MATH 131, Fall 2019
Quiz 11 Solutions

1. Let

$$f(x) = 2x^3 - 3x^2 - \frac{2}{\sqrt{1-x^2}} - 2^x \ln 2\,.$$

Find a general antiderivative $F(x)$ of $f(x)$, valid wherever $f(x)$ is continuous.

Note that $f(x)$ is continuous only on the open interval $(-1, 1)$, so we do not have to worry about constructing a piecewise solution with different constants for different subdomains. Then using linearity, the power rule for anti-differentiation, together with knowledge of derivatives of logarithms and inverse trigonometric functions, we obtain

$$F(x) = \frac{1}{2}x^4 - x^3 + 2\arccos(x) - 2^x + C\,.$$

Note that we have two choices for handling $-2/\sqrt{1-x^2}$, since both $-\arcsin(x)$ and $\arccos(x)$, and any constant shifts of these, derive to $-1/\sqrt{1-x^2}$.

2. A particle moves with acceleration $a(t) = \cos(t) - \sin(t)$, with initial velocity $v_0 = v(0) = 0$ and initial position $s_0 = s(0) = -1$. Find the position function $s(t)$.

We know that velocity is the derivative of position, and acceleration is the derivative of velocity. Recall also that the derivative of $\sin x$ is $\cos x$ and the derivative of $\cos x$ is $-\sin x$. First anti-derive the acceleration to obtain velocity, and use the given initial value to determine the constant:

$$v(t) = \sin(t) + \cos(t) + C\,,$$

$$v(0) = v_0 = 0 = \sin(0) + \cos(0) + C = 0 + 1 + C \implies C = -1\,.$$

Thus the velocity is $v(t) = \sin(t) + \cos(t) - 1$. Anti-deriving this and using the initial position gives

$$s(t) = -\cos(t) + \sin(t) - t + D\,,$$

$$s(0) = s_0 = -1 = -\cos(0) + \sin(0) - 0 + D \implies D = 0\,.$$

Thus the position function is $s(t) = \sin(t) - \cos(t) - t$.