, 1 \rangle + tn$$

  $$= \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle = \langle 2 + 3t, 4 - t, 1 + 5t \rangle.$$

- The **parametric equations** are:
  
  $$x = 2 + 3t$$
  $$y = 4 - t$$
  $$z = 1 + 5t.$$
Find the intersection point of the line $L(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution: By part (a), we have $L$ has parametric equations:

$$x = 2 + 3t, \quad y = 4 - t, \quad z = 1 + 5t.$$  

Plug these $t$-values into equation of plane and solve for $t$:

$$3(2 + 3t) - (4 - t) + 5(1 + 5t) = 77,$$

$$6 + 9t - 4 + t + 5 + 25t = 77,$$

$$35t = 70; \quad \Rightarrow t = 2.$$  

So $L$ intersects the plane at time $t = 2$.

At $t = 2$, the parametric equations give the point:

$$\langle 2 + 3 \cdot 2, 4 - 2, 1 + 5 \cdot 2 \rangle = \langle 8, 2, 11 \rangle.$$
Problem 7(b) - Spring 2008

Find the intersection point of the line $L(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have $L$ has **parametric equations**:
  $$
  x = 2 + 3t \\
  y = 4 - t \\
  z = 1 + 5t.
  $$
Problem 7(b) - Spring 2008

Find the intersection point of the line $L(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

By part (a), we have $L$ has **parametric equations**:

\[ x = 2 + 3t \]
\[ y = 4 - t \]
\[ z = 1 + 5t. \]

Plug these $t$-values into equation of plane and solve for $t$: 
Problem 7(b) - Spring 2008

Find the intersection point of the line $L(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have $L$ has **parametric equations**:
  - $x = 2 + 3t$
  - $y = 4 - t$
  - $z = 1 + 5t$.

- Plug these $t$-values into equation of plane and solve for $t$:

  
  
  
  $3(2 + 3t) - (4 - t) + 5(1 + 5t) = 77$,

  
  
  
  $6 + 9t - 4 + t + 5 + 25t = 77$,

  
  
  
  $35t = 70$;

  
  
  
  \[ t = 2 \]
Problem 7(b) - Spring 2008

Find the intersection point of the line \( \mathbf{L}(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle \) in part (a) and the plane \( 3x - y + 5z = 77 \).

Solution:

- By part (a), we have \( \mathbf{L} \) has **parametric equations**:
  \[
  \begin{align*}
  x & = 2 + 3t \\
  y & = 4 - t \\
  z & = 1 + 5t.
  \end{align*}
  \]

- Plug these \( t \)-values into equation of plane and solve for \( t \):
  \[
  3(2 + 3t) - (4 - t) + 5(1 + 5t) = 77, \\
  6 + 9t - 4 + t + 5 + 25t = 77, \\
  35t = 70; \quad \implies \quad t = 2.
  \]
Find the intersection point of the line $\mathbf{L}(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$. 

Solution:

- By part (a), we have $\mathbf{L}$ has parametric equations:
  
  $\begin{align*}
  x &= 2 + 3t \\
  y &= 4 - t \\
  z &= 1 + 5t.
  \end{align*}$

- Plug these $t$-values into equation of plane and solve for $t$:

  $$3(2 + 3t) - (4 - t) + 5(1 + 5t) = 77,$$

  $$6 + 9t - 4 + t + 5 + 25t = 77,$$

  $$35t = 70; \quad \implies t = 2.$$

  So $\mathbf{L}$ intersects the plane at time $t = 2$. 
Problem 7(b) - Spring 2008

Find the intersection point of the line \( \mathbf{L}(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle \) in part (a) and the plane \( 3x - y + 5z = 77 \).

Solution:

- By part (a), we have \( \mathbf{L} \) has **parametric equations**:
  
  \[
  x = 2 + 3t \\
  y = 4 - t \\
  z = 1 + 5t.
  \]

- Plug these \( t \)-values into equation of plane and solve for \( t \):

  \[
  3(2 + 3t) - (4 - t) + 5(1 + 5t) = 77, \\
  6 + 9t - 4 + t + 5 + 25t = 77, \\
  35t = 70; \quad \implies t = 2.
  \]

  So \( \mathbf{L} \) intersects the plane at time \( t = 2 \).

- At \( t = 2 \), the **parametric equations** give the point:

  \[
  \langle 2 + 3 \cdot 2, 4 - 2, 1 + 5 \cdot 2 \rangle = \langle 8, 2, 11 \rangle.
  \]
Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

\[ u(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \( x \) and \( y \) by eliminating \( t \).
A *plane* curve is given by the graph of the vector function
\[ u(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \(x\) and \(y\) by eliminating \(t\).

**Solution:**

- Rewriting \(u\), we get:
  \[ u(t) = \langle 1 + \cos t, \sin t \rangle \]
Problem 8(a) - Spring 2008

A plane curve is given by the graph of the vector function

\[ u(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \( x \) and \( y \) by eliminating \( t \).

Solution:

- Rewriting \( u \), we get:

\[ u(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle. \]
A plane curve is given by the graph of the vector function

\[ u(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \( x \) and \( y \) by eliminating \( t \).

Solution:

- Rewriting \( u \), we get:

  \[ u(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle. \]

- Since \( \langle \cos t, \sin t \rangle \) is the parametrization of the circle of radius 1 centered at the origin,
Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

\[ u(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \( x \) and \( y \) by eliminating \( t \).

Solution:

- Rewriting \( u \), we get:
  \[ u(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle. \]
- Since \( \langle \cos t, \sin t \rangle \) is the *parametrization* of the circle of radius 1 centered at the origin, then \( u \) is a circle of radius \( r = 1 \) centered at \((1, 0)\).
Problem 8(a) - Spring 2008

A *plane* curve is given by the graph of the vector function

\[ \mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \( x \) and \( y \) by eliminating \( t \).

**Solution:**

- Rewriting \( \mathbf{u} \), we get:

  \[ \mathbf{u}(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle. \]

- Since \( \langle \cos t, \sin t \rangle \) is the *parametrization* of the circle of radius 1 centered at the origin, then \( \mathbf{u} \) is a circle of radius \( r = 1 \) centered at \((1, 0)\).

- So the answer is:

  \[ (x - 1)^2 + (y - 0)^2 = 1^2 \]
A *plane* curve is given by the graph of the vector function

\[ u(t) = \langle 1 + \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi. \]

Find a single equation for the curve in terms of \( x \) and \( y \) by eliminating \( t \).

**Solution:**

- Rewriting \( u \), we get:

  \[ u(t) = \langle 1 + \cos t, \sin t \rangle = \langle 1, 0 \rangle + \langle \cos t, \sin t \rangle. \]

- Since \( \langle \cos t, \sin t \rangle \) is the **parametrization** of the circle of radius 1 centered at the origin, then \( u \) is a circle of radius \( r = 1 \) centered at \((1, 0)\).

- So the answer is:

  \[(x - 1)^2 + (y - 0)^2 = 1^2\]

or

\[(x - 1)^2 + y^2 = 1.\]
Consider the space curve given by the graph of the vector function

$$r(t) = \langle 1 + \cos t, \sin t, t \rangle, \quad 0 \leq t \leq 2\pi.$$  

Sketch the curve and indicate the direction of increasing $t$ in your graph.
Problem 8(b) - Spring 2008

Consider the *space* curve given by the graph of the vector function

\[ \mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle, \quad 0 \leq t \leq 2\pi. \]

Sketch the curve and indicate the direction of increasing \( t \) in your graph.

**Solution:**

The sketch would be the following one translated 1 unit along the \( x \)-axis.
Determine **parametric equations** for the line $T$ tangent to the graph of the space curve for $r(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $T$ tangent to the graph of the *space* curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).

Solution:

- First find the velocity vector $\mathbf{r}'(t)$:
  \[ \mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle \]
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line \( T \) tangent to the graph of the **space curve** for \( r(t) = \left< 1 + \cos t, \sin t, t \right> \) at \( t = \pi/3 \), and sketch \( T \) in the graph obtained in part (b).

**Solution:**

- First find the velocity vector \( r'(t) \):
  \[
  r'(t) = \left< (1 + \cos t)', (\sin t)', 1 \right> = \left< -\sin t, \cos t, 1 \right>.
  \]
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $T$ tangent to the graph of the space curve for $r(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).

**Solution:**

- First find the velocity vector $r'(t)$:
  
  
  $$r'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$

- At $t = \frac{\pi}{3}$,

  $$r\left(\frac{\pi}{3}\right) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle$$
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $T$ tangent to the graph of the space curve for $r(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).

**Solution:**

- First find the velocity vector $r'(t)$:
  
  $$r'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.$$  

- At $t = \pi/3$,
  
  $$r\left(\frac{\pi}{3}\right) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,$$
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line \( T \) tangent to the graph of the space curve for \( r(t) = \langle 1 + \cos t, \sin t, t \rangle \) at \( t = \pi/3 \), and sketch \( T \) in the graph obtained in part (b).

**Solution:**

- First find the velocity vector \( r'(t) \):
  \[
  r'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.
  \]

- At \( t = \frac{\pi}{3} \),
  \[
  r\left(\frac{\pi}{3}\right) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,
  \]
  \[
  r'\left(\frac{\pi}{3}\right) = \langle -\sin \frac{\pi}{3}, \cos \frac{\pi}{3}, 1 \rangle
  \]
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $T$ tangent to the graph of the **space curve** for $r(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).

**Solution:**

- First find the velocity vector $r'(t)$:
  \[
  r'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.
  \]

- At $t = \pi/3$,
  \[
  r\left(\frac{\pi}{3}\right) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,
  \]
  \[
  r'\left(\frac{\pi}{3}\right) = \langle -\sin \frac{\pi}{3}, \cos \frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle.
  \]
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $T$ tangent to the graph of the space curve for $\mathbf{r}(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).

**Solution:**

- First find the velocity vector $\mathbf{r}'(t)$:
  \[
  \mathbf{r}'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.
  \]

- At $t = \pi/3$,
  \[
  \mathbf{r}(\frac{\pi}{3}) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,
  \]
  \[
  \mathbf{r}'(\frac{\pi}{3}) = \langle -\sin \frac{\pi}{3}, \cos \frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle.
  \]

- The vector part of tangent line $T$ is $\mathbf{r}'(\frac{\pi}{3})$ and a point on line is $\mathbf{r}(\frac{\pi}{3})$. 
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line $T$ tangent to the graph of the *space* curve for $r(t) = \langle 1 + \cos t, \sin t, t \rangle$ at $t = \pi/3$, and sketch $T$ in the graph obtained in part (b).

**Solution:**

- First find the velocity vector $r'(t)$:
  
  $r'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle$.

- At $t = \pi/3$,
  
  $r\left(\frac{\pi}{3}\right) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle$,

  
  $r'\left(\frac{\pi}{3}\right) = \langle -\sin \frac{\pi}{3}, \cos \frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle$.

- The vector part of tangent line $T$ is $r'(\frac{\pi}{3})$ and a point on line is $r\left(\frac{\pi}{3}\right)$.

- The **vector equation** is: $T(t) = r\left(\frac{\pi}{3}\right) + tr'\left(\frac{\pi}{3}\right)$. 
Problem 8(c) - Spring 2008

Determine **parametric equations** for the line \( T \) tangent to the graph of the **space curve** for \( r(t) = \langle 1 + \cos t, \sin t, t \rangle \) at \( t = \pi / 3 \), and sketch \( T \) in the graph obtained in part (b).

**Solution:**

- First find the velocity vector \( r'(t) \):
  \[
  r'(t) = \langle (1 + \cos t)', (\sin t)', 1 \rangle = \langle -\sin t, \cos t, 1 \rangle.
  \]
- At \( t = \pi / 3 \),
  \[
  r\left( \frac{\pi}{3} \right) = \langle 1 + \cos \frac{\pi}{3}, \sin \frac{\pi}{3}, \frac{\pi}{3} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3} \rangle,
  \]
  \[
  r'\left( \frac{\pi}{3} \right) = \langle -\sin \frac{\pi}{3}, \cos \frac{\pi}{3}, 1 \rangle = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, 1 \rangle.
  \]
- The vector part of tangent line \( T \) is \( r'(\frac{\pi}{3}) \) and a point on line is \( r\left( \frac{\pi}{3} \right) \).
- The **vector equation** is: \( T(t) = r\left( \frac{\pi}{3} \right) + tr'(\frac{\pi}{3}) \).
- The **parametric equations** are:
  \[
  x = \frac{3}{2} - \frac{\sqrt{3}}{2}t \\
y = \frac{\sqrt{3}}{2} + \frac{1}{2}t \\
z = \frac{\pi}{3} + t.
  \]
Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Determine \( r(t) \) for all \( t \).
Problem 9(a) - Spring 2008

Suppose that \( \mathbf{r}(t) \) has derivative \( \mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Determine \( \mathbf{r}(t) \) for all \( t \).

Solution:

- Find \( \mathbf{r}(t) \) by integration:

\[
\mathbf{r}(t) = \int \mathbf{r}'(t) \, dt = \int \langle -\sin 2t, \cos 2t, 0 \rangle \, dt
\]
Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Determine \( r(t) \) for all \( t \).

Solution:

- Find \( r(t) \) by integration:

\[
r(t) = \int r'(t) \, dt = \int \langle -\sin 2t, \cos 2t, 0 \rangle \, dt
\]

\[
= \left\langle \frac{1}{2} \cos(2t) + x_0, \frac{1}{2} \sin(2t) + y_0, z_0 \right\rangle.
\]
Problem 9(a) - Spring 2008

Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Determine \( r(t) \) for all \( t \).

Solution:

- Find \( r(t) \) by integration:

\[
r(t) = \int r'(t) \, dt = \int \langle -\sin 2t, \cos 2t, 0 \rangle \, dt
\]

\[
= \langle \frac{1}{2} \cos(2t) + x_0, \frac{1}{2} \sin(2t) + y_0, z_0 \rangle.
\]

- Now solve for the point \( (x_0, y_0, z_0) \) using \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \):
Problem 9(a) - Spring 2008

Suppose that \( \mathbf{r}(t) \) has derivative \( \mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Determine \( \mathbf{r}(t) \) for all \( t \).

Solution:

- Find \( \mathbf{r}(t) \) by integration:

\[
\mathbf{r}(t) = \int \mathbf{r}'(t) \, dt = \int \langle -\sin 2t, \cos 2t, 0 \rangle \, dt
\]

\[
= \langle \frac{1}{2} \cos(2t) + x_0, \frac{1}{2} \sin(2t) + y_0, z_0 \rangle.
\]

- Now solve for the point \( (x_0, y_0, z_0) \) using \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \):

\[
(\frac{1}{2} \cos(0) + x_0, \frac{1}{2} \sin(0) + y_0, z_0) = (\frac{1}{2} + x_0, y_0, z_0) = (\frac{1}{2}, 0, 1).
\]
Problem 9(a) - Spring 2008

Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Determine \( r(t) \) for all \( t \).

Solution:

- Find \( r(t) \) by integration:
  \[
  r(t) = \int r'(t) \, dt = \int \langle -\sin 2t, \cos 2t, 0 \rangle \, dt
  \]
  \[
  = \langle \frac{1}{2} \cos(2t) + x_0, \frac{1}{2} \sin(2t) + y_0, z_0 \rangle.
  \]

- Now solve for the point \( (x_0, y_0, z_0) \) using \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \):
  \[
  (\frac{1}{2} \cos(0) + x_0, \frac{1}{2} \sin(0) + y_0, z_0) = (\frac{1}{2} + x_0, y_0, z_0) = (\frac{1}{2}, 0, 1).
  \]
  So \( x_0 = 0, \ y_0 = 0, \ z_0 = 1 \).
Problem 9(a) - Spring 2008

Suppose that \( \mathbf{r}(t) \) has derivative \( \mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \).

Determine \( \mathbf{r}(t) \) for all \( t \).

Solution:

- Find \( \mathbf{r}(t) \) by integration:

  \[
  \mathbf{r}(t) = \int \mathbf{r}'(t) \, dt = \int \langle -\sin 2t, \cos 2t, 0 \rangle \, dt
  \]

  \[
  = \langle \frac{1}{2} \cos(2t) + x_0, \frac{1}{2} \sin(2t) + y_0, z_0 \rangle.
  \]

- Now solve for the point \((x_0, y_0, z_0)\) using \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \):

  \[
  \left( \frac{1}{2} \cos(0) + x_0, \frac{1}{2} \sin(0) + y_0, z_0 \right) = \left( \frac{1}{2} + x_0, y_0, z_0 \right) = \langle \frac{1}{2}, 0, 1 \rangle.
  \]

  So \( x_0 = 0, \ y_0 = 0, \ z_0 = 1 \).

- Thus,

  \[
  \mathbf{r}(t) = \langle \frac{1}{2} \cos 2t, \frac{1}{2} \sin 2t, 1 \rangle.
  \]
Suppose that $r(t)$ has derivative $r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $r(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Show that $r(t)$ is orthogonal to $r'(t)$ for all $t$. 

Solution: By part (a), $r(t) = \langle \frac{1}{2} \cos 2t, \frac{1}{2} \sin 2t, 1 \rangle$, Taking dot products, we get:

$$r(t) \cdot r'(t) = -\frac{1}{2} \cos 2t \sin 2t + \frac{1}{2} \sin 2t \cos 2t + 0 = 0.$$ 

Since the dot product is zero, then for each $t$, $r(t)$ is orthogonal to $r'(t)$. 


Suppose that $r(t)$ has derivative $r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $r(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Show that $r(t)$ is orthogonal to $r'(t)$ for all $t$.

Solution:

- By part (a),

$$r(t) = \langle \frac{1}{2} \cos 2t, \frac{1}{2} \sin 2t, 1 \rangle,$$
Problem 9(b) - Spring 2008

Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Show that \( r(t) \) is orthogonal to \( r'(t) \) for all \( t \).

Solution:

- By part (a),
  \[
  r(t) = \langle \frac{1}{2} \cos 2t, \frac{1}{2} \sin 2t, 1 \rangle,
  \]
- Taking dot products, we get:
  \[
  r(t) \cdot r'(t) = -\frac{1}{2} \cos 2t \sin 2t + \frac{1}{2} \sin 2t \cos 2t + 0 = 0.
  \]
Suppose that $r(t)$ has derivative $r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $r(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Show that $r(t)$ is orthogonal to $r'(t)$ for all $t$.

Solution:

- By part (a),
  \[ r(t) = \langle \frac{1}{2} \cos 2t, \frac{1}{2} \sin 2t, 1 \rangle, \]

- Taking dot products, we get:
  \[ r(t) \cdot r'(t) = -\frac{1}{2} \cos 2t \sin 2t + \frac{1}{2} \sin 2t \cos 2t + 0 = 0. \]

- Since the dot product is zero, then for each $t$, $r(t)$ is orthogonal to $r'(t)$. 
Suppose that \( \mathbf{r}(t) \) has derivative \( \mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( \mathbf{r}(t) \) on the interval \( 0 \leq t \leq 1 \).
Problem 9(c) - Spring 2008

Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( r(t) \) on the interval \( 0 \leq t \leq 1 \).

Solution:

- Recall that the length of \( r(t) \) on the interval \([0, 1]\) is gotten by integrating the speed \( |r'(t)| \).
Suppose that \( \mathbf{r}(t) \) has derivative \( \mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( \mathbf{r}(t) \) on the interval \( 0 \leq t \leq 1 \).

**Solution:**

- Recall that the length of \( \mathbf{r}(t) \) on the interval \([0, 1]\) is gotten by integrating the speed \( |\mathbf{r}'(t)| \).
- Calculating, we get:

\[
L = \int_{0}^{1} |\mathbf{r}'(t)| \, dt = \int_{0}^{1} |\langle -\sin 2t, \cos 2t, 0 \rangle| \, dt
\]
Suppose that $\mathbf{r}(t)$ has derivative $\mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle$ on the interval $0 \leq t \leq 1$. Suppose we know that $\mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle$. Find the arclength $L$ of the graph of the vector function $\mathbf{r}(t)$ on the interval $0 \leq t \leq 1$.

Solution:

- Recall that the length of $\mathbf{r}(t)$ on the interval $[0, 1]$ is gotten by integrating the speed $|\mathbf{r}'(t)|$.
- Calculating, we get:

$$L = \int_{0}^{1} |\mathbf{r}'(t)| \, dt = \int_{0}^{1} |\langle -\sin 2t, \cos 2t, 0 \rangle| \, dt$$

$$= \int_{0}^{1} \sqrt{\sin^2 2t + \cos^2 2t} \, dt$$

$$= \int_{0}^{1} dt$$

$$= 1$$

Thus $L = 1$. 
Problem 9(c) - Spring 2008

Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( r(t) \) on the interval \( 0 \leq t \leq 1 \).

Solution:

- Recall that the length of \( r(t) \) on the interval \([0, 1]\) is gotten by integrating the speed \( |r'(t)| \).
- Calculating, we get:

\[
L = \int_{0}^{1} |r'(t)| \, dt = \int_{0}^{1} |-\sin 2t, \cos 2t, 0| \, dt
\]

\[
= \int_{0}^{1} \sqrt{\sin^2 2t + \cos^2 2t} \, dt = \int_{0}^{1} |1| \, dt
\]
Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( r(t) \) on the interval \( 0 \leq t \leq 1 \).

**Solution:**

- Recall that the length of \( r(t) \) on the interval \([0, 1]\) is gotten by integrating the speed \( |r'(t)| \).
- Calculating, we get:
  \[
  L = \int_0^1 |r'(t)| \, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0 \rangle| \, dt \\
  = \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t} \, dt = \int_0^1 1 \, dt = t \bigg|_0^1 = 1.
  \]
Suppose that \( r(t) \) has derivative \( r'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( r(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( r(t) \) on the interval \( 0 \leq t \leq 1 \).

Solution:

- Recall that the length of \( r(t) \) on the interval \([0, 1]\) is gotten by integrating the speed \( |r'(t)| \).
- Calculating, we get:

\[
L = \int_0^1 |r'(t)| \, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0 \rangle| \, dt
\]

\[
= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t} \, dt = \int_0^1 |1| \, dt = t \bigg|_0^1 = 1.
\]
Suppose that \( \mathbf{r}(t) \) has derivative \( \mathbf{r}'(t) = \langle -\sin 2t, \cos 2t, 0 \rangle \) on the interval \( 0 \leq t \leq 1 \). Suppose we know that \( \mathbf{r}(0) = \langle \frac{1}{2}, 0, 1 \rangle \). Find the arclength \( L \) of the graph of the vector function \( \mathbf{r}(t) \) on the interval \( 0 \leq t \leq 1 \).

**Solution:**

- Recall that the length of \( \mathbf{r}(t) \) on the interval \([0, 1]\) is gotten by integrating the speed \( |\mathbf{r}'(t)| \).
- Calculating, we get:

\[
L = \int_0^1 |\mathbf{r}'(t)| \, dt = \int_0^1 |\langle -\sin 2t, \cos 2t, 0 \rangle| \, dt
\]

\[
= \int_0^1 \sqrt{\sin^2 2t + \cos^2 2t} \, dt = \int_0^1 1 \, dt = t \bigg|_0^1 = 1.
\]

- Thus

\[ L = 1. \]
If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - \left(\frac{1}{3}t^3\right)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),

Find the speed \( s(t) \) and the velocity \( \mathbf{v}(t) \) of the object at time \( t \).
Problem 10(a) - Spring 2008

If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet), find the speed \( s(t) \) and the velocity \( v(t) \) of the object at time \( t \).

Solution:

Recall that the velocity \( v(t) \) vector of \( r(t) \) at time \( t \) is \( r'(t) \) and the speed \( s(t) \) is its length \( |r'(t)| \).
Problem 10(a) - Spring 2008

If $\mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k}$ represents the position vector of a moving object (where $t \geq 0$ is measured in seconds and distance is measured in feet), find the speed $s(t)$ and the velocity $\mathbf{v}(t)$ of the object at time $t$.

Solution:

- Recall that the velocity $\mathbf{v}(t)$ vector of $\mathbf{r}(t)$ at time $t$ is $\mathbf{r}'(t)$ and the speed $s(t)$ is its length $|\mathbf{r}'(t)|$.
- Calculating with $\mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle$:
Problem 10(a) - Spring 2008

If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),
Find the speed \( s(t) \) and the velocity \( \mathbf{v}(t) \) of the object at time \( t \).

Solution:

- Recall that the velocity \( \mathbf{v}(t) \) vector of \( \mathbf{r}(t) \) at time \( t \) is \( \mathbf{r}'(t) \) and the speed \( s(t) \) is its length \( |\mathbf{r}'(t)| \).
- Calculating with \( \mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \):

  \[
  \mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,
  \]

  \[
  s(t) = |\mathbf{r}'(t)| = \sqrt{2^2 + (2t)^2 + (-t^2)^2} = \sqrt{4 + 4t^2 + t^4}.
  \]
Problem 10(a) - Spring 2008

If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet), find the speed \( s(t) \) and the velocity \( \mathbf{v}(t) \) of the object at time \( t \).

Solution:

- Recall that the velocity \( \mathbf{v}(t) \) vector of \( \mathbf{r}(t) \) at time \( t \) is \( \mathbf{r}'(t) \) and the speed \( s(t) \) is its length \( |\mathbf{r}'(t)| \).
- Calculating with \( \mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \):

\[
\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,
\]

\[
s(t) = |\mathbf{r}'(t)|
\]
Problem 10(a) - Spring 2008

If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),
Find the speed \( s(t) \) and the velocity \( \mathbf{v}(t) \) of the object at time \( t \).

Solution:

- Recall that the velocity \( \mathbf{v}(t) \) vector of \( \mathbf{r}(t) \) at time \( t \) is \( \mathbf{r}'(t) \) and the speed \( s(t) \) is its length \( |\mathbf{r}'(t)| \).
- Calculating with \( \mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \):

\[
\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,
\]

\[
s(t) = |\mathbf{r}'(t)| = \sqrt{2^2 + (2t)^2 + (-t^2)^2}
\]
Problem 10(a) - Spring 2008

If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),

Find the speed \( s(t) \) and the velocity \( \mathbf{v}(t) \) of the object at time \( t \).

**Solution:**

- Recall that the velocity \( \mathbf{v}(t) \) vector of \( \mathbf{r}(t) \) at time \( t \) is \( \mathbf{r}'(t) \) and the speed \( s(t) \) is its length \( |\mathbf{r}'(t)| \).

- Calculating with \( \mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \):

\[
\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2, 2t, -t^2 \rangle,
\]

\[
s(t) = |\mathbf{r}'(t)| = \sqrt{2^2 + (2t)^2 + (-t^2)^2} = \sqrt{4 + 4t^2 + t^4}.
\]
If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)

If a second object travels along a path given defined by the graph of the vector function \( w(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects \textit{intersect} at a common point \( P \).
If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)

If a second object travels along a path given defined by the graph of the vector function \( w(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects \textbf{intersect} at a common point \( P \).

**Solution:**

- Note that \( w(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle \) and 
  \( r(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \).
Problem 10(b) - Spring 2008

If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)

If a second object travels along a path given defined by the graph of the vector function \( \mathbf{w}(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects intersect at a common point \( P \).

Solution:

- Note that \( \mathbf{w}(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle \) and \( \mathbf{r}(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \).
- Setting the \( x \) and \( y \)-coordinates of \( \mathbf{w}(s) \) and \( \mathbf{r}(t) \) equal, we obtain:
  \[
  x = 2t = 2 + 2s \implies t = s + 1
  \]
Problem 10(b) - Spring 2008

If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)

If a second object travels along a path given defined by the graph of the vector function \( w(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects intersect at a common point \( P \).

Solution:

- Note that \( w(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle \) and \( r(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \).
- Setting the \( x \) and \( y \)-coordinates of \( w(s) \) and \( r(t) \) equal, we obtain:
  \[
  x = 2t = 2 + 2s \implies t = s + 1
  \]
  \[
  y = t^2 - 6 = 5 - s \implies (s + 1)^2 - 6 = s^2 + 2s - 5 = 5 - s
  \]
Problem 10(b) - Spring 2008

If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)

If a second object travels along a path given defined by the graph of the vector function \( w(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects intersect at a common point \( P \).

Solution:

- Note that \( w(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle \) and \( r(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \).
- Setting the \( x \) and \( y \)-coordinates of \( w(s) \) and \( r(t) \) equal, we obtain:
  
  \[
  x = 2t = 2 + 2s \implies t = s + 1
  \]
  
  \[
  y = t^2 - 6 = 5 - s \implies (s + 1)^2 - 6 = s^2 + 2s - 5 = 5 - s
  \]
  
  \[
  \implies s^2 + 3s - 10 = 0 \implies (s + 5)(s - 2) = 0.
  \]
If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)

If a second object travels along a path given defined by the graph of the vector function \( w(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects intersect at a common point \( P \).

**Solution:**

- Note that \( w(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle \) and \( r(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \).
- Setting the \( x \) and \( y \)-coordinates of \( w(s) \) and \( r(t) \) equal, we obtain:
  \[
  x = 2t = 2 + 2s \implies t = s + 1
  \]
  \[
  y = t^2 - 6 = 5 - s \implies (s + 1)^2 - 6 = s^2 + 2s - 5 = 5 - s
  \]
  \[
  \implies s^2 + 3s - 10 = 0 \implies (s + 5)(s - 2) = 0.
  \]
- So, \((s = 2 \text{ and } t = 3)\) or \((s = -5 \text{ and } t = -4)\).
Problem 10(b) - Spring 2008

If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet.)
If a second object travels along a path given defined by the graph of the vector function \( w(s) = \langle 2, 5, 1 \rangle + s\langle 2, -1, -5 \rangle \), show that the paths of the two objects intersect at a common point \( P \).

Solution:

1. Note that \( w(s) = \langle 2 + 2s, 5 - s, 1 - 5s \rangle \) and \( r(t) = \langle 2t, t^2 - 6, -\frac{1}{3}t^3 \rangle \).
2. Setting the \( x \) and \( y \)-coordinates of \( w(s) \) and \( r(t) \) equal, we obtain:
   \[ x = 2t = 2 + 2s \implies t = s + 1 \]
   \[ y = t^2 - 6 = 5 - s \implies (s + 1)^2 - 6 = s^2 + 2s - 5 = 5 - s \]
   \[ \implies s^2 + 3s - 10 = 0 \implies (s + 5)(s - 2) = 0. \]
3. So, \( (s = 2 \text{ and } t = 3) \) or \( (s = -5 \text{ and } t = -4) \).
4. Since \( r(3) = \langle 6, 3, -9 \rangle = w(2) \),
the paths intersect at \( P = (6, 3, -9) \).
If \( \mathbf{r}(t) = (2t)\mathbf{i} + (t^2 - 6)\mathbf{j} - (\frac{1}{3}t^3)\mathbf{k} \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),

If \( s = t \) in part (b), (i.e. the position of the second object is \( \mathbf{w}(t) \) when the first object is at position \( \mathbf{r}(t) \)), do the two objects ever collide?
Problem 10(c) - Spring 2008

If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),

If \( s = t \) in part (b), (i.e. the position of the second object is \( w(t) \) when the first object is at position \( r(t) \)), do the two objects ever collide?

Solution:

- Set \( t = s \) in part (b).
Problem 10(c) - Spring 2008

If \( r(t) = (2t)i + (t^2 - 6)j - \left(\frac{1}{3}t^3\right)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet),

If \( s = t \) in part (b), (i.e. the position of the second object is \( w(t) \) when the first object is at position \( r(t) \)), do the two objects ever collide?

Solution:

- Set \( t = s \) in part (b).
- Then the \( x \)-coordinate of \( r(t) \) is \( 2t \) and the \( x \)-coordinate of \( w(t) = \langle 2 + 2t, 5 - t, 1 - 5t \rangle \) is \( 2 + 2t \), and \( 2t \neq 2 + 2t \) for all \( t \).
If \( r(t) = (2t)i + (t^2 - 6)j - (\frac{1}{3}t^3)k \) represents the position vector of a moving object (where \( t \geq 0 \) is measured in seconds and distance is measured in feet), If \( s = t \) in part \((b)\), (i.e. the position of the second object is \( w(t) \) when the first object is at position \( r(t) \)), do the two objects ever collide?

**Solution:**

- Set \( t = s \) in part \((b)\).
- Then the \( x \)-coordinate of \( r(t) \) is \( 2t \) and the \( x \)-coordinate of \( w(t) = \langle 2 + 2t, 5 - t, 1 - 5t \rangle \) is \( 2 + 2t \), and \( 2t \neq 2 + 2t \) for all \( t \).
- Since \( r(t) \) and \( w(t) \) have different \( x \)-coordinates for all values of \( t \), then they **never collide**.
Problem 11(a) - Spring 2007

Find **parametric equations** for the line \( L \) which contains \( A(7, 6, 4) \) and \( B(4, 6, 5) \).
Problem 11(a) - Spring 2007

Find **parametric equations** for the line \( L \) which contains \( A(7, 6, 4) \) and \( B(4, 6, 5) \).

**Solution:**

- A vector parallel to the line \( L \) is:

\[
\vec{v} = \overrightarrow{AB} = \langle 4 - 7, 6 - 6, 5 - 4, \rangle = \langle -3, 0, 1 \rangle.
\]
Problem 11(a) - Spring 2007

Find \textbf{parametric equations} for the line \textbf{L} which contains \textbf{A}(7, 6, 4) and \textbf{B}(4, 6, 5).

\textbf{Solution:}

- A vector parallel to the line \textbf{L} is:

\[ \mathbf{v} = \overrightarrow{AB} = \langle 4 - 7, 6 - 6, 5 - 4, \rangle = \langle -3, 0, 1 \rangle. \]

- A point on the line is \textbf{A}(7, 6, 4).
Problem 11(a) - Spring 2007

Find **parametric equations** for the line $L$ which contains $A(7, 6, 4)$ and $B(4, 6, 5)$.

**Solution:**

- A vector parallel to the line $L$ is:

  $$ \mathbf{v} = \overrightarrow{AB} = \langle 4 - 7, 6 - 6, 5 - 4, \rangle = \langle -3, 0, 1 \rangle. $$

- A point on the line is $A(7, 6, 4)$.
- Therefore **parametric equations** for the line $L$ are:

  $$ x = 7 - 3t $$

  $$ y = 6 $$

  $$ z = 4 + t. $$
Find the **parametric equations** for the **line** $L$ of intersection of the planes $x - 2y + z = 5$ and $2x + y - z = 0$. 

**Solution:**

A vector $v$ parallel to the line is the cross product of the normal vectors of the planes:

$$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle$$

$$= \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix}i - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}j + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}k$$

$$= \langle 1, 3, 5 \rangle.$$ 

A point on $L$ is any $(x_0, y_0, z_0)$ that satisfies both of the plane equations. Setting $z = 0$, we obtain the equations $x - 2y = 5$ and $2x + y = 0$ and find such a point $(1, -2, 0)$. Therefore, parametric equations for $L$ are:

$$x = 1 + t, \quad y = -2 + 3t, \quad z = 5t.$$
Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes \( x - 2y + z = 5 \) and \( 2x + y - z = 0 \).

**Solution:**

- A vector \( \mathbf{v} \) parallel to the line is the cross product of the normal vectors of the planes:

\[
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle
\]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -2 & 1 \\
2 & 1 & -1 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
-2 & 1 \\
1 & -1 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
1 & -2 \\
2 & 1 \\
\end{vmatrix}
\]

Therefore, \( \mathbf{v} = \langle 1, 3, 5 \rangle \).

A point on \( \mathbf{L} \) is any \((x_0, y_0, z_0)\) that satisfies both of the plane equations. Setting \( z = 0 \), we obtain the equations \( x - 2y = 5 \) and \( 2x + y = 0 \) and find such a point \((1, -2, 0)\).

Therefore, parametric equations for \( \mathbf{L} \) are:

\[
x = 1 + t
\]

\[
y = -2 + 3t
\]

\[
z = 5t
\]
Problem 11(b) - Spring 2007

Find the parametric equations for the line \( L \) of intersection of the planes \( x - 2y + z = 5 \) and \( 2x + y - z = 0 \).

Solution:

A vector \( \mathbf{v} \) parallel to the line is the cross product of the normal vectors of the planes:

\[
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle
\]
Problem 11(b) - Spring 2007

Find the **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

**Solution:**

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

$$
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} 
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -2 & 1 \\
2 & 1 & -1 
\end{vmatrix}
$$
Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes \(x - 2y + z = 5\) and \(2x + y - z = 0\).

**Solution:**

- A vector \( \mathbf{v} \) parallel to the line is the cross product of the normal vectors of the planes:

\[
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -2 & 1 \\
2 & 1 & -1
\end{vmatrix}
\]

\[
= \begin{vmatrix}
-2 & 1 \\
1 & -1
\end{vmatrix} \mathbf{i} - \begin{vmatrix}
1 & 1 \\
2 & -1
\end{vmatrix} \mathbf{j} + \begin{vmatrix}
1 & -2 \\
2 & 1
\end{vmatrix} \mathbf{k}
\]

\[
= \langle 1, 3, 5 \rangle.
\]

- A point on \( \mathbf{L} \) is any \((x_0, y_0, z_0)\) that satisfies both of the plane equations. Setting \(z = 0\), we obtain the equations \(x - 2y = 5\) and \(2x + y = 0\) and find such a point \((1, -2, 0)\).

Therefore, parametric equations for \( \mathbf{L} \) are:

\[
x = 1 + t
\]

\[
y = -2 + 3t
\]

\[
z = 5t.
\]
Problem 11(b) - Spring 2007

Find the parametric equations for the line $L$ of intersection of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $v$ parallel to the line is the cross product of the normal vectors of the planes:

$$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix}
  i & j & k \\
  1 & -2 & 1 \\
  2 & 1 & -1 \\
\end{vmatrix} = \begin{vmatrix}
  -2 & 1 \\
  1 & -1 \\
\end{vmatrix} i - \begin{vmatrix}
  1 & 1 \\
  2 & -1 \\
\end{vmatrix} j + \begin{vmatrix}
  1 & -2 \\
  2 & 1 \\
\end{vmatrix} k = \langle 1, 3, 5 \rangle.$$
Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line** \( L \) of intersection of the planes \( x - 2y + z = 5 \) and \( 2x + y - z = 0 \).

Solution:

- A vector \( \mathbf{v} \) parallel to the line is the cross product of the normal vectors of the planes:

  \[
  \mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  1 & -2 & 1 \\
  2 & 1 & -1 \\
\end{vmatrix}
  = \begin{vmatrix}
  -2 & 1 \\
  1 & -1 \\
\end{vmatrix} \mathbf{i} - \begin{vmatrix}
  1 & 1 \\
  2 & -1 \\
\end{vmatrix} \mathbf{j} + \begin{vmatrix}
  1 & -2 \\
  2 & 1 \\
\end{vmatrix} \mathbf{k} = \langle 1, 3, 5 \rangle.
  
- A point on \( L \) is any \((x_0, y_0, z_0)\) that satisfies **both** of the plane equations.
Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

**Solution:**

- A vector $v$ parallel to the line is the cross product of the normal vectors of the planes:
  
  $$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} i - \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} j + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} k = \langle 1, 3, 5 \rangle.$$  

- A point on $L$ is any $(x_0, y_0, z_0)$ that satisfies both of the plane equations. Setting $z = 0$, we obtain the equations $x - 2y = 5$ and $2x + y = 0$ and find such a point $(1, -2, 0)$.
Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes \(x - 2y + z = 5\) and \(2x + y - z = 0\).

**Solution:**

- A vector \(v\) parallel to the line is the cross product of the normal vectors of the planes:
  \[
v = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} i & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = \langle 1, 3, 5 \rangle.
\]
- A point on \(L\) is any \((x_0, y_0, z_0)\) that satisfies **both** of the plane equations. Setting \(z = 0\), we obtain the equations \(x - 2y = 5\) and \(2x + y = 0\) and find such a point \((1, -2, 0)\).
- Therefore **parametric equations** for \(L\) are:
  \[
x = 1 + t \\
y = -2 + 3t \\
z = 5t.
\]
Problem 12(a) - Spring 2007

Find an equation of the plane which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

A normal vector to the plane can be found by taking the cross product of any two vectors that lie in the plane. Two vectors that lie in the plane are $\vec{PQ} = \langle 2, -2, -1 \rangle$ and $\vec{PR} = \langle 3, 0, -3 \rangle$. So the normal vector is $\vec{n} = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix} i & j & k \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix} i + \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} j + \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} k = \langle 6, 3, 6 \rangle$.

A point on the plane is $P(-1, 0, 2)$. Therefore, $6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0$, or simplified, $6x + 3y + 6z - 6 = 0$. 
Find an equation of the plane which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

A normal vector to the plane can be found by taking the cross product of any two vectors that lie in the plane.
Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

**Solution:**

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie in the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2, -2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -3 \rangle$. 
Find an equation of the plane which contains the points \( P(-1, 0, 2), \ Q(1, -2, 1) \) and \( R(2, 0, -1) \).

Solution:

- A normal vector to the plane can be found by taking the cross product of \( \vec{PQ} = \langle 2, -2, -1 \rangle \) and \( \vec{PR} = \langle 3, 0, -3 \rangle \).
- So the normal vector is
  \[
  \mathbf{n} = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  2 & -2 & -1 \\
  3 & 0 & -3 \\
\end{vmatrix}
  \]
- A point on the plane is \( P(-1, 0, 2) \).
- Therefore, \( 6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0 \), or simplified, \( 6x + 3y + 6z - 6 = 0 \).
Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

**Solution:**

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie in the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2, -2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -3 \rangle$.

- So the normal vector is

  $$
  \mathbf{n} = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  2 & -2 & -1 \\
  3 & 0 & -3
  \end{vmatrix}
  = \begin{vmatrix}
  -2 & -1 \\
  2 & -1 \\
  3 & -3
  \end{vmatrix} \mathbf{i} - \begin{vmatrix}
  2 & -1 \\
  3 & -3
  \end{vmatrix} \mathbf{j} + \begin{vmatrix}
  2 & -2 \\
  3 & 0
  \end{vmatrix} \mathbf{k} = \langle 6, 3, 6 \rangle.
  $$
Problem 12(a) - Spring 2007

Find an equation of the plane which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of any two vectors that lie in the plane. Two vectors that lie in the plane are $\vec{PQ} = \langle 2, -2, -1 \rangle$ and $\vec{PR} = \langle 3, 0, -3 \rangle$.

- So the normal vector is

$$
\mathbf{n} = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & -2 & -1 \\
3 & 0 & -3 \\
\end{vmatrix} = \\
\begin{vmatrix}
-2 & -1 \\
0 & -3 \\
\end{vmatrix} \mathbf{i} - \\
\begin{vmatrix}
2 & -1 \\
3 & -3 \\
\end{vmatrix} \mathbf{j} + \\
\begin{vmatrix}
2 & -2 \\
3 & 0 \\
\end{vmatrix} \mathbf{k} = \langle 6, 3, 6 \rangle.
$$

- A point on the plane is $P(-1, 0, 2)$. Therefore,

$$
6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0,
$$

or simplified,

$$
6x + 3y + 6z - 6 = 0.
$$
Problem 12(a) - Spring 2007

Find an equation of the plane which contains the points $P(-1, 0, 2), Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of any two vectors that lie in the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2, -2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -3 \rangle$.

- So the normal vector is $n = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \left| \begin{array}{ccc} i & j & k \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{array} \right| = \left| \begin{array}{ccc} -2 & -1 \\ 2 & -1 \\ 3 & -3 \end{array} \right| i - \left| \begin{array}{ccc} 0 & -1 \\ 2 & -1 \\ 3 & -3 \end{array} \right| j + \left| \begin{array}{ccc} 0 & -1 \\ 2 & -2 \\ 3 & 0 \end{array} \right| k = \langle 6, 3, 6 \rangle$.

- A point on the plane is $P(-1, 0, 2)$. Therefore,

$$6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0.$$
Problem 12(a) - Spring 2007

Find an equation of the plane which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of any two vectors that lie in the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2, -2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 0, -3 \rangle$.

- So the normal vector is
  $$n = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} \mathbf{k} = \langle 6, 3, 6 \rangle.$$  

- A point on the plane is $P(-1, 0, 2)$. Therefore,
  $$6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0,$$

or simplified, $$6x + 3y + 6z - 6 = 0.$$
Problem 12(b) - Spring 2007

Find the distance $D$ from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$. 

Solution: The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance $D$ from $(1, 0, -1)$ to the plane is equal to:

$$D = \frac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{|\langle 1, -1, -1 \rangle \cdot \langle 2, 1, -2 \rangle|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{|2 - 1 + 2|}{\sqrt{9}} = 1.$$
Problem 12(b) - Spring 2007

Find the distance $D$ from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane.
Find the distance $D$ from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. 
Problem 12(b) - Spring 2007

Find the distance \( D \) from the point \( P_1 = (1, 0, -1) \) to the plane \( 2x + y - 2z = 1 \).

Solution:

The normal to the plane is \( \mathbf{n} = \langle 2, 1, -2 \rangle \) and the point \( P_0 = (0, 1, 0) \) lies on this plane. Consider the vector from \( P_0 \) to \( P_1 = (1, 0, -1) \) which is \( \mathbf{b} = \langle 1, -1, -1 \rangle \). The distance \( D \) from \((1, 0, -1)\) to the plane is equal to:

\[
|\mathbf{b} \cdot \mathbf{n}| / |\mathbf{n}|
\]
Problem 12(b) - Spring 2007

Find the distance $D$ from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance $D$ from $(1, 0, -1)$ to the plane is equal to:

$$|\text{comp}_\mathbf{n} \mathbf{b}| =$$
Problem 12(b) - Spring 2007

Find the distance $D$ from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $n = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 0, -1)$ which is $b = \langle 1, -1, -1 \rangle$. The distance $D$ from $(1, 0, -1)$ to the plane is equal to:

$$|\text{comp}_n b| = \left| b \cdot \frac{n}{|n|} \right|$$
Problem 12(b) - Spring 2007

Find the distance $D$ from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance $D$ from $(1, 0, -1)$ to the plane is equal to:

$$\left|\text{comp}_n \mathbf{b}\right| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{||\mathbf{n}||} \right| = |\langle 1, -1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle|$$
Problem 12(b) - Spring 2007

Find the distance \( D \) from the point \( P_1 = (1, 0, -1) \) to the plane \( 2x + y - 2z = 1 \).

Solution:

The normal to the plane is \( \mathbf{n} = \langle 2, 1, -2 \rangle \) and the point \( P_0 = (0, 1, 0) \) lies on this plane. Consider the vector from \( P_0 \) to \( P_1 = (1, 0, -1) \) which is \( \mathbf{b} = \langle 1, -1, -1 \rangle \). The distance \( D \) from \( (1,0,-1) \) to the plane is equal to:

\[
|\mathbf{comp}_n \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{||\mathbf{n}||} \right| = \left| \langle 1, -1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle \right| = 1.
\]
Find the point $P$ in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find $P$ or first find the equation of the line passing through $P$ and the point $(1, 0, -1)$ and then solve for $P$.)
Problem 12(c) - Spring 2007

Find the point $P$ in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find $P$ or first find the equation of the line passing through $P$ and the point $(1, 0, -1)$ and then solve for $P$.)

Solution:

1. First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$;
Problem 12(c) - Spring 2007

Find the point \( P \) in the plane \( 2x + y - 2z = 1 \) which is closest to the point \( (1, 0, -1) \). (Hint: You can use part (b) of this problem to help find \( P \) or first find the equation of the line passing through \( P \) and the point \( (1, 0, -1) \) and then solve for \( P \).)

Solution:

- First find the **parametric equations** of the line that goes through the point \( (1, 0, -1) \) that is normal to the plane: \( x = 1 + 2t \), \( y = t \), \( z = -1 - 2t \); here \( n = \langle 2, 1, -2 \rangle \) is a normal to the plane.
Problem 12(c) - Spring 2007

Find the point $P$ in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find $P$ or first find the equation of the line passing through $P$ and the point $(1, 0, -1)$ and then solve for $P$.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $n = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point $P$ in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.
Problem 12(c) - Spring 2007

Find the point \( P \) in the plane \( 2x + y - 2z = 1 \) which is closest to the point \( (1, 0, -1) \). (Hint: You can use part (b) of this problem to help find \( P \) or first find the equation of the line passing through \( P \) and the point \( (1, 0, -1) \) and then solve for \( P \).)

Solution:

- First find the **parametric equations** of the line that goes through the point \( (1, 0, -1) \) that is normal to the plane: \( x = 1 + 2t \), \( y = t \), \( z = -1 - 2t \); here \( n = \langle 2, 1, -2 \rangle \) is a normal to the plane.
- The point \( P \) in the plane closest to \( (1, 0, -1) \) is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation:
  \[
  2(1 + 2t) + (t) - 2(-1 - 2t) = 1.
  \]
Problem 12(c) - Spring 2007

Find the point \( P \) in the plane \( 2x + y - 2z = 1 \) which is closest to the point \((1, 0, -1)\). (Hint: You can use part (b) of this problem to help find \( P \) or first find the equation of the line passing through \( P \) and the point \((1, 0, -1)\) and then solve for \( P \).)

Solution:

- First find the **parametric equations** of the line that goes through the point \((1, 0, -1)\) that is normal to the plane: \( x = 1 + 2t, \ y = t, \ z = -1 - 2t; \) here \( n = \langle 2, 1, -2 \rangle \) is a normal to the plane.
- The point \( P \) in the plane closest to \((1, 0, -1)\) is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation: \( 2(1 + 2t) + (t) - 2(-1 - 2t) = 1 \).

Simplifying and solving for \( t \),
\[
9t + 4 = 1 \implies t = -\frac{1}{3}.
\]
Problem 12(c) - Spring 2007

Find the point $P$ in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find $P$ or first find the equation of the line passing through $P$ and the point $(1, 0, -1)$ and then solve for $P$.)

Solution:

- First find the **parametric equations** of the line that goes through the point $(1, 0, -1)$ that is normal to the plane: $x = 1 + 2t$, $y = t$, $z = -1 - 2t$; here $\mathbf{n} = \langle 2, 1, -2 \rangle$ is a normal to the plane.
- The point $P$ in the plane closest to $(1, 0, -1)$ is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation: $2(1 + 2t) + (t) - 2(-1 - 2t) = 1$.

Simplifying and solving for $t$,

$$9t + 4 = 1 \implies t = -\frac{1}{3}.$$  

- Plugging this $t$-value into the **parametric equations**, we get the coordinates of the point of intersection: $x = 1 + 2(-\frac{1}{3}) = \frac{1}{3}$, $y = -\frac{1}{3}$, $z = -1 - 2(-\frac{1}{3}) = -\frac{1}{3}$.  

So the point on the plane closest to $(1, 0, -1)$ is $P = (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$.  

Problem 12(c) - Spring 2007

Find the point \( P \) in the plane \( 2x + y - 2z = 1 \) which is closest to the point \((1, 0, -1)\). (Hint: You can use part (b) of this problem to help find \( P \) or first find the equation of the line passing through \( P \) and the point \((1, 0, -1)\) and then solve for \( P \).)

Solution:

- First find the **parametric equations** of the line that goes through the point \((1, 0, -1)\) that is normal to the plane: \( x = 1 + 2t, \ y = t, \ z = -1 - 2t; \) here \( n = \langle 2, 1, -2 \rangle \) is a normal to the plane.
- The point \( P \) in the plane closest to \((1, 0, -1)\) is the intersection of this line and the plane.
- Substitute the **parametric equations** of the line into the plane equation: \( 2(1 + 2t) + t - 2(-1 - 2t) = 1. \)

Simplifying and solving for \( t \),

\[
9t + 4 = 1 \implies t = -\frac{1}{3}.
\]

- Plugging this \( t \)-value into the **parametric equations**, we get the coordinates of the point of intersection: \( x = 1 + 2(-\frac{1}{3}) = \frac{1}{3}, \)
\( y = -\frac{1}{3}, \ z = -1 - 2(-\frac{1}{3}) = -\frac{1}{3}. \)
- So the point on the plane closest to \((1, 0, -1)\) is \( P = (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}) \).
Problem 13(a) - Spring 2007

Consider the two space curves
\[ r_1(t) = \langle \cos(t - 1), t^2 - 1, 2t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \]
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).
Consider the two space curves
\( \mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \)
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:
- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( \mathbf{r}_1 \) and \( s \)-value \( s = 1 \) for \( \mathbf{r}_2 \).
Consider the two space curves
\( r_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad r_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle, \)
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:

- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( r_1 \) and \( s \)-value \( s = 1 \) for \( r_2 \).
- \( r_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle \) is the tangent vector to \( r_1(t) \).
Consider the two space curves
\[ r_1(t) = \langle \cos(t-1), t^2-1, 2t^4 \rangle, \quad r_2(s) = \langle 1+\ln s, s^2-2s+1, 2s^2 \rangle, \]
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:
- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( r_1 \) and \( s \)-value \( s = 1 \) for \( r_2 \).
- \( r_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle \) is the tangent vector to \( r_1(t) \).
- At \( t = 1 \), \( r_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle \)
Consider the two space curves
\( \mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle \), where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

**Solution:**

- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( \mathbf{r}_1 \) and \( s \)-value \( s = 1 \) for \( \mathbf{r}_2 \).
- \( \mathbf{r}_1'(t) = \langle -\sin(t - 1), 2t, 8t^3 \rangle \) is the tangent vector to \( \mathbf{r}_1(t) \).
- At \( t = 1 \), \( \mathbf{r}_1'(1) = \langle -\sin(1 - 1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle \).
Problem 13(a) - Spring 2007

Consider the two space curves
\( \mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \)
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:

- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( \mathbf{r}_1 \) and \( s \)-value \( s = 1 \) for \( \mathbf{r}_2 \).
- \( \mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle \) is the tangent vector to \( \mathbf{r}_1(t) \).
- At \( t = 1 \), \( \mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle \).
- \( \mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s - 2, 4s \rangle \) is the tangent vector to \( \mathbf{r}_2(s) \).
Consider the two space curves 
\[ \mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \]
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:
- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( \mathbf{r}_1 \) and \( s \)-value \( s = 1 \) for \( \mathbf{r}_2 \).
- \( \mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle \) is the tangent vector to \( \mathbf{r}_1(t) \).
- At \( t = 1 \), \( \mathbf{r}_1'(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle \).
- \( \mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s - 2, 4s \rangle \) is the tangent vector to \( \mathbf{r}_2(s) \).
- At \( s = 1 \), \( \mathbf{r}_2'(1) = \langle \frac{1}{1}, 2(1) - 2, 4(1) \rangle \)
Problem 13(a) - Spring 2007

Consider the two space curves
\[ \mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \]
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:

- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( \mathbf{r}_1 \) and \( s \)-value \( s = 1 \) for \( \mathbf{r}_2 \).
- \( \mathbf{r}'_1(t) = \langle -\sin(t - 1), 2t, 8t^3 \rangle \) is the tangent vector to \( \mathbf{r}_1(t) \).
- At \( t = 1 \), \( \mathbf{r}'_1(1) = \langle -\sin(1 - 1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle \).
- \( \mathbf{r}'_2(s) = \langle \frac{1}{s}, 2s - 2, 4s \rangle \) is the tangent vector to \( \mathbf{r}_2(s) \).
- At \( s = 1 \), \( \mathbf{r}'_2(1) = \langle \frac{1}{1}, 2(1) - 2, 4(1) \rangle = \langle 1, 0, 4 \rangle \).
Problem 13(a) - Spring 2007

Consider the two space curves

\[ r_1(t) = \langle \cos(t-1), t^2 - 1, 2t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \]

where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

Solution:

- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( r_1 \) and \( s \)-value \( s = 1 \) for \( r_2 \).
- \( r'_1(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle \) is the tangent vector to \( r_1(t) \).
- At \( t = 1 \), \( r'_1(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle \).
- \( r'_2(s) = \langle \frac{1}{s}, 2s - 2, 4s \rangle \) is the tangent vector to \( r_2(s) \).
- At \( s = 1 \), \( r'_2(1) = \langle \frac{1}{1}, 2(1) - 2, 4(1) \rangle = \langle 1, 0, 4 \rangle \).
- Therefore,

\[
\cos(\theta) = \frac{\langle 0, 2, 8 \rangle \cdot \langle 1, 0, 4 \rangle}{\|\langle 0, 2, 8 \rangle\| \|\langle 1, 0, 4 \rangle\|}
\]
Consider the two space curves
\[ \mathbf{r}_1(t) = \langle \cos(t-1), t^2 - 1, 2t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, 2s^2 \rangle, \]
where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle \( \theta \) between the tangent vectors of the two curves at the intersection point \((1, 0, 2)\).

**Solution:**

- The point \((1, 0, 2)\) corresponds to the \( t \)-value \( t = 1 \) for \( \mathbf{r}_1 \) and \( s \)-value \( s = 1 \) for \( \mathbf{r}_2 \).
- \( \mathbf{r}'_1(t) = \langle -\sin(t-1), 2t, 8t^3 \rangle \) is the tangent vector to \( \mathbf{r}_1(t) \).
- At \( t = 1 \), \( \mathbf{r}'_1(1) = \langle -\sin(1-1), 2(1), 8(1^3) \rangle = \langle 0, 2, 8 \rangle \).
- \( \mathbf{r}'_2(s) = \langle \frac{1}{s}, 2s - 2, 4s \rangle \) is the tangent vector to \( \mathbf{r}_2(s) \).
- At \( s = 1 \), \( \mathbf{r}'_2(1) = \langle \frac{1}{1}, 2(1) - 2, 4(1) \rangle = \langle 1, 0, 4 \rangle \).
- Therefore,
\[
\cos(\theta) = \frac{\langle 0, 2, 8 \rangle \cdot \langle 1, 0, 4 \rangle}{||\langle 0, 2, 8 \rangle||||\langle 1, 0, 4 \rangle||} = \frac{32}{\sqrt{68} \sqrt{17}}.
\]
Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

\[ x^2 + y^2 + 2y + z^2 + 4z = 20. \]

**Solution:**

Completing the square in the \( y \) and \( z \) variables, we get

\[ x^2 + (y^2 + 2y + 1) + (z^2 + 4z + 4) = 20 + 1 + 4. \]

Rewriting, we have

\[ x^2 + (y + 1)^2 + (z + 2)^2 = 25 = 5^2. \]

Hence, the **center** is \( C = (0, -1, -2) \) and the **radius** is \( r = 5 \).
Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

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- Completing the square in the \( y \) and \( z \) variables, we get

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Find the **center** and **radius** of the sphere

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Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

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**Solution:**

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- Rewriting, we have

\[ x^2 + (y + 1)^2 + (z + 2)^2 = 25 = 5^2. \]

- Hence, the **center** is \( C = (0, -1, -2) \) and the **radius** is \( r = 5 \).
The velocity vector of a particle moving in space equals \( v(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k} \) at any time \( t \geq 0 \).

At the time \( t = 4 \), this particle is at the point \((0, 5, 4)\). Find an equation of the tangent line \( T \) to the position curve \( r(t) \) at the time \( t = 4 \).
The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k} \) at any time \( t \geq 0 \).

At the time \( t = 4 \), this particle is at the point \((0, 5, 4)\). Find an equation of the tangent line \( T \) to the position curve \( \mathbf{r}(t) \) at the time \( t = 4 \).

Solution:

- This line goes through the point \((0, 5, 4)\) and has vector part parallel to the tangent vector \( \mathbf{v}(4) = \langle 8, -8, 4 \rangle \).
Problem 14(a) - Spring 2007

The velocity vector of a particle moving in space equals
\[ \mathbf{v}(t) = 2t \mathbf{i} - 2t \mathbf{j} + t \mathbf{k} \]
at any time \( t \geq 0 \).

At the time \( t = 4 \), this particle is at the point \((0, 5, 4)\). Find an equation of the tangent line \( T \) to the position curve \( \mathbf{r}(t) \) at the time \( t = 4 \).

Solution:

- This line goes through the point \((0, 5, 4)\) and has vector part parallel to the tangent vector \( \mathbf{v}(4) = \langle 8, -8, 4 \rangle \).
- The vector equation is: \( T(t) = \langle 0, 5, 4 \rangle + t \langle 8, -8, 4 \rangle \).
The velocity vector of a particle moving in space equals \( v(t) = 2ti - 2tj + tk \) at any time \( t \geq 0 \).

At the time \( t = 4 \), this particle is at the point \((0, 5, 4)\). Find an equation of the tangent line \( T \) to the position curve \( r(t) \) at the time \( t = 4 \).

**Solution:**

- This line goes through the point \((0, 5, 4)\) and has vector part parallel to the tangent vector \( v(4) = \langle 8, -8, 4 \rangle \).
- The vector equation is: \( T(t) = \langle 0, 5, 4 \rangle + t\langle 8, -8, 4 \rangle \)
- So the line \( T \) has the parametric equations:
  \[
  x = 8t \\
  y = 5 - 8t \\
  z = 4 + 4t. 
  \]
Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals 
\( \mathbf{v}(t) = 2t \mathbf{i} - 2t \mathbf{j} + t \mathbf{k} \) at any time \( t \geq 0 \).

Find the length \( L \) of the arc traveled from time \( t = 2 \) to time \( t = 4 \).
Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $v(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$. Find the length $L$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$L = \int_{2}^{4} |v(t)|\, dt = \int_{2}^{4} \sqrt{(2t)^2 + (-2t)^2 + t^2}\, dt$$
The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k} \) at any time \( t \geq 0 \).

Find the length \( L \) of the arc traveled from time \( t = 2 \) to time \( t = 4 \).

Solution:

Using the arclength formula,

\[
L = \int_2^4 |\mathbf{v}(t)| \, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2} \, dt
\]

\[
= \int_2^4 \sqrt{9t^2} \, dt
\]

\[
= \int_2^4 3t \, dt
\]

\[
= \left[ \frac{3}{2} t^2 \right]_2^4 = \frac{3}{2} (16 - 4) = 18.
\]
The velocity vector of a particle moving in space equals $v(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$. Find the length $L$ of the arc traveled from time $t = 2$ to time $t = 4$.

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Using the arclength formula,

\[
L = \int_{2}^{4} |v(t)| \, dt = \int_{2}^{4} \sqrt{(2t)^2 + (-2t)^2 + t^2} \, dt
\]

\[
= \int_{2}^{4} \sqrt{9t^2} \, dt = \int_{2}^{4} 3t \, dt
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The velocity vector of a particle moving in space equals

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Find the length \( L \) of the arc traveled from time \( t = 2 \) to time \( t = 4 \).

**Solution:**

Using the arclength formula,

\[ L = \int_{2}^{4} |\mathbf{v}(t)| \, dt = \int_{2}^{4} \sqrt{(2t)^2 + (-2t)^2 + t^2} \, dt \]

\[ = \int_{2}^{4} \sqrt{9t^2} \, dt = \int_{2}^{4} 3t \, dt \]

\[ = \frac{3}{2} t^2 \bigg|_{2}^{4} \]

\[ = \frac{3}{2} (16 - 4) = 18 \]
The velocity vector of a particle moving in space equals 

\[ \mathbf{v}(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k} \] 
at any time \( t \geq 0 \).

Find the length \( L \) of the arc traveled from time \( t = 2 \) to time \( t = 4 \).

**Solution:**

Using the arclength formula,

\[
L = \int_2^4 |\mathbf{v}(t)| \, dt = \int_2^4 \sqrt{(2t)^2 + (-2t)^2 + t^2} \, dt
\]

\[
= \int_2^4 \sqrt{9t^2} \, dt = \int_2^4 3t \, dt
\]

\[
= \frac{3}{2} t^2 \bigg|_2^4 = \frac{3}{2} (16 - 4)
\]
Problem 14(b) - Spring 2007

The velocity vector of a particle moving in space equals $v(t) = 2t\mathbf{i} - 2t\mathbf{j} + t\mathbf{k}$ at any time $t \geq 0$. Find the length $L$ of the arc traveled from time $t = 2$ to time $t = 4$.

Solution:

Using the arclength formula,

$$L = \int_{2}^{4} |v(t)| \, dt = \int_{2}^{4} \sqrt{(2t)^2 + (-2t)^2 + t^2} \, dt$$

$$= \int_{2}^{4} \sqrt{9t^2} \, dt = \int_{2}^{4} 3t \, dt$$

$$= \frac{3}{2} t^2 \bigg|_{2}^{4} = \frac{3}{2} (16 - 4) = 18.$$
Problem 14(c) - Spring 2007

Find a vector function \( \mathbf{r}(t) \) which represents the **curve of intersection** of the cylinder \( x^2 + y^2 = 1 \) and the plane \( x + 2y + z = 4 \).
Problem 14(c) - Spring 2007

Find a vector function $\mathbf{r}(t)$ which represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + 2y + z = 4$.

Solution:

- Since the first equation is the equation of a circular cylinder, parametrize the $x$ and $y$ coordinates by setting $x = \cos(t)$ and $y = \sin(t)$. 

- Next, use the second equation $z = 4 - x - 2y$ to solve for $z$ in terms of $t$: 

$z = 4 - x - 2y = 4 - \cos(t) - 2\sin(t)$.

- Therefore, $\mathbf{r}(t) = \langle \cos(t), \sin(t), 4 - \cos(t) - 2\sin(t) \rangle$. 

Problem 14(c) - Spring 2007

Find a vector function \( r(t) \) which represents the **curve of intersection** of the cylinder \( x^2 + y^2 = 1 \) and the plane \( x + 2y + z = 4 \).

**Solution:**

- Since the first equation is the equation of a circular cylinder, parametrize the \( x \) and \( y \) coordinates by setting \( x = \cos(t) \) and \( y = \sin(t) \).
- Next use the second equation \( z = 4 - x - 2y \) to solve for \( z \) in terms of \( t \):
Problem 14(c) - Spring 2007

Find a vector function \( \mathbf{r}(t) \) which represents the curve of intersection of the cylinder \( x^2 + y^2 = 1 \) and the plane \( x + 2y + z = 4 \).

Solution:

- Since the first equation is the equation of a circular cylinder, parametrize the \( x \) and \( y \) coordinates by setting \( x = \cos(t) \) and \( y = \sin(t) \).
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\[
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\]
Problem 14(c) - Spring 2007

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Solution:

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- Next use the second equation \( z = 4 - x - 2y \) to solve for \( z \) in terms of \( t \):

\[
  z = 4 - x - 2y = 4 - \cos(t) - 2\sin(t).
\]

- Therefore,

\[
  \mathbf{r}(t) = \langle \cos(t), \sin(t), 4 - \cos(t) - 2\sin(t) \rangle.
\]
Consider the points $A(2, 1, 0)$, $B(1, 0, 2)$ and $C(0, 2, 1)$. Find the area $A$ of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:
The area of the parallelogram is $|\vec{AB} \times \vec{AC}| = |\begin{vmatrix} i & j & k \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}| = |\begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix}i + |\begin{vmatrix} -1 & -2 \\ -1 & 1 \end{vmatrix}j + |\begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix}k| = |\langle -3, -3, -3 \rangle| = \sqrt{27}$. 

So the area of the triangle $ABC$ is $A = \sqrt{27}/2$. 
Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area $A$ of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\vec{AB} \times \vec{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \sqrt{27}.$$
Problem 15(a) - Spring 2008

Consider the points $A(2, 1, 0)$, $B(1, 0, 2)$ and $C(0, 2, 1)$. Find the area $A$ of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

The area of the parallelogram is

$$|\vec{AB} \times \vec{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -2 & 1 \\ -2 & 1 \end{vmatrix} \mathbf{k}$$

$$= \sqrt{27}.$$
Consider the points \( A(2, 1, 0), \ B(1, 0, 2) \) and \( C(0, 2, 1) \). Find the area \( A \) of the triangle \( ABC \). (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

The area of the parallelogram is

\[
\left| \overrightarrow{AB} \times \overrightarrow{AC} \right| = \left| \begin{array}{ccc}
    \mathbf{i} & \mathbf{j} & \mathbf{k} \\
    -1 & -1 & 2 \\
    -2 & 1 & 1 \\
\end{array} \right|
\]

\[
= \left| \begin{array}{ccc}
    -1 & 2 & \mathbf{i} \\
    1 & 1 & \mathbf{j} \\
    -2 & 1 & \mathbf{k} \\
\end{array} \right|
\]

\[
= |\langle -3, -3, -3 \rangle|
\]
Problem 15(a) - Spring 2008

Consider the points $A(2, 1, 0)$, $B(1, 0, 2)$ and $C(0, 2, 1)$. Find the area $A$ of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

The area of the parallelogram is

$$|\vec{AB} \times \vec{AC}| = \begin{vmatrix}
    \mathbf{i} & \mathbf{j} & \mathbf{k} \\
    -1 & -1 & 2 \\
    -2 & 1 & 1 \\
\end{vmatrix} = \begin{vmatrix}
    -1 & 2 \\
    1 & 1 \\
\end{vmatrix} \mathbf{i} - \begin{vmatrix}
    2 & 1 \\
    1 & 1 \\
\end{vmatrix} \mathbf{j} + \begin{vmatrix}
    -1 & -1 \\
    -2 & 1 \\
\end{vmatrix} \mathbf{k}
$$

$$= |\langle -3, -3, -3 \rangle| = \sqrt{27}.$$
Consider the points $A(2, 1, 0)$, $B(1, 0, 2)$ and $C(0, 2, 1)$. Find the area $A$ of the triangle $ABC$. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

Solution:

- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} \mathbf{k}$$

$$= |\langle -3, -3, -3 \rangle| = \sqrt{27}.$$

- So the area of the triangle $ABC$ is

$$A = \frac{\sqrt{27}}{2}.$$
Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).
Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- We find \( \mathbf{r}(t) \) by integrating \( \mathbf{r}'(t) = \mathbf{v}(t) \):

\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(t) \, dt + \mathbf{r}(0)
\]
Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- We find \( \mathbf{r}(t) \) by integrating \( \mathbf{r}'(t) = \mathbf{v}(t) \):

\[ \mathbf{r}(t) = \int_0^t \mathbf{v}(t) \, dt + \mathbf{r}(0) = \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle \bigg|_0^t + \langle 1, 2, 0 \rangle \]
Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

We find \( \mathbf{r}(t) \) by integrating \( \mathbf{r}'(t) = \mathbf{v}(t) \):

\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(t) \, dt + \mathbf{r}(0) = \left. \langle - \cos t, \frac{1}{2} \sin 2t, e^t \rangle \right|_0^t + \langle 1, 2, 0 \rangle
\]

\[
= \langle - \cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle - \cos 0, \frac{1}{2} \sin 0, e^0 \rangle + \langle 1, 2, 0 \rangle
\]

\[
= \langle - \cos t, \frac{1}{2} \sin 2t, e^t \rangle + \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle
\]

\[
= \langle - \cos t - 1 + 1, \frac{1}{2} \sin 2t, e^t + 1 \rangle
\]

\[
= \langle - \cos t, \frac{1}{2} \sin 2t, e^t + 1 \rangle
\]
Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

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\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(t) \, dt + \mathbf{r}(0) = \left. \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle \right|_0^t + \langle 1, 2, 0 \rangle
\]

\[
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle -\cos 0, \frac{1}{2} \sin 0, e^0 \rangle + \langle 1, 2, 0 \rangle
\]

\[
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle
\]
Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:
- We find \( \mathbf{r}(t) \) by integrating \( \mathbf{r}'(t) = \mathbf{v}(t) \):

\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(t) \, dt + \mathbf{r}(0) = \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle \bigg|_0^t + \langle 1, 2, 0 \rangle
\]

\[
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle -\cos 0, \frac{1}{2} \sin 0, e^0 \rangle + \langle 1, 2, 0 \rangle
\]

\[
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle
\]

\[
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle + \langle 2, 2, -1 \rangle.
\]
Problem 15(b) - Spring 2008

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- We find \( \mathbf{r}(t) \) by integrating \( \mathbf{r}'(t) = \mathbf{v}(t) \):

\[
\mathbf{r}(t) = \int_0^t \mathbf{v}(t) \, dt + \mathbf{r}(0) = \left[ -\cos t, \frac{1}{2} \sin 2t, e^t \right]_0^t + \langle 1, 2, 0 \rangle \\
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle -\cos 0, \frac{1}{2} \sin 0, e^0 \rangle + \langle 1, 2, 0 \rangle \\
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle - \langle -1, 0, 1 \rangle + \langle 1, 2, 0 \rangle \\
= \langle -\cos t, \frac{1}{2} \sin 2t, e^t \rangle + \langle 2, 2, -1 \rangle.
\]

- So,

\[ \mathbf{r}(t) = \langle 2 - \cos t, 2 + \frac{1}{2} \sin 2t, -1 + e^t \rangle. \]
Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

\[ x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and} \]
\[ x = 4 - t, \quad y = 3 + 2t, \quad z = 1 \]

Solution:

To find the equation of a plane, we need to find its normal \( \mathbf{n} \) and a point on it.

Setting \( t = 0 \), we find the point \((4, 3, 1)\) on the first line.

The part vector \( \mathbf{v}_1 \) of the first line is \( \langle -4, -1, 5 \rangle \) and the vector part \( \mathbf{v}_2 \) of the second line is \( \langle -1, 2, 0 \rangle \).

Since the vector \( \mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 \) is orthogonal to both \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \), it is the normal to the plane.

The equation of the plane is:

\[
\langle -10, -5, -9 \rangle \cdot \langle x - 4, y - 3, z - 1 \rangle = -10(x - 4) - 5(y - 3) - 9(z - 1) = 0
\]
Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

\[
\begin{align*}
  x &= 4 - 4t, & y &= 3 - t, & z &= 1 + 5t \\
  x &= 4 - t, & y &= 3 + 2t, & z &= 1
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\]

Solution:

- To find the equation of a plane, we need to find its normal \( \mathbf{n} \) and a point on it.
Problem 16 - Fall 2007

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The part vector \( \mathbf{v}_1 \) of the first line is \( \langle -4, -1, 5 \rangle \) and the vector part \( \mathbf{v}_2 \) of the second line is \( \langle -1, 2, 0 \rangle \).

Since the vector
\[
\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -1 & 5 \\ -1 & 2 & 0 \end{vmatrix} = \langle -10, -5, -9 \rangle,
\]
is orthogonal to both \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \), it is the normal to the plane.
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\[ x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and} \]
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  \]
Problem 17 - Fall 2007

Find the distance $D$ from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$. 

Solution:

Recall the distance formula $D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.

In order to apply the formula, rewrite the equation of the plane in standard form: $4x - 6y - z - 5 = 0$.

So, the distance from $(3, -2, 7)$ to the plane is:

$D = \frac{|4 \cdot 3 + (-6) \cdot (-2) + (-1) \cdot 7 - 5|}{\sqrt{4^2 + (-6)^2 + (-1)^2}} = \frac{12}{\sqrt{53}}$. 

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$$D = \frac{|(4 \cdot 3) + (-6 \cdot -2) + (-1 \cdot 7) - 5|}{\sqrt{4^2 + (-6)^2 + (-1)^2}}$$
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Problem 18 - Fall 2007

Determine whether the lines $L_1$ and $L_2$ given below are parallel, skew or intersecting. If they intersect, find the point of intersection.

$L_1: \frac{x}{1} = \frac{y - 1}{2} = \frac{z - 2}{3}$

$L_2: \frac{x - 3}{-4} = \frac{y - 2}{-3} = \frac{z - 1}{2}$

Solution:
Rewrite these lines as vector equations:

$L_1(\mathbf{t}) = \langle t, 2t + 1, 3t + 2 \rangle$

$L_2(\mathbf{s}) = \langle -4s + 3, -3s + 2, 2s + 1 \rangle$

Equating $x$ and $y$-coordinates:

$x = t = -4s + 3$

$y = 2t + 1 = -3s + 2$

Solving gives $s = 1$ and $t = -1$.

$L_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = L_2(1)$.

So these lines do not intersect.

Since the lines are clearly not parallel (the direction vectors $\langle 1, 2, 3 \rangle$ and $\langle -4, -3, 2 \rangle$ are not parallel), the lines are skew.
Problem 18 - Fall 2007

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\begin{align*}
L_1 & : \frac{x}{1} = \frac{y - 1}{2} = \frac{z - 2}{3} \\
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Solution:

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\end{align*}$

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\end{align*}$

Solution:

- Rewrite these lines as vector equations:

  $L_1(t) = \langle t, 2t + 1, 3t + 2 \rangle$

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x &= \frac{1}{2} \\
y - 1 &= \frac{2}{3} \\
z - 2 &= \frac{3}{4}
\end{align*}$

$L_2 : \begin{align*}
x - 3 &= \frac{-4}{-3} \\
y - 2 &= \frac{-3}{2} \\
z - 1 &= \frac{2}{1}
\end{align*}$

Solution:

- Rewrite these lines as vector equations:
  
  $L_1(t) = \langle t, 2t + 1, 3t + 2 \rangle$

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Solution:

- Rewrite these lines as vector equations:
  \[ L_1(t) = \langle t, 2t + 1, 3t + 2 \rangle \]
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  \[ x = t = -4s + 3 \]
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- Solving gives $s = 1$ and $t = -1$.
- $L_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = L_2(1)$. So these lines do not intersect.
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- Since the lines are clearly not parallel (the direction vectors $\langle 1, 2, 3 \rangle$ and $\langle -4, -3, 2 \rangle$ are not parallel), the lines are skew.
Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity
\[ \mathbf{v}(t) = \langle 3t^2, 2 \sin(2t), e^t \rangle. \]
Find the \textit{acceleration} of the particle. Write down a formula for the \textit{speed} of the particle (you do not need to simplify the expression algebraically).
Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity
\[ \mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]
Find the acceleration of the particle. Write down a formula for the speed of the particle (you do not need to simplify the expression algebraically).

Solution:
- Recall the acceleration vector \( \mathbf{a}(t) = \mathbf{v}'(t) \).
Suppose a particle moving in space has the velocity
\[ \mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]
Find the acceleration of the particle. Write down a formula for the speed of the particle (you do not need to simplify the expression algebraically).

Solution:
- Recall the acceleration vector \( \mathbf{a}(t) = \mathbf{v}'(t) \). Hence,
  \[ \mathbf{a}(t) = \langle 6t, 4\cos(2t), e^t \rangle. \]
Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity
\[ v(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]
Find the **acceleration** of the particle. Write down a formula for the **speed** of the particle (you do not need to simplify the expression algebraically).

**Solution:**

- Recall the **acceleration vector** \( a(t) = v'(t) \). Hence,
  \[ a(t) = \langle 6t, 4\cos(2t), e^t \rangle. \]
- Recall that the **speed** \( t \) is the length of the velocity vector.
Problem 19(a) - Fall 2007

Suppose a particle moving in space has the velocity
\[ v(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]
Find the acceleration of the particle. Write down a formula for the speed of the particle (you do not need to simplify the expression algebraically).

Solution:

- Recall the acceleration vector \( a(t) = v'(t) \). Hence,
  \[ a(t) = \langle 6t, 4\cos(2t), e^t \rangle. \]

- Recall that the speed \( t \) is the length of the velocity vector. Hence,
  \[ \text{speed}(t) = \sqrt{9t^4 + 4\sin^2(2t) + e^{2t}}. \]
Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

\[ v(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]

If initially the particle has the position \( r(0) = \langle 0, -1, 2 \rangle \), what is the position at time \( t \)?
Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

\[ \mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]

If initially the particle has the position \( \mathbf{r}(0) = \langle 0, -1, 2 \rangle \), what is the position at time \( t \)?

Solution:

- To find the position \( \mathbf{r}(t) \), we first integrate the velocity \( \mathbf{v}(t) \) and second use the initial position value \( \mathbf{r}(0) = \langle 0, -1, 2 \rangle \) to solve for the constants of integration.
Suppose a particle moving in space has the velocity
\[ \mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]

If initially the particle has the position \( \mathbf{r}(0) = \langle 0, -1, 2 \rangle \), what is the position at time \( t \)?

**Solution:**

To find the position \( \mathbf{r}(t) \), we first integrate the velocity \( \mathbf{v}(t) \) and second use the initial position value \( \mathbf{r}(0) = \langle 0, -1, 2 \rangle \) to solve for the constants of integration.

\[ \mathbf{r}(t) = \int \langle 3t^2, 2\sin 2t, e^t \rangle \, dt \]
Suppose a particle moving in space has the velocity

\[ \mathbf{v}(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]

If initially the particle has the position \( \mathbf{r}(0) = \langle 0, -1, 2 \rangle \), what is the position at time \( t \)?

**Solution:**

To find the position \( \mathbf{r}(t) \), we *first* integrate the velocity \( \mathbf{v}(t) \) and *second* use the initial position value \( \mathbf{r}(0) = \langle 0, -1, 2 \rangle \) to solve for the **constants of integration**.

\[ \mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int \langle 3t^2, 2\sin(2t), e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle. \]
Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

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\]

- Plugging in the position at \( t = 0 \), we get:

\[
\langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle
\]
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Suppose a particle moving in space has the velocity

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- Plugging in the position at \( t = 0 \), we get:

\[ \langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle = \langle 0, -1, 2 \rangle. \]
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\langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle = \langle 0, -1, 2 \rangle.
\]
Thus, \( x_0 = 0, y_0 = 0 \) and \( z_0 = 1 \).
Problem 19(b) - Fall 2007

Suppose a particle moving in space has the velocity

\[ v(t) = \langle 3t^2, 2\sin(2t), e^t \rangle. \]

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- To find the position \( r(t) \), we first integrate the velocity \( v(t) \) and second use the initial position value \( r(0) = \langle 0, -1, 2 \rangle \) to solve for the constants of integration.

\[
 r(t) = \int v(t) \, dt = \int \langle 3t^2, 2\sin(2t), e^t \rangle \, dt = \langle t^3 + x_0, -\cos(2t) + y_0, e^t + z_0 \rangle.
\]

- Plugging in the position at \( t = 0 \), we get:

\[
 \langle 0^3 + x_0, -\cos(0) + y_0, e^0 + z_0 \rangle = \langle x_0, -1 + y_0, 1 + z_0 \rangle = \langle 0, -1, 2 \rangle.
\]

Thus, \( x_0 = 0 \), \( y_0 = 0 \) and \( z_0 = 1 \).

- Hence,

\[
 r(t) = \langle t^3, -\cos 2t, e^t + 1 \rangle.
\]
Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the area of the parallelogram.
Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the area of the parallelogram.

Solution:

Consider the vectors $PQ = \langle 0, 2, -1 \rangle$ and $PR = \langle 3, 2, 0 \rangle$. 
Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are \( P(0, -1, 1), \) \( Q(0, 1, 0) \) and \( R(3, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the area of the parallelogram.

Solution:

Consider the vectors \( \vec{PQ} = \langle 0, 2, -1 \rangle \) and \( \vec{PR} = \langle 3, 2, 0 \rangle \). Then the area of the parallelogram spanned by \( PQ \) and \( PR \) is:
Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the area of the parallelogram.

**Solution:**

Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0 \rangle$. Then the area of the parallelogram spanned by $PQ$ and $PR$ is:

$$\text{Area}(\Delta) = |\overrightarrow{PQ} \times \overrightarrow{PR}|$$
Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0 \rangle$. Then the area of the parallelogram spanned by $PQ$ and $PR$ is:

$$\text{Area}(\Delta) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{vmatrix} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$
Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0 \rangle$. Then the area of the parallelogram spanned by $PQ$ and $PR$ is:

$$\text{Area}(\Delta) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \left| \begin{array}{ccc} i & j & k \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{array} \right|$$

$$= |\langle 2, -3, -6 \rangle|$$
Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the area of the parallelogram.

Solution:

Consider the vectors $\vec{PQ} = \langle 0, 2, -1 \rangle$ and $\vec{PR} = \langle 3, 2, 0 \rangle$. Then the area of the parallelogram spanned by $PQ$ and $PR$ is:

$$\text{Area}(\Delta) = |\vec{PQ} \times \vec{PR}| = \begin{vmatrix} i & j & k \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{vmatrix}$$

$$= |\langle 2, -3, -6 \rangle| = \sqrt{4 + 9 + 36} = 7$$
Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are \( P(0, -1, 1), \) 
\( Q(0, 1, 0) \) and \( R(3, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the cosine of the angle between the vector \( PQ \) and \( PR \).

Solution:
Note that:
\[
-\vec{PQ} = \langle 0, 2, -1 \rangle \\
-\vec{PR} = \langle 3, 2, 0 \rangle.
\]

By our formula for dot products:
\[
\cos \theta = \frac{-\vec{PQ} \cdot -\vec{PR}}{|-\vec{PQ}| \, |\, -\vec{PR}|} = \frac{\langle 0, 2, -1 \rangle \cdot \langle 3, 2, 0 \rangle}{\sqrt{5} \sqrt{13}} = \frac{4 \sqrt{5}}{\sqrt{13}}.
\]
Three of the four vertices of a parallelogram are \( P(0, -1, 1) \), \( Q(0, 1, 0) \) and \( R(3, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the cosine of the angle between the vector \( PQ \) and \( PR \).

**Solution:**

Note that:

\[
\vec{PQ} = \langle 0, 2, -1 \rangle \quad \vec{PR} = \langle 3, 2, 0 \rangle.
\]
Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the cosine of the angle between the vector $PQ$ and $PR$.

Solution:

Note that:

$$\overrightarrow{PQ} = \langle 0, 2, -1 \rangle \quad \overrightarrow{PR} = \langle 3, 2, 0 \rangle.$$

By our formula for dot products:

$$\cos \theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}| |\overrightarrow{PR}|}.$$
Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are \( P(0, -1, 1), Q(0, 1, 0) \) and \( R(3, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the cosine of the angle between the vector \( PQ \) and \( PR \).

Solution:

- Note that:

\[
\vec{PQ} = \langle 0, 2, -1 \rangle \quad \vec{PR} = \langle 3, 2, 0 \rangle.
\]

- By our formula for dot products:

\[
\cos \theta = \frac{\vec{PQ} \cdot \vec{PR}}{|\vec{PQ}| |\vec{PR}|} = \frac{\langle 0, 2, -1 \rangle \cdot \langle 3, 2, 0 \rangle}{\sqrt{5} \sqrt{13}}.
\]
Three of the four vertices of a parallelogram are \( P(0, -1, 1) \), \( Q(0, 1, 0) \) and \( R(3, 1, 1) \). Two of the sides are \( PQ \) and \( PR \). Find the cosine of the angle between the vector \( PQ \) and \( PR \).

Solution:

- Note that:
  \[
PQ = \langle 0, 2, -1 \rangle \quad PR = \langle 3, 2, 0 \rangle.
  \]

- By our formula for dot products:
  \[
  \cos \theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{||\overrightarrow{PQ}|| \ ||\overrightarrow{PR}||} = \frac{\langle 0, 2, -1 \rangle \cdot \langle 3, 2, 0 \rangle}{\sqrt{5} \sqrt{13}} = \frac{4}{\sqrt{5} \sqrt{13}}.
  \]
Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the coordinates of the fourth vertex.

Solution:
Denote the fourth vertex by $S$. Then $\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0, 1, 0 \rangle + \langle 3, 2, 0 \rangle = \langle 3, 3, 0 \rangle$, where $O$ is the origin. That is, $S = (3, 3, 0)$. 
Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $S$. 
Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $S$. Then

$$\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0, 1, 0 \rangle + \langle 3, 2, 0 \rangle = \langle 3, 3, 0 \rangle,$$

where $O$ is the origin.
Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are $PQ$ and $PR$. Find the coordinates of the fourth vertex.

**Solution:**

Denote the fourth vertex by $S$. Then

$$\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0, 1, 0 \rangle + \langle 3, 2, 0 \rangle = \langle 3, 3, 0 \rangle,$$

where $O$ is the origin. That is,

$$S = (3, 3, 0).$$
Let $C$ be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$ 

Determine the point(s) of intersection of $C$ with the $xz$-plane.

Solution:

The points of intersection of $C$ with the $xz$-plane correspond to the points where the $y$-coordinate of $C$ is 0. When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$. Hence, $$\langle 2 - \left(\frac{1}{2}\right)^2, 2 \cdot \frac{1}{2} - 1, \ln \frac{1}{2} \rangle = \langle \frac{13}{4}, 0, -\ln 2 \rangle$$ is the unique point of the intersection of $C$ with the $xz$-plane.
Problem 21(a) - Fall 2007

Let $C$ be the parametric curve

\[ x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t. \]

Determine the point(s) of intersection of $C$ with the $xz$-plane.

Solution:

- The points of intersection of $C$ with the $xz$-plane correspond to the points where the $y$-coordinate of $C$ is 0.
Problem 21(a) - Fall 2007

Let $C$ be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$ 

Determine the point(s) of intersection of $C$ with the $xz$-plane.

Solution:

- The points of intersection of $C$ with the $xz$-plane correspond to the points where the $y$-coordinate of $C$ is 0.
- When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$. 

Hence, $\langle 2 - \left(\frac{1}{2}\right)^2, 2 \cdot \frac{1}{2} - 1, \ln \frac{1}{2} \rangle = \langle \frac{13}{4}, 0, -\ln 2 \rangle$ is the unique point of the intersection of $C$ with the $xz$-plane.
Problem 21(a) - Fall 2007

Let $C$ be the parametric curve

\[ x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t. \]

Determine the point(s) of intersection of $C$ with the $xz$-plane.

**Solution:**

- The points of intersection of $C$ with the $xz$-plane correspond to the points where the $y$-coordinate of $C$ is 0.
- When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$.
- Hence,

\[ \langle 2 - \left(\frac{1}{2}\right)^2, 2 \cdot \frac{1}{2} - 1, \ln \frac{1}{2} \rangle = \langle \frac{3}{4}, 0, -\ln 2 \rangle \]

is the unique point of the intersection of $C$ with $xz$-plane.
Problem 21(b) - Fall 2007

Let \( C \) be the parametric curve

\[
    x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.
\]

Determine **parametric equations** of tangent line to \( C \) at \((1, 1, 0)\).
Problem 21(b) - Fall 2007

Let $C$ be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$ 

Determine parametric equations of tangent line to $C$ at $(1, 1, 0)$.

Solution:

- Using the $y$-coordinate of $C$, note that $t = 1$ when $(1, 1, 0) \in C$. 

Let $C$ be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$  

Determine **parametric equations** of tangent line to $C$ at $(1, 1, 0)$.

**Solution:**

- Using the $y$-coordinate of $C$, note that $t = 1$ when $(1, 1, 0) \in C$.
- The velocity vector to

  $$C(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$$

  is:
Problem 21(b) - Fall 2007

Let \( \mathbf{C} \) be the parametric curve

\[
\begin{align*}
x &= 2 - t^2, \\
y &= 2t - 1, \\
z &= \ln t.
\end{align*}
\]

Determine \textbf{parametric equations} of tangent line to \( \mathbf{C} \) at \((1, 1, 0)\).

Solution:

• Using the \( y \)-coordinate of \( \mathbf{C} \), note that \( t = 1 \) when \((1, 1, 0) \in \mathbf{C}\).

• The velocity vector to

\[
\mathbf{C}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.
\]

This is the vector part of the tangent line to \( \mathbf{C} \) at \((1, 1, 0)\).
Problem 21(b) - Fall 2007

Let \( C \) be the parametric curve
\[
\begin{align*}
    x &= 2 - t^2, \\
    y &= 2t - 1, \\
    z &= \ln t.
\end{align*}
\]
Determine **parametric equations** of tangent line to \( C \) at \((1, 1, 0)\).

**Solution:**

- Using the \( y \)-coordinate of \( C \), note that \( t = 1 \) when \((1, 1, 0) \in C\).
- The velocity vector to
  \[
  C(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle
  \]
  is:
  \[
  C'(t) = \langle -2t, 2, \frac{1}{t} \rangle.
  \]
- Thus,
  \[
  C'(1) = \langle -2, 2, 1 \rangle
  \]
  is the vector part of the tangent line to \( C \) at \((1, 1, 0)\).
Problem 21(b) - Fall 2007

Let \( C \) be the parametric curve

\[
\begin{align*}
x &= 2 - t^2, \\
y &= 2t - 1, \\
z &= \ln t.
\end{align*}
\]

Determine **parametric equations** of tangent line to \( C \) at \((1, 1, 0)\).

Solution:

- Using the \( y \)-coordinate of \( C \), note that \( t = 1 \) when \((1, 1, 0) \in C\).
- The velocity vector to \( C(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle \) is:
  \[
  C'(t) = \langle -2t, 2, \frac{1}{t} \rangle.
  \]
- Thus, \( C'(1) = \langle -2, 2, 1 \rangle \) is the vector part of the tangent line to \( C \) at \((1, 1, 0)\).
- The **parametric equations** are:
  \[
  \begin{align*}
x &= 1 - 2t \\
y &= 1 + 2t \\
z &= t.
\end{align*}
\]
Let $\mathbf{C}$ be the parametric curve

$$x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.$$ 

Set up, but not solve, a formula that will determine the length $L$ of $\mathbf{C}$ for $1 \leq t \leq 2$. 

Solution:

The vector equation of $\mathbf{C}$ is

$$\mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$$

with velocity vector

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.$$ 

Since the length of $L$ is the integral of the speed $|\mathbf{v}(t)|$,

$$L = \int_1^2 \sqrt{(-2t)^2 + 2^2 + \left(\frac{1}{t}\right)^2} \, dt.$$
Problem 21(c) - Fall 2007

Let \( \mathbf{C} \) be the parametric curve

\[
x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t.
\]

Set up, but not solve, a formula that will determine the length \( L \) of \( \mathbf{C} \) for \( 1 \leq t \leq 2 \).

Solution:

- The vector equation of \( \mathbf{C} \) is \( \mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle \) with velocity vector

\[
\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.
\]
Problem 21(c) - Fall 2007

Let \( C \) be the parametric curve

\[
\begin{align*}
  x &= 2 - t^2, \\
  y &= 2t - 1, \\
  z &= \ln t.
\end{align*}
\]

Set up, but not solve, a formula that will determine the length \( L \) of \( C \) for \( 1 \leq t \leq 2 \).

Solution:

- The vector equation of \( C \) is \( \mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle \) with velocity vector

  \[
  \mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle.
  \]

- Since the length of \( L \) is the integral of the speed \( |\mathbf{r}'(t)| \),

  \[
  L = \int_{1}^{2} \left| \langle -2t, 2, \frac{1}{t} \rangle \right| \, dt
  \]
Let $C$ be the parametric curve

\[ x = 2 - t^2, \quad y = 2t - 1, \quad z = \ln t. \]

Set up, but not solve, a formula that will determine the length $L$ of $C$ for $1 \leq t \leq 2$.

Solution:

- The vector equation of $C$ is $\mathbf{r}(t) = \langle 2 - t^2, 2t - 1, \ln t \rangle$ with velocity vector $\mathbf{v}(t) = \mathbf{r}'(t) = \langle -2t, 2, \frac{1}{t} \rangle$.

- Since the length of $L$ is the integral of the speed $|\mathbf{r}'(t)|$,

\[ L = \int_{1}^{2} |\langle -2t, 2, \frac{1}{t} \rangle| \, dt = \int_{1}^{2} \sqrt{4t^2 + 4 + \frac{1}{t^2}} \, dt. \]
Find **parametric equations** for the line \( r \) which contains \( A(2, 0, 1) \) and \( B(-1, 1, -1) \).
Problem 22(a) - Fall 2006

Find **parametric equations** for the line \( r \) which contains \( A(2, 0, 1) \) and \( B(-1, 1, -1) \).

**Solution:**

- Note that \( \overrightarrow{AB} = \langle -3, 1, -2 \rangle \) and the **vector equation** is:

\[
r(t) = \vec{A} + t\overrightarrow{AB}
\]
Problem 22(a) - Fall 2006

Find parametric equations for the line \( r \) which contains \( A(2, 0, 1) \) and \( B(-1, 1, -1) \).

Solution:

Note that \( \overrightarrow{AB} = \langle -3, 1, -2 \rangle \) and the vector equation is:

\[
\mathbf{r}(t) = \mathbf{A} + t\mathbf{AB} = \langle 2, 0, 1 \rangle + t\langle -3, 1, -2 \rangle
\]
Problem 22(a) - Fall 2006

Find **parametric equations** for the line \( r \) which contains \( A(2, 0, 1) \) and \( B(-1, 1, -1). \)

**Solution:**

Note that \( \overrightarrow{AB} = \langle -3, 1, -2 \rangle \) and the **vector equation** is:

\[
\mathbf{r}(t) = \mathbf{A} + t\overrightarrow{AB} = \langle 2, 0, 1 \rangle + t\langle -3, 1, -2 \rangle = \langle 2 - 3t, t, 1 - 2t \rangle.
\]
Problem 22(a) - Fall 2006

Find **parametric equations** for the line \( r \) which contains \( A(2, 0, 1) \) and \( B(-1, 1, -1) \).

**Solution:**

- Note that \( \overrightarrow{AB} = \langle -3, 1, -2 \rangle \) and the **vector equation** is:

  \[
  r(t) = \vec{A} + t\overrightarrow{AB} = \langle 2, 0, 1 \rangle + t\langle -3, 1, -2 \rangle = \langle 2 - 3t, t, 1 - 2t \rangle.
  \]

- The **parametric equations** are:

  \[
  x = 2 - 3t \\
  y = t \\
  z = 1 - 2t.
  \]
Problem 22(b) - Fall 2006
Determine whether the lines \( L_1 : x = 1 + 2t, \ y = 3t, \ z = 2 - t \)
and \( L_2 : x = -1 + s, \ y = 4 + s, \ z = 1 + 3s \) are parallel, skew or intersecting.

Solution:
Vector part of line \( L_1 \) is \( \vec{v}_1 = \langle 2, 3, -1 \rangle \) and for line \( L_2 \) is \( \vec{v}_2 = \langle 1, 1, 3 \rangle \).
Clearly, \( \vec{v}_1 \) is not a scalar multiple of \( \vec{v}_2 \) and so these lines are not parallel.

If these lines intersect, then for some values of \( t \) and \( s \):
\[
\begin{align*}
x &= 1 + 2t = -1 + s \\
y &= 3t = 4 + s \\
z &= 2 - t = 1 + 3s
\end{align*}
\]
Solving yields: \( t = 6 \) and \( s = 14 \).
Plugging these values into \( z = 2 - t = 1 + 3s \) yields the inequality \( -4 \neq 43 \), which means the \( z \)-coordinates are never equal and the lines do not intersect.
Thus, the lines are skew.
Problem 22(b) - Fall 2006

Determine whether the lines \( L_1 : x = 1 + 2t, \ y = 3t, \ z = 2 - t \) and \( L_2 : x = -1 + s, \ y = 4 + s, \ z = 1 + 3s \) are parallel, skew or intersecting.

Solution:

- Vector part of line \( L_1 \) is \( \mathbf{v}_1 = \langle 2, 3, -1 \rangle \) and for line \( L_2 \) is \( \mathbf{v}_2 = \langle 1, 1, 3 \rangle \).
Problem 22(b) - Fall 2006

Determine whether the lines $L_1 : x = 1 + 2t, y = 3t, z = 2 - t$ and $L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$ are parallel, skew or intersecting.

Solution:

- Vector part of line $L_1$ is $v_1 = \langle 2, 3, -1 \rangle$ and for line $L_2$ is $v_2 = \langle 1, 1, 3 \rangle$. Clearly, $v_1$ is not a scalar multiple of $v_2$ and so these lines are not parallel.
Problem 22(b) - Fall 2006

Determine whether the lines $L_1 : x = 1 + 2t, y = 3t, z = 2 - t$ and $L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$ are parallel, skew or intersecting.

Solution:
- Vector part of line $L_1$ is $v_1 = \langle 2, 3, -1 \rangle$ and for line $L_2$ is $v_2 = \langle 1, 1, 3 \rangle$. Clearly, $v_1$ is not a scalar multiple of $v_2$ and so these lines are not parallel.
- If these lines intersect, then for some values of $t$ and $s$:
  
  $$x = 1 + 2t = -1 + s$$

  Solving yields: $t = 6$ and $s = 14$. Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means the $z$-coordinates are never equal and the lines do not intersect. Thus, the lines are skew.
Problem 22(b) - Fall 2006

Determine whether the lines \( L_1 : x = 1 + 2t, \ y = 3t, \ z = 2 - t \) and \( L_2 : x = -1 + s, \ y = 4 + s, \ z = 1 + 3s \) are parallel, skew or intersecting.

Solution:

- Vector part of line \( L_1 \) is \( \mathbf{v}_1 = \langle 2, 3, -1 \rangle \) and for line \( L_2 \) is \( \mathbf{v}_2 = \langle 1, 1, 3 \rangle \). Clearly, \( \mathbf{v}_1 \) is not a scalar multiple of \( \mathbf{v}_2 \) and so these lines are not parallel.
- If these lines intersect, then for some values of \( t \) and \( s \):
  \[
  x = 1 + 2t = -1 + s \quad \implies \quad 2t = -2 + s,
  \]
  which does not imply  \( t = s \).

Thus, the lines are skew.
Problem 22(b) - Fall 2006

Determine whether the lines $L_1 : x = 1 + 2t, y = 3t, z = 2 - t$ and $L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$ are parallel, skew or intersecting.

Solution:
- Vector part of line $L_1$ is $v_1 = \langle 2, 3, -1 \rangle$ and for line $L_2$ is $v_2 = \langle 1, 1, 3 \rangle$. Clearly, $v_1$ is not a scalar multiple of $v_2$ and so these lines are not parallel.
- If these lines intersect, then for some values of $t$ and $s$:
  \[
  x = 1 + 2t = -1 + s \implies 2t = -2 + s, \\
  y = 3t = 4 + s
  \]

Thus, the lines are skew.
Problem 22(b) - Fall 2006

Determine whether the lines \( L_1 : x = 1 + 2t, y = 3t, z = 2 - t \) and \( L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s \) are parallel, skew or intersecting.

Solution:

- Vector part of line \( L_1 \) is \( \mathbf{v}_1 = \langle 2, 3, -1 \rangle \) and for line \( L_2 \) is \( \mathbf{v}_2 = \langle 1, 1, 3 \rangle \). Clearly, \( \mathbf{v}_1 \) is not a scalar multiple of \( \mathbf{v}_2 \) and so these lines are not parallel.
- If these lines intersect, then for some values of \( t \) and \( s \):
  \[
  x = 1 + 2t = -1 + s \quad \implies \quad 2t = -2 + s,
  \]
  \[
  y = 3t = 4 + s \quad \implies \quad 3t = 4 + s.
  \]
Problem 22(b) - Fall 2006

Determine whether the lines $L_1 : x = 1 + 2t, y = 3t, z = 2 - t$ and $L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$ are parallel, skew or intersecting.

Solution:

- Vector part of line $L_1$ is $v_1 = \langle 2, 3, -1 \rangle$ and for line $L_2$ is $v_2 = \langle 1, 1, 3 \rangle$. Clearly, $v_1$ is not a scalar multiple of $v_2$ and so these lines are not parallel.

- If these lines intersect, then for some values of $t$ and $s$:
  \[
  x = 1 + 2t = -1 + s \quad \Rightarrow \quad 2t = -2 + s, \\
  y = 3t = 4 + s \quad \Rightarrow \quad 3t = 4 + s.
  \]

  Solving yields: $t = 6$ and $s = 14$. Thus, the lines are skew.
Problem 22(b) - Fall 2006
Determine whether the lines \( L_1 : x = 1 + 2t, \ y = 3t, \ z = 2 - t \) and \( L_2 : x = -1 + s, \ y = 4 + s, \ z = 1 + 3s \) are parallel, skew or intersecting.

Solution:
- Vector part of line \( L_1 \) is \( \mathbf{v}_1 = \langle 2, 3, -1 \rangle \) and for line \( L_2 \) is \( \mathbf{v}_2 = \langle 1, 1, 3 \rangle \). Clearly, \( \mathbf{v}_1 \) is not a scalar multiple of \( \mathbf{v}_2 \) and so these lines are not parallel.
- If these lines intersect, then for some values of \( t \) and \( s \):
  \[
  x = 1 + 2t = -1 + s \quad \Rightarrow \quad 2t = -2 + s, \\
  y = 3t = 4 + s \quad \Rightarrow \quad 3t = 4 + s.
  \]
  Solving yields: \( t = 6 \) and \( s = 14 \).
  
  Plugging these values into \( z = 2 - t = 1 + 3s \) yields the inequality \(-4 \neq 43\), which means the \( z \)-coordinates are never equal and the lines do not intersect.
Determine whether the lines $L_1 : x = 1 + 2t, y = 3t, z = 2 - t$ and $L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$ are parallel, skew or intersecting.

Solution:

- Vector part of line $L_1$ is $v_1 = \langle 2, 3, -1 \rangle$ and for line $L_2$ is $v_2 = \langle 1, 1, 3 \rangle$. Clearly, $v_1$ is not a scalar multiple of $v_2$ and so these lines are not parallel.
- If these lines intersect, then for some values of $t$ and $s$:
  $$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$
  $$y = 3t = 4 + s \implies 3t = 4 + s.$$

  Solving yields: $t = 6$ and $s = 14$.

  Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means the $z$-coordinates are never equal and the lines do not intersect.

  Thus, the lines are skew.
Problem 23(a) - Fall 2006

Find an equation of the plane which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.
Problem 23(a) - Fall 2006

Find an equation of the plane which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.
Problem 23(a) - Fall 2006

Find an equation of the plane which contains the points \( P(-1, 2, 1), \ Q(1, -2, 1) \) and \( R(1, 1, -1) \).

Solution:

- Consider the vectors \( \vec{PQ} = \langle 2, -4, 0 \rangle \) and \( \vec{PR} = \langle 2, -1, -2 \rangle \) which are parallel to the plane.
- The normal vector to the plane is:

\[
\mathbf{n} = \vec{PQ} \times \vec{PR}
\]
Find an **equation of the plane** which contains the points  
\( P(-1, 2, 1) \), \( Q(1, -2, 1) \) and \( R(1, 1, -1) \).

**Solution:**

- Consider the vectors \( \overrightarrow{PQ} = \langle 2, -4, 0 \rangle \) and \( \overrightarrow{PR} = \langle 2, -1, -2 \rangle \) which are parallel to the plane.
- The normal vector to the plane is:

\[
\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}
\]

Since \( P(-1, 2, 1) \) lies on the plane, the equation of the plane is:

\[
\langle 8, 4, 6 \rangle \cdot \langle x+1, y-2, z-1 \rangle = 8(x+1) + 4(y-2) + 6(z-1) = 0
\]
Problem 23(a) - Fall 2006

Find an equation of the plane which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\vec{PQ} = \langle 2, -4, 0 \rangle$ and $\vec{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$ n = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}. $$
Find an equation of the plane which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.

- The normal vector to the plane is:

$$n = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$ 

- Since $P(-1, 2, 1)$ lies on the plane, the equation of the plane is:
Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points
$P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.

- The normal vector to the plane is:

$$
n = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8i + 4j + 6k.
$$

- Since $P(-1, 2, 1)$ lies on the plane, the **equation of the plane** is:

$$
\langle 8, 4, 6 \rangle \cdot \langle x+1, y-2, z-1 \rangle = 8(x+1) + 4(y-2) + 6(z-1) = 0.
$$
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$. 
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $\mathbf{(1, 2, -1)}$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane.
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. 

The distance $D$ from $(1, 2, -1)$ to the plane is equal to:

$$D = \left| \frac{\mathbf{b} \cdot \mathbf{n}}{\|\mathbf{n}\|} \right| = \frac{|\langle 1, 1, -1 \rangle \cdot \langle 2, 1, -2 \rangle|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{5}{\sqrt{9}} = \frac{5}{3}.$$
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance $D$ from $(1, 2, -1)$ to the plane is equal to:
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance $D$ from $(1, 2, -1)$ to the plane is equal to:

$$|\text{comp}_n \mathbf{b}| =$$
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 2, -1)$ which is $\mathbf{b} = \langle 1, 1, -1 \rangle$. The distance $D$ from $(1, 2, -1)$ to the plane is equal to:

$$|\text{comp}_n \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $n = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 2, -1)$ which is $b = \langle 1, 1, -1 \rangle$. The distance $D$ from $(1, 2, -1)$ to the plane is equal to:

$$|\text{comp}_n b| = \left| b \cdot \frac{n}{|n|} \right| = |\langle 1, 1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle|$$
Problem 23(b) - Fall 2006

Find the distance $D$ from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.

Solution:

The normal to the plane is $n = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from $P_0$ to $P_1 = (1, 2, -1)$ which is $b = \langle 1, 1, -1 \rangle$. The distance $D$ from $(1, 2, -1)$ to the plane is equal to:

$$|\text{comp}_n b| = \left| b \cdot \frac{n}{|n|} \right| = |\langle 1, 1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle| = \frac{5}{3}.$$
Problem 24(a) - Fall 2006

Let two space curves
\[ \mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \]
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \((1, 0, 1)\).
Problem 24(a) - Fall 2006

Let two space curves
\[ r_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \]
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \((1, 0, 1)\).

Solution:

- When \( r_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
Let two space curves
\( \mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \)
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \((1, 0, 1)\).

Solution:
- When \( \mathbf{r}_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
- When \( \mathbf{r}_2(s) = \langle 1, 0, 1 \rangle \), then \( s = 1 \).
Problem 24(a) - Fall 2006

Let two space curves 
\( r_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \)
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \((1, 0, 1)\).

Solution:

- When \( r_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
- When \( r_2(s) = \langle 1, 0, 1 \rangle \), then \( s = 1 \).
- Calculating derivatives, we obtain:
  
  \[
  r'_1(t) = \langle -\sin(t - 1), 2t, 4t^3 \rangle \\
  r'_1(1) = \langle 0, 2, 4 \rangle \\
  r'_2(s) = \langle \frac{1}{s}, 2s - 2, 2s \rangle \\
  r'_2(1) = \langle 1, 0, 2 \rangle.
  \]
Let two space curves
\[ r_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \]
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \( (1, 0, 1) \).

Solution:
- When \( r_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
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  \[ r'_1(1) = \langle 0, 2, 4 \rangle \]
  \[ r'_2(s) = \langle \frac{1}{s}, 2s - 2, 2s \rangle \]
  \[ r'_2(1) = \langle 1, 0, 2 \rangle. \]
- Hence,
  \[ \cos \theta = \frac{r'_1(1) \cdot r'_2(1)}{|r'_1(1)||r'_2(1)|} \]
Problem 24(a) - Fall 2006

Let two space curves
\[ r_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \]
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \((1, 0, 1)\).

Solution:

- When \( r_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
- When \( r_2(s) = \langle 1, 0, 1 \rangle \), then \( s = 1 \).
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  \[ r'_1(t) = \langle -\sin(t - 1), 2t, 4t^3 \rangle \]
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  \[ r'_2(s) = \langle \frac{1}{s}, 2s - 2, 2s \rangle \]
  \[ r'_2(1) = \langle 1, 0, 2 \rangle. \]
- Hence,
  \[ \cos \theta = \frac{r'_1(1) \cdot r'_2(1)}{|r'_1(1)||r'_2(1)|} = \frac{\langle 0, 2, 4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{20} \sqrt{5}} = \frac{8}{10} = \frac{4}{5}. \]
Problem 24(a) - Fall 2006

Let two space curves
\[ r_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \]
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \( (1, 0, 1) \).

Solution:

- When \( r_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
- When \( r_2(s) = \langle 1, 0, 1 \rangle \), then \( s = 1 \).
- Calculating derivatives, we obtain:
  \[ r'_1(t) = \langle -\sin(t - 1), 2t, 4t^3 \rangle \]
  \[ r'_1(1) = \langle 0, 2, 4 \rangle \]
  \[ r'_2(s) = \langle \frac{1}{s}, 2s - 2, 2s \rangle \]
  \[ r'_2(1) = \langle 1, 0, 2 \rangle. \]
- Hence,
  \[ \cos \theta = \frac{r'_1(1) \cdot r'_2(1)}{|r'_1(1)||r'_2(1)|} = \frac{\langle 0, 2, 4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{20} \sqrt{5}} \]
  \[ = \frac{1}{\sqrt{100}} (0 \cdot 1 + 2 \cdot 0 + 4 \cdot 2) \]
Let two space curves
\( r_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad r_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \)
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \((1, 0, 1)\).

Solution:
- When \( r_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
- When \( r_2(s) = \langle 1, 0, 1 \rangle \), then \( s = 1 \).
- Calculating derivatives, we obtain:
  \[ r_1'(t) = \langle -\sin(t - 1), 2t, 4t^3 \rangle \]
  \[ r_1'(1) = \langle 0, 2, 4 \rangle \]
  \[ r_2'(s) = \langle \frac{1}{s}, 2s - 2, 2s \rangle \]
  \[ r_2'(1) = \langle 1, 0, 2 \rangle \]
- Hence,
  \[
  \cos \theta = \frac{r_1'(1) \cdot r_2'(1)}{|r_1'(1)||r_2'(1)|} = \frac{\langle 0, 2, 4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{20}\sqrt{5}}
  \]
  \[
  = \frac{1}{\sqrt{100}}(0 \cdot 1 + 2 \cdot 0 + 4 \cdot 2) = \frac{8}{10}
  \]
Let two space curves
\[ \mathbf{r}_1(t) = \langle \cos(t - 1), t^2 - 1, t^4 \rangle, \quad \mathbf{r}_2(s) = \langle 1 + \ln s, s^2 - 2s + 1, s^2 \rangle, \]
be given where \( t \) and \( s \) are two independent real parameters. Find the cosine of the angle between the tangent vectors of the two curves at the intersection point \( (1, 0, 1) \).

**Solution:**
- When \( \mathbf{r}_1(t) = \langle 1, 0, 1 \rangle \), then \( t = 1 \).
- When \( \mathbf{r}_2(s) = \langle 1, 0, 1 \rangle \), then \( s = 1 \).
- Calculating derivatives, we obtain:
  \[ \mathbf{r}_1'(t) = \langle -\sin(t - 1), 2t, 4t^3 \rangle \]
  \[ \mathbf{r}_1'(1) = \langle 0, 2, 4 \rangle \]
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  \[ \mathbf{r}_2'(1) = \langle 1, 0, 2 \rangle. \]
- Hence,
  \[ \cos \theta = \frac{\mathbf{r}_1'(1) \cdot \mathbf{r}_2'(1)}{|\mathbf{r}_1'(1)||\mathbf{r}_2'(1)|} = \frac{\langle 0, 2, 4 \rangle \cdot \langle 1, 0, 2 \rangle}{\sqrt{20} \sqrt{5}} \]
  \[ = \frac{1}{\sqrt{100}} (0 \cdot 1 + 2 \cdot 0 + 4 \cdot 2) = \frac{8}{10} = \frac{4}{5}. \]
Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):
  \[ \mathbf{r}(t) = \int \mathbf{v}(t) \, dt \]
Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):

  \[
  \mathbf{r}(t) = \int \mathbf{v}(t) \, dt
  = \int \langle \sin t, \cos 2t, e^t \rangle \, dt
  = \langle -\cos(t) + x_0, \frac{1}{2} \sin(2t) + y_0, e^t + z_0 \rangle.
  \]
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):
  \[
  \mathbf{r}(t) = \int \mathbf{v}(t) \, dt
  \]
  \[
  = \int \langle \sin t, \cos 2t, e^t \rangle \, dt = \langle -\cos(t) + x_0, \frac{1}{2} \sin(2t) + y_0, e^t + z_0 \rangle.
  \]
- Now use the initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \) to solve for \( x_0, y_0, z_0 \).
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):

  \[
  \mathbf{r}(t) = \int \mathbf{v}(t) \, dt
  = \int \langle \sin t, \cos 2t, e^t \rangle \, dt = \langle -\cos(t) + x_0, \frac{1}{2} \sin(2t) + y_0, e^t + z_0 \rangle.
  \]

- Now use the initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \) to solve for \( x_0, y_0, z_0 \).

  \[
  -\cos(0) + x_0 = -1 + x_0 = 1 \implies x_0 = 2.
  \]
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity

\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]

and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):

  \[ \mathbf{r}(t) = \int \mathbf{v}(t) \, dt \]

  \[ = \int \langle \sin t, \cos 2t, e^t \rangle \, dt = \langle -\cos(t) + x_0, \frac{1}{2} \sin(2t) + y_0, e^t + z_0 \rangle. \]

- Now use the initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \) to solve for \( x_0, y_0, z_0 \).

  \[ -\cos(0) + x_0 = -1 + x_0 = 1 \implies x_0 = 2. \]

  \[ \frac{1}{2} \sin(0) + y_0 = 0 + y_0 = 2 \implies y_0 = 2. \]
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
\[ v(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( r(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( r(t) \).

Solution:

- The **position vector function** \( r(t) \) is the integral of its derivative \( r'(t) = v(t) \):
  \[
  r(t) = \int v(t) \, dt = \int \langle \sin t, \cos 2t, e^t \rangle \, dt = \langle -\cos(t) + x_0, \frac{1}{2} \sin(2t) + y_0, e^t + z_0 \rangle.
  \]

- Now use the initial position \( r(0) = \langle 1, 2, 0 \rangle \) to solve for \( x_0, y_0, z_0 \).
  
  \[-\cos(0) + x_0 = -1 + x_0 = 1 \implies x_0 = 2.\]

  \[
  \frac{1}{2} \sin(0) + y_0 = 0 + y_0 = 2 \implies y_0 = 2.
  \]

  
  \[e^0 + z_0 = 1 + z_0 = 0 \implies z_0 = -1.\]
Problem 24(b) - Fall 2006

Suppose a particle moving in space has velocity
\[ \mathbf{v}(t) = \langle \sin t, \cos 2t, e^t \rangle \]
and initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \). Find the position vector function \( \mathbf{r}(t) \).

Solution:

- The **position vector function** \( \mathbf{r}(t) \) is the integral of its derivative \( \mathbf{r}'(t) = \mathbf{v}(t) \):

  \[
  \mathbf{r}(t) = \int \mathbf{v}(t) \, dt
  = \int \langle \sin t, \cos 2t, e^t \rangle \, dt
  = \langle - \cos(t) + x_0, \frac{1}{2} \sin(2t) + y_0, e^t + z_0 \rangle.
  \]

- Now use the initial position \( \mathbf{r}(0) = \langle 1, 2, 0 \rangle \) to solve for \( x_0, y_0, z_0 \).

  \[ - \cos(0) + x_0 = -1 + x_0 = 1 \implies x_0 = 2. \]

  \[ \frac{1}{2} \sin(0) + y_0 = 0 + y_0 = 2 \implies y_0 = 2. \]

  \[ e^0 + z_0 = 1 + z_0 = 0 \implies z_0 = -1. \]

- Hence,

  \[ \mathbf{r}(t) = \langle - \cos(t) + 2, \frac{1}{2} \sin(2t) + 2, e^t - 1 \rangle. \]
Problem 25(a) - Fall 2006

Let \( f(x, y) = e^{x^2 - y} + x\sqrt{4 - y^2} \). Find partial derivatives \( f_x, f_y \) and \( f_{xy} \).

Problem 25(b) - Fall 2006

Find an equation for the tangent plane of the graph of

\[ f(x, y) = \sin(2x + y) + 1 \]

at the point (0, 0, 1).

Problem 26(a) - Fall 2006

Let \( g(x, y) = ye^x \). Estimate \( g(0.1, 1.9) \) using the linear approximation of \( g(x, y) \) at \( (x, y) = (0, 2) \).

Solutions to these problems:

These types of problems will not be on this exam.
Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$. 

Solution:

Complete the square in order to put the equation in the form:

\[(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2.\]

We get:

\[x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16,\]

This gives the equation

\[\left(x - 0\right)^2 + \left(y - 0\right)^2 + \left(z + 3\right)^2 = 25 = 5^2.\]

Hence, the **center** is $C = (0, 0, -3)$ and the **radius** is $r = 5$. 

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere \( x^2 + y^2 + z^2 + 6z = 16 \).

**Solution:**

- Complete the square in order to put the equation in the form:

  \[
  (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2.
  \]
Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

**Solution:**

- Complete the square in order to put the equation in the form:
  $$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2.$$

- We get:
  $$x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.$$
Find the **center** and **radius** of the sphere \( x^2 + y^2 + z^2 + 6z = 16 \).

**Solution:**

- Complete the square in order to put the equation in the form:
  \[
  (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2.
  \]

- We get:
  \[
  x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.
  \]

- This gives the equation
  \[
  (x - 0)^2 + (y - 0)^2 + (z + 3)^2 = 25 = 5^2.
  \]
Find the center and radius of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Solution:

- Complete the square in order to put the equation in the form:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2.$$ 

- We get:

$$x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.$$ 

- This gives the equation

$$(x - 0)^2 + (y - 0)^2 + (z + 3)^2 = 25 = 5^2.$$ 

Hence, the center is $C = (0, 0, -3)$ and the radius is $r = 5$. 


Problem 26(c) - Fall 2006

Let \( f(x, y) = \sqrt{16 - x^2 - y^2} \). Draw a contour map of level curves \( f(x, y) = k \) with \( k = 1, 2, 3 \). Label the level curves by the corresponding values of \( k \).

Solution:

A problem of this type will not be on this exam.
Problem 27

Consider the line $L$ through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line $L$ and the plane given by $2x - 3y + 4z = 13$. 

Solution:

The vector part of $L$ is $\vec{AB} = \langle 3, 2, -1 \rangle$ and the point $A$ is on the line.

The vector equation of $L$ is:

$$L = \vec{A} + t \vec{AB} = \langle 2, 1, -1 \rangle + t \langle 3, 2, -1 \rangle = \langle 2 + 3t, 1 + 2t, -1 - t \rangle.$$ 

Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the equation of the plane gives:

$$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13 = \Rightarrow -4t = 16 = \Rightarrow t = -4.$$ 

So, the point of intersection is:

$$L(-4) = \langle 2 - 12, 1 - 8, -1 + 4 \rangle = \langle -10, -7, 3 \rangle.$$
Consider the line $L$ through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line $L$ and the plane given by $2x - 3y + 4z = 13$.

**Solution:**

- The vector part of $L$ is $\vec{AB} = \langle 3, 2, -1 \rangle$ and the point $A$ is on the line.
Consider the line \( \mathbf{L} \) through points \( A = (2, 1, -1) \) and \( B = (5, 3, -2) \). Find the intersection of the line \( \mathbf{L} \) and the plane given by \( 2x - 3y + 4z = 13 \).

**Solution:**

- The vector part of \( \mathbf{L} \) is \( \overrightarrow{AB} = \langle 3, 2, -1 \rangle \) and the point \( A \) is on the line.
- The **vector equation** of \( \mathbf{L} \) is:
  
  \[ \mathbf{L} = \overrightarrow{\mathbf{A}} + t\overrightarrow{AB} \]
Problem 27
Consider the line $L$ through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the intersection of the line $L$ and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of $L$ is $\vec{AB} = \langle 3, 2, -1 \rangle$ and the point $A$ is on the line.
- The vector equation of $L$ is:
  
  $$L = \vec{A} + t\vec{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle$$
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Consider the line \( L \) through points \( A = (2, 1, -1) \) and \( B = (5, 3, -2) \). Find the **intersection** of the line \( L \) and the plane given by \( 2x - 3y + 4z = 13 \).

Solution:

- The vector part of \( L \) is \( \vec{AB} = \langle 3, 2, -1 \rangle \) and the point \( A \) is on the line.
- The **vector equation** of \( L \) is:
  \[
  L = \vec{A} + t\vec{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2 + 3t, 1 + 2t, -1 - t \rangle.
  \]
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Consider the line \( L \) through points \( A = (2, 1, -1) \) and \( B = (5, 3, -2) \). Find the intersection of the line \( L \) and the plane given by \( 2x - 3y + 4z = 13 \).

Solution:

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  \]
- Plugging \( x = 2 + 3t, y = 1 + 2t \) and \( z = -1 - t \) into the equation of the plane gives:
Problem 27
Consider the line $L$ through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the intersection of the line $L$ and the plane given by $2x - 3y + 4z = 13$.

Solution:

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  $$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13.$$
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  $$L = \vec{A} + t\vec{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$
- Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the equation of the plane gives:
  $$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$
  $$\implies -4t = 16$$
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Consider the line \( L \) through points \( A = (2, 1, -1) \) and \( B = (5, 3, -2) \). Find the **intersection** of the line \( L \) and the plane given by \( 2x - 3y + 4z = 13 \).

**Solution:**

- The vector part of \( L \) is \( \overrightarrow{AB} = \langle 3, 2, -1 \rangle \) and the point \( A \) is on the line.
- The **vector equation** of \( L \) is:
  \[
  L = \vec{A} + t\overrightarrow{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2 + 3t, 1 + 2t, -1 - t \rangle.
  \]
- Plugging \( x = 2 + 3t, \ y = 1 + 2t \) and \( z = -1 - t \) into the **equation of the plane** gives:
  \[
  2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13
  \]
  \[
  \implies -4t = 16 \implies t = -4.
  \]
Problem 27

Consider the line $L$ through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the intersection of the line $L$ and the plane given by $2x - 3y + 4z = 13$.

Solution:

• The vector part of $L$ is $\vec{AB} = \langle 3, 2, -1 \rangle$ and the point $A$ is on the line.

• The vector equation of $L$ is:

$$L = \vec{A} + t\vec{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2+3t, 1+2t, -1-t \rangle.$$ 

• Plugging $x = 2 + 3t$, $y = 1 + 2t$ and $z = -1 - t$ into the equation of the plane gives:

$$2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13$$

$$\implies -4t = 16 \implies t = -4.$$ 

• So, the point of intersection is:

$$L(-4) = \langle 2 - 12, 1 - 8, -1 - (-4) \rangle$$
Problem 27
Consider the line \( L \) through points \( A = (2, 1, -1) \) and \( B = (5, 3, -2) \). Find the intersection of the line \( L \) and the plane given by \( 2x - 3y + 4z = 13 \).

Solution:

- The vector part of \( L \) is \( \vec{AB} = \langle 3, 2, -1 \rangle \) and the point \( A \) is on the line.
- The vector equation of \( L \) is:
  \[
  \vec{L} = \vec{A} + t\vec{AB} = \langle 2, 1, -1 \rangle + t\langle 3, 2, -1 \rangle = \langle 2 + 3t, 1 + 2t, -1 - t \rangle.
  \]
- Plugging \( x = 2 + 3t, y = 1 + 2t \) and \( z = -1 - t \) into the equation of the plane gives:
  \[
  2(2 + 3t) - 3(1 + 2t) + 4(-1 - t) = -4t - 3 = 13
  \]
  \[
  \implies -4t = 16 \implies t = -4.
  \]
- So, the point of intersection is:
  \[
  \vec{L}(-4) = \langle 2 - 12, 1 - 8, -1 - (-4) \rangle = \langle -10, -7, 3 \rangle.
  \]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ \mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.
Problem 28(a)
Two masses travel through space along space curve described by the two vector functions

\[ \mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.
Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

Solution:
- Equate the \( x \) and \( z \)-coordinates:
\[
x = t = 3 - s
\]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ \mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

Solution:

- Equate the \( x \) and \( z \)-coordinates:

  \[ x = t = 3 - s \]

  \[ z = 3 + t^2 = 3 + (3 - s)^2 \]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ \mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters. Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

Solution:

- Equate the \( x \) and \( z \)-coordinates:

\[ x = t = 3 - s \]

\[ z = 3 + t^2 = 3 + (3 - s)^2 = 3 + 9 - 6s + s^2 \]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters. Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

Solution:

- Equate the \( x \) and \( z \)-coordinates:

  \[ x = t = 3 - s \]

  \[ z = 3 + t^2 = 3 + (3 - s)^2 = 3 + 9 - 6s + s^2 = s^2 \]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters. Show that the two space curves \textbf{intersect} by finding the point of intersection and the parameter values where this occurs.

Solution:

1. Equate the \( x \) and \( z \)-coordinates:
   
   \[ x = t = 3 - s \]
   
   \[ z = 3 + t^2 = 3 + (3 - s)^2 = 3 + 9 - 6s + s^2 = s^2 \]

2. Thus, the \textbf{parameter values} are:
   
   \[ 12 - 6s = 0 \]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ \mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

Solution:

- Equate the \( x \) and \( z \)-coordinates:

\[ x = t = 3 - s \]

\[ z = 3 + t^2 = 3 + (3 - s)^2 = 3 + 9 - 6s + s^2 = s^2 \]

- Thus, the parameter values are:

\[ 12 - 6s = 0 \iff (s = 2 \text{ and } t = 1). \]
Problem 28(a)

Two masses travel through space along space curve described by the two vector functions

\[ \mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Show that the two space curves intersect by finding the point of intersection and the parameter values where this occurs.

Solution:

- Equate the \( x \) and \( z \)-coordinates:

  \[ x = t = 3 - s \]

  \[ z = 3 + t^2 = 3 + (3 - s)^2 = 3 + 9 - 6s + s^2 = s^2 \]

- Thus, the parameter values are:

  \[ 12 - 6s = 0 \implies (s = 2 \text{ and } t = 1). \]

- So, \( \mathbf{r}_1(1) = \langle 1, 0, 4 \rangle = \mathbf{r}_2(2) \) is the desired intersection point.
Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

\[\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad \mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle\]

where \( t \) and \( s \) are two independent real parameters.
Find parametric equation for the tangent line to the space curve \( \mathbf{r}_1(t) \) at the intersection point. (Use the value \( t = 1 \) in part (a)).
Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Find **parametric equation** for the tangent line to the space curve \( r_1(t) \) at the intersection point. (Use the value \( t = 1 \) in part (a)).

Solution:

- The velocity vector of \( r_1(t) \) at the intersection point is \( r_1'(1) \).
Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Find **parametric equation** for the tangent line to the space curve \( r_1(t) \) at the intersection point. (Use the value \( t = 1 \) in part (a)).

**Solution:**

- The velocity vector of \( r_1(t) \) at the intersection point is \( r'_1(1) \).
- Since

\[
 r'_1(t) = \langle 1, -1, 2t \rangle,
\]

\[
 r'_1(1) = \langle 1, -1, 2 \rangle.
\]
Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Find **parametric equation** for the tangent line to the space curve \( r_1(t) \) at the intersection point. (Use the value \( t = 1 \) in part (a)).

Solution:

- The velocity vector of \( r_1(t) \) at the intersection point is \( r'_1(1) \).
- Since

  \[ r'_1(t) = \langle 1, -1, 2t \rangle, \]

  \[ r'_1(1) = \langle 1, -1, 2 \rangle. \]

- The **vector equation** of the tangent line is:

  \[ T(t) = r_1(1) + t\langle 1, -1, 2 \rangle = \langle 1, 0, 4 \rangle + t\langle 1, -1, 2 \rangle \]
Problem 28(b)

Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters.

Find \textbf{parametric equation} for the tangent line to the space curve \( r_1(t) \) at the intersection point. (Use the value \( t = 1 \) in part \( (a) \)).

Solution:

- The velocity vector of \( r_1(t) \) at the intersection point is \( r_1'(1) \).

Since

\[ r_1'(t) = \langle 1, -1, 2t \rangle, \]

\[ r_1'(1) = \langle 1, -1, 2 \rangle. \]

- The \textbf{vector equation} of the tangent line is:

\[ T(t) = r_1(1) + t\langle 1, -1, 2 \rangle = \langle 1, 0, 4 \rangle + t\langle 1, -1, 2 \rangle = \langle 1 + t, -t, 4 + 2t \rangle. \]
Two masses travel through space along space curve described by the two vector functions

\[ r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle, \quad r_2(s) = \langle 3 - s, s - 2, s^2 \rangle \]

where \( t \) and \( s \) are two independent real parameters. Find **parametric equation** for the tangent line to the space curve \( r_1(t) \) at the intersection point. (Use the value \( t = 1 \) in part (a)).

**Solution:**

- The velocity vector of \( r_1(t) \) at the intersection point is \( r_1'(1) \).
- Since \( r_1'(t) = \langle 1, -1, 2t \rangle \),
  \[ r_1'(1) = \langle 1, -1, 2 \rangle. \]

- The **vector equation** of the tangent line is:
  \[ T(t) = r_1(1) + t\langle 1, -1, 2 \rangle = \langle 1, 0, 4 \rangle + t\langle 1, -1, 2 \rangle = \langle 1 + t, -t, 4 + 2t \rangle. \]

- The **parametric equations** are:
  \[ \begin{align*}
x &= 1 + t \\
y &= -t \\
z &= 4 + 2t
\end{align*} \]
Problem 29
Consider the parallelogram with vertices $A, B, C, D$ such that $B$ and $C$ are adjacent to $A$. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point $C$.

Solution:
After drawing a picture, the point $C$ is easily seen to be: $\vec{OA} + \vec{BD} = \langle 2, 5, 1 \rangle + \langle 2, 1, -7 \rangle = \langle 4, 6, -6 \rangle$, where $O$ is the origin.
Problem 29
Consider the parallelogram with vertices $A, B, C, D$ such that $B$ and $C$ are adjacent to $A$. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point $C$.

Solution:
After drawing a picture, the point $C$ is easily seen to be:

$$\overrightarrow{OA} + \overrightarrow{BD}$$
Problem 29

Consider the parallelogram with vertices $A, B, C, D$ such that $B$ and $C$ are adjacent to $A$. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, −3)$, find the point $C$.

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After drawing a picture, the point $C$ is easily seen to be:

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Problem 29
Consider the parallelogram with vertices \( A, B, C, D \) such that \( B \) and \( C \) are adjacent to \( A \). If \( A = (2, 5, 1), B = (3, 1, 4), D = (5, 2, -3) \), find the point \( C \).

Solution:
After drawing a picture, the point \( C \) is easily seen to be:

\[
\overrightarrow{OA} + \overrightarrow{BD} = \langle 2, 5, 1 \rangle + \langle 2, 1, -7 \rangle = \langle 4, 6, -6 \rangle,
\]
where \( O \) is the origin.
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** \( \text{proj}_{\vec{AB}}(\vec{AC}) \) of the vector $\vec{AC}$ onto the vector $\vec{AB}$. 
Problem 30(a)
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the \textbf{orthogonal projection} $\text{proj}_{\vec{AB}}(\vec{AC})$ of the vector $\vec{AC}$ onto the vector $\vec{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \vec{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \vec{AC} = \langle -2, 1, 1 \rangle$ into the formula:
Problem 30(a)
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the orthogonal projection $\text{proj}_{\vec{AB}}(\vec{AC})$ of the vector $\vec{AC}$ onto the vector $\vec{AB}$.

Solution:
We just plug in the vectors $\mathbf{a} = \vec{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \vec{AC} = \langle -2, 1, 1 \rangle$ into the formula:
\[
\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \mathbf{a}.
\]
Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the orthogonal projection $\text{proj}_{\overrightarrow{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1 \rangle$ into the formula:

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \mathbf{a}.$$  

- Plugging in, we get:

$$\text{proj}_{\overrightarrow{AB}}(\overrightarrow{AC})$$
Problem 30(a)

Consider the points \( A = (2, 1, 0), \ B = (1, 0, 2) \) and \( C = (0, 2, 1) \).
Find the **orthogonal projection** \( \text{proj}_{\vec{AB}}(\vec{AC}) \) of the vector \( \vec{AC} \) onto the vector \( \vec{AB} \).

Solution:

- We just plug in the vectors \( \mathbf{a} = \vec{AB} = \langle -1, -1, 2 \rangle \) and \( \mathbf{b} = \vec{AC} = \langle -2, 1, 1 \rangle \) into the formula:

\[
\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \mathbf{a}.
\]

- Plugging in, we get:

\[
\text{proj}_{\vec{AB}}(\vec{AC}) = \frac{\langle -1, -1, 2 \rangle \cdot \langle -2, 1, 1 \rangle}{\langle -1, -1, 2 \rangle \cdot \langle -1, -1, 2 \rangle} \langle -1, -1, 2 \rangle
\]
Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the orthogonal projection $\text{proj}_{\overrightarrow{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

Solution:

- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1 \rangle$ into the formula:

  \[
  \text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \mathbf{a}.
  \]

- Plugging in, we get:

  \[
  \text{proj}_{\overrightarrow{AB}}(\overrightarrow{AC}) = \frac{\langle -1, -1, 2 \rangle \cdot \langle -2, 1, 1 \rangle}{\langle -1, -1, 2 \rangle \cdot \langle -1, -1, 2 \rangle} \langle -1, -1, 2 \rangle = \frac{1}{2} \langle -1, -1, 2 \rangle.
  \]
Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle $ABC$. 

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\triangle ABC$ can be found by taking the area of the parallelogram spanned by $\vec{AB}$ and $\vec{AC}$ and dividing by 2. Thus:

$$\text{Area} \left( \triangle ABC \right) = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right|$$

$$\vec{AB} = (1 - 2, 0 - 1, 2 - 0) = (-1, -1, 2)$$

$$\vec{AC} = (0 - 2, 2 - 1, 1 - 0) = (-2, 1, 1)$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-1 & -1 & 2 \\
-2 & 1 & 1
\end{vmatrix} = (-1 - 2)\mathbf{i} - (-2 - 4)\mathbf{j} + (1 + 2)\mathbf{k} = (-3, 6, 3)$$

$$\left| \vec{AB} \times \vec{AC} \right| = \sqrt{(-3)^2 + 6^2 + 3^2} = \sqrt{9 + 36 + 9} = \sqrt{54}$$

$$\text{Area} \left( \triangle ABC \right) = \frac{1}{2} \times \sqrt{54} = \frac{1}{2} \times 3\sqrt{6} = \frac{3\sqrt{6}}{2}$$
Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle $ABC$.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Thus:

$$\text{Area}(\triangle) = \frac{1}{2} |\langle -3, -3, -3 \rangle| = \frac{1}{2} \sqrt{27}.$$
Problem 30(b)
Consider the points \( A = (2, 1, 0), \ B = (1, 0, 2) \) and \( C = (0, 2, 1) \). Find the area of triangle \( ABC \).

Solution:
Consider the points \( A = (2, 1, 0), \ B = (1, 0, 2) \) and \( C = (0, 2, 1) \). Then the area of the triangle \( \triangle \) with these vertices can be found by taking the area of the parallelogram spanned by \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) and dividing by 2.
Problem 30(b)
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle $ABC$.

Solution:
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2}$$
Problem 30(b)
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle $ABC$.

Solution:
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left| \begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
-1 & -1 & 2 \\
-2 & 1 & 1 
\end{array} \right|$$
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle $ABC$.

**Solution:**

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \\ \end{vmatrix} \right|$$

$$= \frac{1}{2} |\langle -3, -3, -3 \rangle|$$
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle $ABC$.

**Solution:**

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\Delta$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \left| \begin{array}{ccc} i & j & k \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{array} \right|$$

$$= \frac{1}{2} |\langle -3, -3, -3 \rangle| = \frac{1}{2} \sqrt{9 + 9 + 9}$$
Problem 30(b)

Consider the points \( A = (2, 1, 0), \ B = (1, 0, 2) \) and \( C = (0, 2, 1) \). Find the area of triangle \( ABC \).

Solution:

Consider the points \( A = (2, 1, 0), \ B = (1, 0, 2) \) and \( C = (0, 2, 1) \). Then the area of the triangle \( \Delta \) with these vertices can be found by taking the area of the parallelogram spanned by \( \overrightarrow{AB} \) and \( \overrightarrow{AC} \) and dividing by 2. Thus:

\[
\text{Area}(\Delta) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} |\langle -3, -3, -3 \rangle| = \frac{1}{2} \sqrt{9 + 9 + 9} = \frac{1}{2} \sqrt{27}.
\]
Problem 30(c)

Consider the points \( A = (2, 1, 0) \), \( B = (1, 0, 2) \) and \( C = (0, 2, 1) \). Find the distance \( d \) from the point \( C \) to the line \( L \) that contains points \( A \) and \( B \).
Problem 30(c)
Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance $d$ from the point $C$ to the line $L$ that contains points $A$ and $B$.

Solution:
- From the figure drawn on the blackboard, we see that the distance $d$ from $C$ to $L$ is the absolute value of the scalar projection of $\vec{AC}$ in the direction $\vec{v} = \vec{AC} - \text{proj}_{\vec{AB}} \vec{AC}$.
Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance $d$ from the point $C$ to the line $L$ that contains points $A$ and $B$.

Solution:

- From the figure drawn on the blackboard, we see that the distance $d$ from $C$ to $L$ is the absolute value of the scalar projection of $\vec{AC}$ in the direction $\vec{v} = \vec{AC} - \text{proj}_{\vec{AB}} \vec{AC}$.

- The vector $\vec{v}$ lies in the plane containing $A, B, C$ and is perpendicular to $\vec{AB}$.
Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance $d$ from the point $C$ to the line $L$ that contains points $A$ and $B$.

Solution:

- From the figure drawn on the blackboard, we see that the distance $d$ from $C$ to $L$ is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction $\overrightarrow{v} = \overrightarrow{AC} - \text{proj}_{\overrightarrow{AB}} \overrightarrow{AC}$.

- The vector $\overrightarrow{v}$ lies in the plane containing $A$, $B$, $C$ and is perpendicular to $\overrightarrow{AB}$.

- Hence,

$$d = |\text{comp}_v \overrightarrow{AC}|.$$
Problem 30(c)
Consider the points \( A = (2, 1, 0), \ B = (1, 0, 2) \) and \( C = (0, 2, 1) \). Find the distance \( d \) from the point \( C \) to the line \( L \) that contains points \( A \) and \( B \).

Solution:
- From the figure drawn on the blackboard, we see that the distance \( d \) from \( C \) to \( L \) is the absolute value of the scalar projection of \( \vec{AC} \) in the direction \( \vec{v} = \vec{AC} - \text{proj}_{\vec{AB}} \vec{AC} \).
- The vector \( \vec{v} \) lies in the plane containing \( A, B, C \) and is perpendicular to \( \vec{AB} \).
- Hence,
  \[
  d = |\text{comp}_\vec{v} \vec{AC}|.
  \]
- Next, you the student, do the algebraic calculation of \( d \).
Problem 31

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$. 

Solution:

The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence $v$ can be taken to be:

$$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \left| \begin{array}{ccc} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{array} \right| = -3i + j + 5k.$$ 

Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we get:

$$x - 2y = 1 \quad 2x + y = 1.$$ 

Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $P = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle$ lies on the line $L$.

The parametric equations are:

$$x = \frac{3}{5} - 3t \quad y = -\frac{1}{5} + t \quad z = 5t.$$
Problem 31

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. 
Problem 31

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 1 \) and \( 2x + y + z = 1 \).

**Solution:**

- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, 1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:

\[
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle
\]

Choose \( P \in L \) so the \( z \)-coordinate of \( P \) is zero. Setting \( z = 0 \), we get:

\[
\begin{align*}
x &- 2y = 1 \\
2x + y &= 1
\end{align*}
\]

Solving, we find that \( x = \frac{3}{5} \) and \( y = -\frac{1}{5} \). Hence, \( P = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle \) lies on the line \( L \).

The parametric equations are:

\[
\begin{align*}
x &= \frac{3}{5} - \frac{3}{5}t \\
y &= -\frac{1}{5} + t \\
z &= 5t
\end{align*}
\]
Problem 31

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 1 \) and \( 2x + y + z = 1 \).

Solution:

- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, 1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:

  \[
  \mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  1 & -2 & 1 \\
  2 & 1 & 1 
\end{vmatrix}
  \]

- Choose \( P \in L \) so the \( z \)-coordinate of \( P \) is zero. Setting \( z = 0 \), we get:

  \[
  x - 2y = 1 \\
  2x + y = 1
  \]

  Solving, we find that \( x = \frac{3}{5} \) and \( y = -\frac{1}{5} \).

  Hence, \( P = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle \) lies on the line \( L \).

  The parametric equations are:

  \[
  x = \frac{3}{5} - \frac{3}{5}t \\
  y = -\frac{1}{5} + t \\
  z = 5t
  \]
Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

**Solution:**

1. The vector part $\mathbf{v}$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence $\mathbf{v}$ can be taken to be:

   $$
   \mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix}
   \mathbf{i} & \mathbf{j} & \mathbf{k} \\
   1 & -2 & 1 \\
   2 & 1 & 1 
   \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.
   $$

2. Choose $\mathbf{P} \in L$ so the $z$-coordinate of $\mathbf{P}$ is zero. Setting $z = 0$, we get:

   $$
   x - 2y = 1 \quad \text{and} \quad 2x + y = 1.
   $$

   Solving, we find that $x = \frac{3}{5}$ and $y = \frac{-1}{5}$.

   Hence, $\mathbf{P} = \langle \frac{3}{5}, \frac{-1}{5}, 0 \rangle$ lies on the line $L$.

   The parametric equations are:

   $$
   x = \frac{3}{5} - 3t \quad y = \frac{-1}{5} + t \quad z = 5t.
   $$
Problem 31

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 1 \) and \( 2x + y + z = 1 \).

**Solution:**

- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, 1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:

\[
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \mathbf{i} \begin{vmatrix} j & k \\ 1 & -2 \\ 2 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.
\]

- Choose \( \mathbf{P} \in L \) so the z-coordinate of \( \mathbf{P} \) is zero.
Problem 31

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 1 \) and \( 2x + y + z = 1 \).

**Solution:**

- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, 1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:

\[
\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.
\]

- Choose \( \mathbf{P} \in L \) so the \( z \)-coordinate of \( \mathbf{P} \) is zero. Setting \( z = 0 \), we get:

\[
\begin{align*}
x - 2y &= 1 \\
2x + y &= 1.
\end{align*}
\]
Problem 31

Find **parametric equations** for the line \( L \) of intersection of the planes \( x - 2y + z = 1 \) and \( 2x + y + z = 1 \).

Solution:

- The vector part \( \mathbf{v} \) of the line \( L \) of intersection is orthogonal to the normal vectors \( \langle 1, -2, 1 \rangle \) and \( \langle 2, 1, 1 \rangle \). Hence \( \mathbf{v} \) can be taken to be:

  \[
  \mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  1 & -2 & 1 \\
  2 & 1 & 1
  \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.
  \]

- Choose \( \mathbf{P} \in L \) so the \( z \)-coordinate of \( \mathbf{P} \) is zero. Setting \( z = 0 \), we get:

  \[
  \begin{align*}
  x - 2y &= 1 \\
  2x + y &= 1
  \end{align*}
  \]

  Solving, we find that \( x = \frac{3}{5} \) and \( y = -\frac{1}{5} \).
Problem 31

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

**Solution:**

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence $v$ can be taken to be:

  $$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3i + j + 5k.$$  

- Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we get:

  $$x - 2y = 1$$
  $$2x + y = 1.$$  

Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $P = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle$ lies on the line $L$. 
Problem 31

Find parametric equations for the line $L$ of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

Solution:

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence $v$ can be taken to be:

$$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}.$$ 

- Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we get:

$$x - 2y = 1$$
$$2x + y = 1.$$ 

Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $P = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle$ lies on the line $L$.

- The parametric equations are:
Problem 31

Find **parametric equations** for the line $L$ of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

**Solution:**

- The vector part $v$ of the line $L$ of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$. Hence $v$ can be taken to be:

  $$v = \langle 1, -2, 1 \rangle \times \langle 2, 1, 1 \rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3i + j + 5k.$$

- Choose $P \in L$ so the $z$-coordinate of $P$ is zero. Setting $z = 0$, we get:

  $$x - 2y = 1$$
  $$2x + y = 1.$$

  Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$. Hence, $P = \langle \frac{3}{5}, -\frac{1}{5}, 0 \rangle$ lies on the line $L$.

- The **parametric equations** are:

  $$x = \frac{3}{5} - 3t$$
  $$y = -\frac{1}{5} + t$$
  $$z = 5t.$$
Problem 32

Let $L_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $L_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $L_1$ and $L_2$ intersect? If not, are they skew or parallel?
Problem 32

Let $L_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $L_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $L_1$ and $L_2$ intersect? If not, are they skew or parallel?

Solution:

- The vector equations of the lines are:
  
  $L_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle$

  Equating $z$-coordinates, we find $1 = -1 - 2s \Rightarrow s = -1$.

  Equating $y$-coordinates with $s = -1$, we find $4t = 3 - 1 = 2 \Rightarrow t = 1/2$.

  Equating $x$-coordinates with $s = -1$ and $t = 1/2$, we find:
  
  $L_1(1/2) = \langle 0, 2, 1 \rangle = L_2(-1)$.

  Hence, the lines intersect.
Problem 32
Let \( L_1 \) denote the line through the points \((1, 0, 1)\) and \((-1, 4, 1)\) and let \( L_2 \) denote the line through the points \((2, 3, -1)\) and \((4, 4, -3)\). Do the lines \( L_1 \) and \( L_2 \) intersect? If not, are they skew or parallel?

Solution:

- The vector equations of the lines are:
  \[
  L_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle = \langle 1 - 2t, 4t, 1 \rangle
  \]
  
  Equating \( z \)-coordinates, we find \( 1 = -1 - 2s \) which implies \( s = -1 \).
  
  Equating \( y \)-coordinates with \( s = -1 \), we find \( 4t = 3 - 1 = 2 \) which implies \( t = \frac{1}{2} \).
  
  Equating \( x \)-coordinates with \( s = -1 \) and \( t = \frac{1}{2} \), we find:
  \[
  L_1(\frac{1}{2}) = \langle 0, 2, 1 \rangle = L_2(-1)
  \]
  
  Hence, the lines intersect.
Problem 32
Let $L_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $L_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $L_1$ and $L_2$ intersect? If not, are they skew or parallel?

Solution:

The vector equations of the lines are:

$L_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle = \langle 1 - 2t, 4t, 1 \rangle$

$L_2(s) = \langle 2, 3, -1 \rangle + s\langle 2, 1, -2 \rangle = \langle 2 + 2s, 3 + s, -1 - 2s \rangle$
Problem 32
Let $L_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $L_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $L_1$ and $L_2$ intersect? If not, are they skew or parallel?

Solution:

- The vector equations of the lines are:
  
  $L_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle = \langle 1 - 2t, 4t, 1 \rangle$
  
  $L_2(s) = \langle 2, 3, -1 \rangle + s\langle 2, 1, -2 \rangle = \langle 2 + 2s, 3 + s, -1 - 2s \rangle$

- Equating z-coordinates, we find $1 = -1 - 2s \implies s = -1$. Therefore, the lines intersect.
Problem 32

Let $\mathbf{L}_1$ denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let $\mathbf{L}_2$ denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ intersect? If not, are they skew or parallel?

Solution:

- The vector equations of the lines are:
  
  $\mathbf{L}_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle = \langle 1 - 2t, 4t, 1 \rangle$
  
  $\mathbf{L}_2(s) = \langle 2, 3, -1 \rangle + s\langle 2, 1, -2 \rangle = \langle 2 + 2s, 3 + s, -1 - 2s \rangle$

- Equating $z$-coordinates, we find $1 = -1 - 2s \implies s = -1$.

- Equating $y$-coordinates with $s = -1$, we find $4t = 3 - 1 \implies t = \frac{1}{2}$.

Hence, the lines intersect.
Problem 32

Let \( L_1 \) denote the line through the points \((1, 0, 1)\) and \((-1, 4, 1)\) and let \( L_2 \) denote the line through the points \((2, 3, -1)\) and \((4, 4, -3)\). Do the lines \( L_1 \) and \( L_2 \) intersect? If not, are they skew or parallel?

Solution:

- The vector equations of the lines are:
  \[
  L_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle = \langle 1 - 2t, 4t, 1 \rangle
  
  L_2(s) = \langle 2, 3, -1 \rangle + s\langle 2, 1, -2 \rangle = \langle 2 + 2s, 3 + s, -1 - 2s \rangle
  
- Equating z-coordinates, we find \( 1 = -1 - 2s \Rightarrow s = -1 \).
- Equating y-coordinates with \( s = -1 \), we find \( 4t = 3 - 1 \Rightarrow t = \frac{1}{2} \).
- Equating x-coordinates with \( s = -1 \) and \( t = \frac{1}{2} \), we find:
  \[
  L_1\left(\frac{1}{2}\right) = \langle 0, 2, 1 \rangle = L_2(-1).
  
Hence, the lines intersect.
Problem 32
Let \( L_1 \) denote the line through the points \((1, 0, 1)\) and \((-1, 4, 1)\) and let \( L_2 \) denote the line through the points \((2, 3, -1)\) and \((4, 4, -3)\). Do the lines \( L_1 \) and \( L_2 \) intersect? If not, are they skew or parallel?

Solution:

The vector equations of the lines are:

\[
L_1(t) = \langle 1, 0, 1 \rangle + t\langle -2, 4, 0 \rangle = \langle 1 - 2t, 4t, 1 \rangle
\]

\[
L_2(s) = \langle 2, 3, -1 \rangle + s\langle 2, 1, -2 \rangle = \langle 2 + 2s, 3 + s, -1 - 2s \rangle
\]

Equating \( z \)-coordinates, we find \( 1 = -1 - 2s \implies s = -1. \)

Equating \( y \)-coordinates with \( s = -1 \), we find \( 4t = 3 - 1 \implies t = \frac{1}{2}. \)

Equating \( x \)-coordinates with \( s = -1 \) and \( t = \frac{1}{2} \), we find:

\[
L_1\left(\frac{1}{2}\right) = \langle 0, 2, 1 \rangle = L_2(-1).
\]

Hence, the lines intersect.
Problem 33(a)

Find the volume $V$ of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices $B$, $C$, $D$ are all adjacent to the vertex $A$. 

Solution:

The volume $V$ is equal to the absolute value of the determinant of the matrix with rows $\vec{AB} = \langle 2, -3, -4 \rangle$, $\vec{AC} = \langle 3, -1, -5 \rangle$, $\vec{AD} = \langle 0, -4, -3 \rangle$.

$$V = | 2 \cdot (-17) + (-3) \cdot (-3) + (-4) \cdot (-12) | = | -13 | = 13.$$
Problem 33(a)

Find the volume \( V \) of the parallelepiped such that the following four points \( A = (1, 4, 2), \ B = (3, 1, -2), \ C = (4, 3, -3), \ D = (1, 0, -1) \) are vertices and the vertices \( B, C, D \) are all adjacent to the vertex \( A \).

Solution:

The volume \( V \) is equal to the absolute value of the determinant of the matrix with rows \( \vec{AB} = \langle 2, -3, -4 \rangle, \ \vec{AC} = \langle 3, -1, -5 \rangle, \ \vec{AD} = \langle 0, -4, -3 \rangle \).
Problem 33(a)

Find the volume $V$ of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices $B, C, D$ are all adjacent to the vertex $A$.

Solution:

The volume $V$ is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

$$
V = \left| \begin{array}{ccc}
2 & -3 & -4 \\
3 & -1 & -5 \\
0 & -4 & -3 \\
\end{array} \right| = 13
$$
Problem 33(a)

Find the volume $V$ of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices $B$, $C$, $D$ are all adjacent to the vertex $A$.

Solution:

The volume $V$ is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

\[
V = \begin{vmatrix}
2 & -3 & -4 \\
3 & -1 & -5 \\
0 & -4 & -3 \\
\end{vmatrix}
\]

\[
= |2 \cdot (-17) + (-3) \cdot (-9) + (-4) \cdot (-12)|
\]

\[
= |34 + 27 + 48| = 109
\]
Problem 33(a)

Find the volume $V$ of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices $B, C, D$ are all adjacent to the vertex $A$.

Solution:

The volume $V$ is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

\[
V = \begin{vmatrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{vmatrix}
\]

\[
= |2 \cdot (-17) + (-3) \cdot (-9) + (-4) \cdot (-12)| = |-13|
\]
Problem 33(a)

Find the volume $V$ of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices $B, C, D$ are all adjacent to the vertex $A$.

Solution:

The volume $V$ is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

\[
V = \begin{vmatrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{vmatrix} = |2 \cdot (-17) + (-3) \cdot (-9) + (-4) \cdot (-12)| = |-13| = 13.
\]
Problem 33(b)

Find an equation of the plane through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$. 

Solution:
Consider the vectors $\vec{AB} = \langle 2, -3, -4 \rangle$ and $\vec{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.
The normal vector is:
$n = \vec{AB} \times \vec{AC} = \left| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{array} \right| = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$.

Since $A = (1, 4, 2)$ is on the plane, then the equation of the plane is given by:
$11(x - 1) - 2(y - 4) + 7(z - 2) = 0$. 

Problem 33(b)

Find an **equation of the plane** through
\[ A = (1, 4, 2), \ B = (3, 1, -2), \ C = (4, 3, -3). \]

Solution:

- Consider the vectors \( \vec{AB} = \langle 2, -3, -4 \rangle \) and \( \vec{AC} = \langle 3, -1, -5 \rangle \) which lie parallel to the plane.
Problem 33(b)

Find an **equation of the plane** through 
\(A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)\).

Solution:

- Consider the vectors \(\vec{AB} = \langle 2, -3, -4 \rangle\) and \(\vec{AC} = \langle 3, -1, -5 \rangle\) which lie parallel to the plane.
- The normal vector is:

\[
\mathbf{n} = \vec{AB} \times \vec{AC}
\]
Problem 33(b)

Find an equation of the plane through \( A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3) \).

Solution:

Consider the vectors \( \overrightarrow{AB} = \langle 2, -3, -4 \rangle \) and \( \overrightarrow{AC} = \langle 3, -1, -5 \rangle \) which lie parallel to the plane.

The normal vector is:

\[
\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & -3 & -4 \\
3 & -1 & -5 \\
\end{vmatrix}
\]

Since \( A = (1, 4, 2) \) is on the plane, then the equation of the plane is given by:

\[
11(x - 1) - 2(y - 4) + 7(z - 2) = 0
\]
Problem 33(b)

Find an equation of the plane through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

Solution:

Consider the vectors $\vec{AB} = \langle 2, -3, -4 \rangle$ and $\vec{AC} = \langle 3, -1, -5 \rangle$ which lie parallel to the plane.

The normal vector is:

$$\mathbf{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix} = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}.$$
Problem 33(b)

Find an **equation of the plane** through 
\( A = (1, 4, 2), \ B = (3, 1, -2), \ C = (4, 3, -3). \)

Solution:

- Consider the vectors \( \overrightarrow{AB} = \langle 2, -3, -4 \rangle \) and \( \overrightarrow{AC} = \langle 3, -1, -5 \rangle \) which lie parallel to the plane.
- The normal vector is:
  \[
  \mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & -4 \\ 3 & -1 & -5 \end{vmatrix} = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}.
  \]
- Since \( A = (1, 4, 2), \) is on the plane, then the **equation of the plane** is given by:
Problem 33(b)

Find an equation of the plane through 
\( A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3) \).

Solution:

- Consider the vectors 
  \( \overrightarrow{AB} = \langle 2, -3, -4 \rangle \) and 
  \( \overrightarrow{AC} = \langle 3, -1, -5 \rangle \) which lie parallel to the plane.

- The normal vector is:
  \[
  n = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix}
    \mathbf{i} & \mathbf{j} & \mathbf{k} \\
    2 & -3 & -4 \\
    3 & -1 & -5 
  \end{vmatrix} = 11\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}.
  \]

- Since \( A = (1, 4, 2) \), is on the plane, then the equation of the plane is given by:
  \[
  11(x - 1) - 2(y - 4) + 7(z - 2) = 0.
  \]
Problem 33(c)

Find the angle between the plane through
\( A = (1, 4, 2), \ B = (3, 1, -2), \ C = (4, 3 - 3) \) and the \( xy \)-plane.

Solution:
The normal vectors of these planes are
\( n_1 = \langle 0, 0, 1 \rangle, \ n_2 = \langle 11, -2, 7 \rangle \).

If \( \theta \) is the angle between the planes, then:

\[
\cos \theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{7\sqrt{112}}{\sqrt{11^2 + (-2)^2 + 7^2}} = \frac{7\sqrt{174}}{\sqrt{174}}.
\]

\[
\theta = \cos^{-1} \left( \frac{1}{\sqrt{174}} \right).
\]
Problem 33(c)

Find the angle between the plane through $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3 - 3)$ and the $xy$-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$. 

$\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{||\mathbf{n}_1|| ||\mathbf{n}_2||} = \frac{7}{\sqrt{174}}$.

$\theta = \cos^{-1} \left( \frac{1}{\sqrt{174}} \right)$. 
Problem 33(c)

Find the angle between the plane through 
\(A = (1, 4, 2),\ B = (3, 1, -2),\ C = (4, 3, -3)\) and the \(xy\)-plane.

Solution:

- The normal vectors of these planes are \(\mathbf{n}_1 = \langle 0, 0, 1 \rangle\), 
  \(\mathbf{n}_2 = \langle 11, -2, 7 \rangle\).
- If \(\theta\) is the angle between the planes, then:
Problem 33(c)

Find the angle between the plane through
\( A = (1, 4, 2), \ B = (3, 1, -2), \ C = (4, 3 - 3) \) and the \( xy \)-plane.

Solution:

- The normal vectors of these planes are \( \mathbf{n}_1 = \langle 0, 0, 1 \rangle \),
  \( \mathbf{n}_2 = \langle 11, -2, 7 \rangle \).
- If \( \theta \) is the angle between the planes, then:
  \[
  \cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{||\mathbf{n}_1|| ||\mathbf{n}_2||}
  \]
Problem 33(c)

Find the angle between the plane through $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3 - 3)$ and the $xy$-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If $\theta$ is the angle between the planes, then:

$$\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{||\mathbf{n}_1|| ||\mathbf{n}_2||} = \frac{7}{\sqrt{11^2 + (-2)^2 + 7^2}}$$
Problem 33(c)

Find the angle between the plane through $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3 - 3)$ and the $xy$-plane.

Solution:

- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.
- If $\theta$ is the angle between the planes, then:

$$
\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} = \frac{7}{\sqrt{11^2 + (-2)^2 + 7^2}} = \frac{7}{\sqrt{174}}.
$$
Problem 33(c)
Find the angle between the plane through
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Solution:
- The normal vectors of these planes are \( n_1 = \langle 0, 0, 1 \rangle \), \( n_2 = \langle 11, -2, 7 \rangle \).
- If \( \theta \) is the angle between the planes, then:
  \[
  \cos \theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{7}{\sqrt{11^2 + (-2)^2 + 7^2}} = \frac{7}{\sqrt{174}}.
  \]
- \( \theta = \cos^{-1} \left( \frac{1}{\sqrt{174}} \right) \).
Problem 34(a)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).
Problem 34(a)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((−1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).
Problem 34(a)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).

\[
\mathbf{r}(t) = \int \mathbf{v}(t) \, dt
\]
Problem 34(a)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).

\[
\mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int \langle 2t, 2t^{1/2}, 1 \rangle \, dt
\]
Problem 34(a)

The velocity vector of a particle moving in space equals 
\[ \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \] at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).

\[
\mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int \langle 2t, 2t^{1/2}, 1 \rangle \, dt \\
= \langle t^2 + x_0, \frac{4}{3} t^{3/2} + y_0, t + z_0 \rangle.
\]
Problem 34(a)

The velocity vector of a particle moving in space equals\[ \mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k} \] at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).

\[
\mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int \langle 2t, 2t^{1/2}, 1 \rangle \, dt
\]

\[
= \langle t^2 + x_0, \frac{4}{3} t^{3/2} + y_0, t + z_0 \rangle.
\]

- Now use the initial position \( \mathbf{r}(0) = \langle -1, 5, 4 \rangle \) to find \( x_0 = -1; \ y_0 = 5; \ z_0 = 4 \).
Problem 34(a)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).
  \[
  \mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int \langle 2t, 2t^{1/2}, 1 \rangle \, dt
  \]
  \[
  = \langle t^2 + x_0, \frac{4}{3} t^{3/2} + y_0, t + z_0 \rangle.
  \]
- Now use the initial position \( \mathbf{r}(0) = \langle -1, 5, 4 \rangle \) to find
  \( x_0 = -1; \, y_0 = 5; \, z_0 = 4 \).
- Thus,
  \[
  \mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3} t^{3/2} + 5, t + 4 \rangle
  \]
Problem 34(a)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). At the time \( t = 0 \) this particle is at the point \((-1, 5, 4)\). Find the position vector \( \mathbf{r}(t) \) of the particle at the time \( t = 4 \).

Solution:

- To find the position \( \mathbf{r}(t) \), integrate the velocity vector field \( \mathbf{r}'(t) = \mathbf{v}(t) \).
  \[
  \mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \int \langle 2t, 2t^{1/2}, 1 \rangle \, dt
  = \langle t^2 + x_0, \frac{4}{3} t^{3/2} + y_0, t + z_0 \rangle.
  \]

- Now use the initial position \( \mathbf{r}(0) = \langle -1, 5, 4 \rangle \) to find \( x_0 = -1; \, y_0 = 5; \, z_0 = 4 \).

- Thus,
  \[
  \mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3} t^{3/2} + 5, t + 4 \rangle
  \]
  \[
  \mathbf{r}(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle.
  \]
Problem 34(b)

The velocity vector of a particle moving in space equals 
\( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).

Find an equation of the tangent line \( T \) to the curve at the time \( t = 4 \).
Problem 34(b)

The velocity vector of a particle moving in space equals 
\( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).
Find an equation of the tangent line \( \mathbf{T} \) to the curve at the time \( t = 4 \).

Solution:

- Vector equation of the tangent line \( \mathbf{T} \) to \( \mathbf{r}(t) \) at \( t = 4 \) is:

\[
\mathbf{T}(s) = \mathbf{r}(4) + s \mathbf{r}'(4) = \mathbf{r}(4) + s \mathbf{v}(4).
\]
Problem 34(b)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).
Find an equation of the tangent line \( \mathbf{T} \) to the curve at the time \( t = 4 \).

Solution:

• Vector equation of the tangent line \( \mathbf{T} \) to \( \mathbf{r}(t) \) at \( t = 4 \) is:

\[
\mathbf{T}(s) = \mathbf{r}(4) + s\mathbf{r}'(4) = \mathbf{r}(4) + s\mathbf{v}(4).
\]

• By part (a), \( \mathbf{r}(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle \).
Problem 34(b)

The velocity vector of a particle moving in space equals $v(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k}$ at any time $t \geq 0$. Find an equation of the tangent line $T$ to the curve at the time $t = 4$.

Solution:

- Vector equation of the tangent line $T$ to $r(t)$ at $t = 4$ is:

  $$T(s) = r(4) + sr'(4) = r(4) + sv(4).$$

- By part (a), $r(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle$.

- Since $v(4) = 8\mathbf{i} + 4\mathbf{j} + \mathbf{k} = \langle 8, 4, 1 \rangle$, 
The velocity vector of a particle moving in space equals \( v(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).

Find an equation of the tangent line \( T \) to the curve at the time \( t = 4 \).

**Solution:**

- Vector equation of the tangent line \( T \) to \( r(t) \) at \( t = 4 \) is:
  \[
  T(s) = r(4) + sr'(4) = r(4) + sv(4).
  \]

- By part (a), \( r(4) = \langle 15, \frac{32}{3} + 5, 8 \rangle \).

- Since \( v(4) = 8 \mathbf{i} + 4 \mathbf{j} + \mathbf{k} = \langle 8, 4, 1 \rangle \),

  then
  \[
  T(s) = \langle 15, \frac{32}{3} + 5, 8 \rangle + s\langle 8, 4, 1 \rangle.
  \]
Problem 34(c)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).

Does the particle ever pass through the point \( P = (80, 41, 13) \)?
Problem 34(c)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).
Does the particle ever pass through the point \( P = (80, 41, 13) \) ?

Solution:

- From part (a), we have

\[
\mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3} t^{3/2} + 5, t + 4 \rangle.
\]
Problem 34(c)

The velocity vector of a particle moving in space equals 
\[ \mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k} \] at any time \( t \geq 0 \).
Does the particle ever pass through the point \( P = (80, 41, 13) \) ?

Solution:

- From part (a), we have 
  \[ \mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3} t^{3/2} + 5, t + 4 \rangle. \]

- If \( \mathbf{r}(t) = \langle 80, 41, 13 \rangle \), then \( t + 4 = 13 \implies t = 9 \).
Problem 34(c)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \). Does the particle ever pass through the point \( P = (80, 41, 13) \) ?

Solution:

- From part (a), we have
  \[
  \mathbf{r}(t) = \langle t^2 - 1, \frac{4}{3} t^{3/2} + 5, t + 4 \rangle.
  \]
- If \( \mathbf{r}(t) = \langle 80, 41, 13 \rangle \), then \( t + 4 = 13 \implies t = 9 \).
- Hence the point
  \[
  \mathbf{r}(9) = \langle 80, 41, 13 \rangle
  \]
  is on the curve \( \mathbf{r}(t) \).
Problem 34(d)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t\mathbf{i} + 2t^{1/2}\mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).
Find the length of the arc traveled from time \( t = 1 \) to time \( t = 2 \).
Problem 34(d)

The velocity vector of a particle moving in space equals \( \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \) at any time \( t \geq 0 \).
Find the length of the arc traveled from time \( t = 1 \) to time \( t = 2 \).

Solution:

\[
\text{Length} = \int_{1}^{2} |\mathbf{v}(t)| \, dt = \int_{1}^{2} \sqrt{4t^2 + 4t + 1} \, dt.
\]
Problem 34(d)

The velocity vector of a particle moving in space equals
\[ \mathbf{v}(t) = 2t \mathbf{i} + 2t^{1/2} \mathbf{j} + \mathbf{k} \] at any time \( t \geq 0 \).
Find the length of the arc traveled from time \( t = 1 \) to time \( t = 2 \).

Solution:

\[
\text{Length} = \int_{1}^{2} |\mathbf{v}(t)| \, dt = \int_{1}^{2} \sqrt{4t^2 + 4t + 1} \, dt.
\]

Since we are not using calculators on our exam, then this is the final answer.
Consider the surface \( x^2 + 3y^2 - 2z^2 = 1. \)
What are the traces in \( x = k, y = k, z = k? \) Sketch a few.
Problem 35(a)

Consider the surface \( x^2 + 3y^2 - 2z^2 = 1 \).

What are the traces in \( x = k, y = k, z = k \)? Sketch a few.

Solution:

- For \( x = k \neq 1 \), we get the hyperbolas \( 3y^2 - 2z^2 = k \).
- For \( x = 1 \), we get the 2 lines \( y = \pm \frac{3}{2}z \).
- For \( z = 0 \), we get the ellipse \( x^2 + 3y^2 = 1 \).
- For \( z = 1 \), we get the ellipse \( x^2 + 3y^2 = 3 \).
- I am leaving it to you to do the sketches!
<table>
<thead>
<tr>
<th>Problem 35(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consider the surface $x^2 + 3y^2 - 2z^2 = 1$. Sketch the surface in the space.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution:</th>
</tr>
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<tbody>
<tr>
<td>Sorry, you need to do the sketch.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Problem 36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find an equation for the tangent plane to the graph of $f(x, y) = y \ln x$ at $(1, 4, 0)$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution:</th>
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<tr>
<td>A problem of this type will not be on this exam.</td>
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</table>
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane.
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

Solution:

The normal to the first plane is $n = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from $P_0$ to $P_1$ which is $b = \langle 0, 0, \frac{5}{2} \rangle$. 

Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from $P_0$ to $P_1$ which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance $D$ from $P_1$ to the first plane is equal to:

$$D = \left| \frac{\mathbf{b} \cdot \mathbf{n}}{||\mathbf{n}||} \right| = \left| \frac{0 \cdot 2 + 0 \cdot 1 + \frac{5}{2} \cdot -1}{\sqrt{2^2 + 1^2 + (-1)^2}} \right| = \frac{\frac{5}{2}}{\sqrt{6}} = \frac{5}{2\sqrt{6}}.$$
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

Solution:

The normal to the first plane is $n = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from $P_0$ to $P_1$ which is $b = \langle 0, 0, \frac{5}{2} \rangle$. The distance $D$ from $P_1$ to the first plane is equal to:

$$\left| \text{comp}_n b \right| =$$
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

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$$|\text{comp}_n b| = \left| b \cdot \frac{n}{|n|} \right|$$
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from $P_0$ to $P_1$ which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance $D$ from $P_1$ to the first plane is equal to:

$$|\text{comp}_n \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{||\mathbf{n}||} \right| = |\langle 0, 0, \frac{5}{2} \rangle \cdot \frac{1}{\sqrt{6}} \langle 2, 1, -1 \rangle|$$
Problem 37

Find the distance $D$ between the given parallel planes

$$z = 2x + y - 1, \quad -4x - 2y + 2z = 3.$$ 

Solution:

The normal to the first plane is $\mathbf{n} = \langle 2, 1, -1 \rangle$ and the point $P_0 = (0, 0, -1)$ lies on this plane. The point $P_1 = \langle 0, 0, \frac{3}{2} \rangle$ lies on the second plane. Consider the vector from $P_0$ to $P_1$ which is $\mathbf{b} = \langle 0, 0, \frac{5}{2} \rangle$. The distance $D$ from $P_1$ to the first plane is equal to:

$$|\text{comp}_n \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 0, 0, \frac{5}{2} \rangle \cdot \frac{1}{\sqrt{6}} \langle 2, 1, -1 \rangle| = \frac{5}{2\sqrt{6}}.$$
Problem 38

Identify the surface given by the equation

\[ 4x^2 + 4y^2 - 8y - z^2 = 0. \]

Draw the traces and sketch the curve.

Solution:

Sorry, no sketch given.
Problem 39(a)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. Write an equation for the acceleration vector.

Solution:
Since the force due to gravity acts downward, we have
\[ F = ma = -mg \hat{j}, \]
where \( g = |a| \approx 9.8 \text{ m/s}^2 \).
Thus \( a = -g \hat{j} \).
Problem 39(a)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. Write an equation for the acceleration vector.

Solution:

Since the force due to gravity acts downward, we have

\[ \mathbf{F} = m \mathbf{a} = -mg \mathbf{j}, \]

where \( g = |a| \approx 9.8 \text{ m/s}^2. \)
Problem 39(a)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. Write an equation for the acceleration vector.

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Since the force due to gravity acts downward, we have

\[ \mathbf{F} = m \mathbf{a} = -mg \mathbf{j}, \]

where \( g = |a| \approx 9.8 \text{ m/s}^2 \). Thus \( \mathbf{a} = -g \mathbf{j}. \)
Problem 39(b) and 34(c)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.

(b) Write a vector for initial velocity \( \mathbf{v}(0) \).
(c) Write a vector for the initial position \( \mathbf{r}(0) \)
Problem 39(b) and 34(c)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.

(b) Write a vector for initial velocity \( \mathbf{v}(0) \).

(c) Write a vector for the initial position \( \mathbf{r}(0) \)

Solution:

- Initial velocity is:

\[
\mathbf{v}(0) = 100(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = 50\sqrt{3} \mathbf{i} + 50 \mathbf{j},
\]

in units of m/s.
Problem 39(b) and 34(c)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s.

(b) Write a vector for initial velocity \( \mathbf{v}(0) \).

(c) Write a vector for the initial position \( \mathbf{r}(0) \)

Solution:

- Initial velocity is:
  
  \[ \mathbf{v}(0) = 100(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = 50\sqrt{3}\mathbf{i} + 50\mathbf{j}, \]

  in units of m/s.

- The initial position is:

  \[ \mathbf{r}(0) = 5\mathbf{j}, \]

  in units of meters \( m \).
Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. At what time does the projectile hit the ground?
A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. At what time does the projectile hit the ground?

Solution:

- We first find the velocity \( r'(t) \) and position \( r(t) \) functions.
  \[
  r'(t) = \mathbf{v}(t) = -gt \mathbf{j} + \mathbf{v}(0)
  \]
  \[
  r(t) = -\frac{1}{2}gt^2 \mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.
  \]
Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. At what time does the projectile hit the ground?

Solution:

We first find the velocity \( r(t) \) and position \( r(t) \) functions.

\[
\begin{align*}
    r'(t) &= v(t) = -gtj + v(0) \\
    r(t) &= -\frac{1}{2}gt^2j + tv(0) + D.
\end{align*}
\]

Since \( D = r(0) = 5j \), then \( r(t) = -\frac{1}{2}gt^2j + tv(0) + 5j \).
Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. At what time does the projectile hit the ground?

Solution:

- We first find the velocity \( \mathbf{r}'(t) \) and position \( \mathbf{r}(t) \) functions.

\[
\mathbf{r}'(t) = \mathbf{v}(t) = -gt \mathbf{j} + \mathbf{v}(0)
\]

\[
\mathbf{r}(t) = -\frac{1}{2}gt^2 \mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.
\]

Since \( \mathbf{D} = \mathbf{r}(0) = 5\mathbf{j} \), then

\[
\mathbf{r}(t) = -\frac{1}{2}gt^2 \mathbf{j} + t\mathbf{v}(0) + 5\mathbf{j}.
\]

- Hence,

\[
\mathbf{r}(t) = 50\sqrt{3}t \mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}.
\]
Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. At what time does the projectile hit the ground?

Solution:

- We first find the velocity $\mathbf{r}(t)$ and position $\mathbf{r}(t)$ functions.
  
  $r'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}(0)$

  $r(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + \mathbf{D}.$

  Since $\mathbf{D} = r(0) = 5\mathbf{j}$, then $r(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}(0) + 5\mathbf{j}$.

- Hence,
  
  $r(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j}.$

- The projectile hits the ground when $50t - \frac{1}{2}gt^2 + 5 = 0$. 

Problem 39(d)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. At what time does the projectile hit the ground?

Solution:

- We first find the velocity \( r(t) \) and position \( r(t) \) functions.
  \[
  r'(t) = v(t) = -gt\hat{j} + v(0)
  \]
  \[
  r(t) = -\frac{1}{2}gt^2\hat{j} + tv(0) + \mathbf{D}.
  \]

Since \( \mathbf{D} = r(0) = 5\hat{j} \), then \( r(t) = -\frac{1}{2}gt^2\hat{j} + tv(0) + 5\hat{j} \).

- Hence, \( r(t) = 50\sqrt{3}\hat{i} + [50t - \frac{1}{2}gt^2 + 5]\hat{j} \).

- The projectile hits the ground when \( 50t - \frac{1}{2}gt^2 + 5 = 0 \). Applying the quadratic formula, we find
  \[
  t = \frac{100 + \sqrt{100^2 + 40g}}{2g}.
  \]
Problem 39(e)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. How far did it travel, horizontally, before it hit the ground?

Solution:

Recall \( r(t) = 50\sqrt{3} t i + [50t - \frac{1}{2} gt^2 + 5] j \) and the projectile hits the ground when \( t = \frac{-v_0 \sin \theta}{g} \). The horizontal distance \( d \) traveled is the value of the \( x \)-coordinate of \( r(t) \) at \( t = \frac{-v_0 \sin \theta}{g} \):

\[ d = 50\sqrt{3} \left( \frac{-v_0 \sin \theta}{g} \right) \].

Problem 39(e)
A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. How far did it travel, horizontally, before it hit the ground?

Solution:
Recall \( r(t) = 50\sqrt{3}i + [50t - \frac{1}{2}gt^2 + 5]j \) and the projectile hits the ground when \( t = \frac{100 + \sqrt{100^2 + 40g}}{2g} \).
Problem 39(e)

A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. How far did it travel, horizontally, before it hit the ground?

Solution:

1. Recall \( r(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j} \) and the projectile hits the ground when \( t = \frac{100 + \sqrt{100^2 + 40g}}{2g} \).
2. The horizontal distance \( d \) traveled is the value of the \( x \)-coordinate of \( r(t) \) at \( t = \frac{100 + \sqrt{100^2 + 40g}}{2g} \).
A projectile is fired from a point 5 m above the ground at an angle of 30 degrees and an initial speed of 100 m/s. How far did it travel, horizontally, before it hit the ground?

Solution:

- Recall \( r(t) = 50\sqrt{3}t\mathbf{i} + [50t - \frac{1}{2}gt^2 + 5]\mathbf{j} \) and the projectile hits the ground when \( t = \frac{100 + \sqrt{100^2 + 40g}}{2g} \).

- The horizontal distance \( d \) traveled is the value of the \( x \)-coordinate of \( r(t) \) at \( t = \frac{100 + \sqrt{100^2 + 40g}}{2g} \):

  \[
  d = 50\sqrt{3} \left( \frac{100 + \sqrt{100^2 + 40g}}{2g} \right).
  \]
Problem 40

Explain why the limit of \( f(x, y) = \frac{3x^2 y^2}{2x^4 + y^4} \) does not exist as \((x, y)\) approaches \((0, 0)\).

Solution:

A problem of this type will not be on this exam.
Problem 41

Find an equation of the plane that passes through the point $P(1, 1, 0)$ and contains the line given by parametric equations $x = 2 + 3t, y = 1 - t, z = 2 + 2t$. 
Problem 41

Find an equation of the plane that passes through the point \( P(1, 1, 0) \) and contains the line given by parametric equations
\[ x = 2 + 3t, \quad y = 1 - t, \quad z = 2 + 2t. \]

Solution:

- The direction vector \( \mathbf{a} = \langle 3, -1, 2 \rangle \) of the line is parallel to the plane.
Problem 41

Find an **equation of the plane** that passes through the point \(P(1, 1, 0)\) and contains the line given by **parametric equations**
\[ x = 2 + 3t, \quad y = 1 - t, \quad z = 2 + 2t. \]

**Solution:**

1. The direction vector \(a = \langle 3, -1, 2 \rangle\) of the line is parallel to the plane.
2. For \(t = 0\), the point \(Q = \langle 2, 1, 2 \rangle\) on the line and the plane.
Problem 41

Find an **equation of the plane** that passes through the point $P(1,1,0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

**Solution:**

- The direction vector $a = \langle 3, -1, 2 \rangle$ of the line is parallel to the plane.
- For $t = 0$, the point $Q = \langle 2, 1, 2 \rangle$ on the line and the plane.
- So $b = PQ = \langle 1, 0, 2 \rangle$ is also parallel to the plane.

To find a normal vector to the plane, take cross products:

$$n = a \times b = \langle -2, -4, 1 \rangle.$$ 

Since $(1,1,0)$ is on the plane, the equation of the plane is:

$$-2(x - 1) - 4(y - 1) + z = 0.$$
Problem 41

Find an **equation of the plane** that passes through the point $P(1, 1, 0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

**Solution:**

- The direction vector $\mathbf{a} = \langle 3, -1, 2 \rangle$ of the line is parallel to the plane.
- For $t = 0$, the point $Q = \langle 2, 1, 2 \rangle$ on the line and the plane.
- So $\mathbf{b} = \mathbf{PQ} = \langle 1, 0, 2 \rangle$ is also parallel to the plane.
- To find a normal vector to the plane, take cross products:

$$
n = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = \langle -2, -4, 1 \rangle.
$$
Problem 41

Find an equation of the plane that passes through the point \( P(1, 1, 0) \) and contains the line given by parametric equations \( x = 2 + 3t, \ y = 1 - t, \ z = 2 + 2t \).

Solution:

- The direction vector \( \mathbf{a} = \langle 3, -1, 2 \rangle \) of the line is parallel to the plane.
- For \( t = 0 \), the point \( \mathbf{Q} = \langle 2, 1, 2 \rangle \) on the line and the plane.
- So \( \mathbf{b} = \mathbf{PQ} = \langle 1, 0, 2 \rangle \) is also parallel to the plane.
- To find a normal vector to the plane, take cross products:

\[
\mathbf{n} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & -1 & 2 \\
1 & 0 & 2
\end{vmatrix} = \langle -2, -4, 1 \rangle.
\]

- Since \( (1, 1, 0) \) is on the plane, the equation of the plane is:

\[
\langle -2, -4, 1 \rangle \cdot \langle x - 1, y - 1, z \rangle = -2(x - 1) - 4(y - 1) + z = 0.
\]
<table>
<thead>
<tr>
<th>Problem 42(a)</th>
<th>Find all of the first order and second order partial derivatives of the function. ( f(x, y) = x^3 - xy^2 + y )</th>
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<td>Solution:</td>
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<table>
<thead>
<tr>
<th>Problem 42(b)</th>
<th>Find all of the first order and second order partial derivatives of the function. ( f(x, y) = \ln(x + \sqrt{x^2 + y^2}) )</th>
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<table>
<thead>
<tr>
<th>Problem 43</th>
<th>Find the linear approximation of the function ( f(x, y) = xye^x ) at ( (x, y) = (1, 1) ), and use it to estimate ( f(1.1, 0.9) ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution:</td>
<td>There is no problem of this type on this exam.</td>
</tr>
</tbody>
</table>
Problem 44

Find a vector function \( r(t) \) which represents the curve of intersection of the paraboloid \( z = 2x^2 + y^2 \) and the parabolic cylinder \( y = x^2 \).
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Solution:

- Set \( t = x \).
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- Since $y = x^2 = t^2$, we get from the equation of the paraboloid a vector function $r(t)$ which represents the curve of intersection:
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$$r(t) = \langle t, t^2, 2t^2 + (t^2)^2 \rangle$$
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\[
r(t) = \langle t, t^2, 2t^2 + (t^2)^2 \rangle = \langle t, t^2, 2t^2 + t^4 \rangle.
\]